Calibrating CAT Bonds for Mexican Earthquakes

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Mexico is exposed to earthquake risk (EQ):

- EQ disasters are huge and volatile
- An 8.1 Mw EQ hit Mexico in 1985: estimated payouts of 4 billion dollars

Figure 1: Location of epicenters

Calibrating CAT Bonds for Mexican Earthquakes
Figure 2: Number of EQs higher than 6.5 Mw in Mexico during 1900-2003.
CAT bonds

- Reconstruction can be financed by transferring the risk with CAT bonds
  - From insurers, reinsurance and corporations (sponsors) to capital market investors
- Alternative or complement to traditional reinsurance
- Supply protection against natural catastrophes without credit risk present in reinsurance
- Offer attractive returns and reduce the portfolio risk
- Attractive surplus alternatives
Calibrating CAT bonds

*Pure parametric index trigger*: payouts are triggered by the occurrence of a catastrophic event with certain defined physical parameters.

- The intensity rate ($\lambda$) describes the flow process of EQ:
  - Reinsurance market ($\lambda_1$): Ceding & Reinsurance company
  - Capital market ($\lambda_2$): SPV & investors
  - Historical data ($\lambda_3$): real intensity of EQ

- Comparative analysis: is $\lambda_1 = \lambda_2 = \lambda_3$? Fair?
Pricing CAT bonds

Modeled – index trigger: the physical parameters of the catastrophe are used to estimate the expected losses to the ceding company’s portfolio.

- Different variables affect the value of the loss: physical parameters, building material, construction design, impact on main cities, etc.
- Minimization of basis risk borne by the sponsor, while remaining non-indemnity based.
Calibrating CAT bonds

- Does the trigger mechanism matter in pricing?
- What are the differences in pricing?
- What are the stochastic properties that influence the valuation?
Outline

1. Motivation ✓
2. What are CAT bonds?
3. Calibrating the parametric Mexican CAT Bond
4. Pricing a Modeled-index CAT bond
CAT bonds

- Ease the transfer of catastrophic insurance risk
- Coupons and principal depend on the performance of a pool or index of natural catastrophe risks
- Parties: Sponsor, SPV, collateral & investors
- If there is no event: SPV gives the principal back to the investors with the final coupon
- If there is an event: investors sacrifices fully or partially their principal plus interest and the SPV pays the insured loss
Structure of Cash flows

Figure 3: Cash Flows Diagram. Event (red), no event (blue)

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Trigger mechanisms

1. **Indemnity**: Actual loss of the ceding company
2. **Modeled loss**: A third party projects the expected losses to the ceding company´s portfolio
3. **Industry index**: The ceding recovers a % of total industry losses in excess of a predetermined point
4. **Parametric index**: weighting boxes exposure
   - Hurricane index value = $K \sum_{i=1}^{I} w_i(v_i - L)^n$
5. **Pure parametric index**: Richter Scale
## CAT-MEX bond

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Issue Date</td>
<td>May-06</td>
</tr>
<tr>
<td>Sponsor</td>
<td>Mexican government</td>
</tr>
<tr>
<td>SPV</td>
<td>CAT-Mex Ltd</td>
</tr>
<tr>
<td>Reinsurer</td>
<td>Swiss Re</td>
</tr>
<tr>
<td>Total size (P)</td>
<td>$160 mio.</td>
</tr>
<tr>
<td>Risk Period</td>
<td>3 year</td>
</tr>
<tr>
<td>Risk</td>
<td>Earthquake</td>
</tr>
<tr>
<td>Structure</td>
<td>Pure Parametric</td>
</tr>
<tr>
<td>Spread (z)</td>
<td>LIBOR plus 235 basis points</td>
</tr>
<tr>
<td>Total coverage</td>
<td>$450 mio.</td>
</tr>
<tr>
<td>Premiums</td>
<td>$26 mio.</td>
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</tbody>
</table>

Table 1: Mexican parametric CAT bond
**Cash flows CAT-MEX**

Figure 4: The cash flows diagram for the mexican CAT bond.
Air Worldwide Corporation modeled the seismic risk.

Given the federal governmental budget plan: 3 zones.

**Figure 5: Map of regions**

Calibrating CAT Bonds for Mexican Earthquakes
The CAT bond payment would be triggered if:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Threshold $u$ in $Mw \geq$ to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>8</td>
</tr>
<tr>
<td>Zone 2</td>
<td>8</td>
</tr>
<tr>
<td>Zone 5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Table 2: Thresholds $u$’s of the Mexican parametric CAT bond

In case of a trigger event:
- Swiss Re pays the covered insured amount to the government
- Investors sacrifice full principal and coupons

Premium & proceeds are used to pay coupons to bondholders
Assumptions

The arrival process of EQs $N_t$, $t \geq 0$ uses the times between EQ $\tau_i = T_i - T_{i-1}$:

$$N_t = \sum_{n=1}^{\infty} 1(T_n < t)$$

EQ suffer loss of memory: $P(X > x + y | X > y) = P(X > x)$

$N_t$ is a Homogeneous Poisson Process (HPP) with intensity rate $\lambda > 0$:

- $N_t$ is governed by the Poisson law
- The waiting times $\tau_i$ are i.i.d. $\exp(\lambda)$
Assumptions

The probability of occurrence of an EQ in the interval $(0, t]$ is:
\[ P(\tau_i < t) = 1 - P(\tau_i \geq t) = 1 - e^{-\lambda t} \]

Define *stopping time* equal to:
\[ \tau = \min \{ t : N_t > 0 \} \]

where \( f_\tau(t) = \lambda e^{-\lambda t} \) is the density of occurrence of event
Calibrating Parametric CAT bond

The intensity rate ($\lambda$) describes the flow process of an EQ:
- Reinsurance market ($\lambda_1$): Ceding & Reinsurance company
- Capital market ($\lambda_2$): SPV & investors
- Historical data ($\lambda_3$): real intensity of EQ

Assumptions:
- Flat term structure of continuously compounded discount interest rates
- $N_t$ is a HPP
Reinsurance market intensity: $\lambda_1$

Let $H$ be the total premium paid by the government (26 mio.) and let $J = 450 \cdot 1(\tau < 3)$ be the payoff.

A compounded discounted *actuarially fair insurance price* at $t = 0$ is:

$$H = E\left[J e^{-\tau r_\tau}\right]$$

$$= E\left[450 \cdot 1(\tau < 3) e^{-\tau r_\tau}\right]$$

$$= 450 \int_0^3 e^{-rt} f_\tau(t) dt$$

where $f_\tau(t) = \lambda_1 e^{-\lambda_1 t}$ is the density of occurrence of event.
Calibrating the Mexican CAT Bond

LIBOR in May 2006 $r_t = \log(1.0541)$,

\[ 26 = 450 \int_0^3 e^{-\log(1.0541)t} \lambda_1 e^{-\lambda_1 t} dt \]  \hspace{1cm} (1)

Hence $\lambda_1 = 0.0215$, i.e. Swiss Re expects:

- 2.15 events in 100 years
- Probability of occurrence of an event in 3 years equal to 0.0624

Calibrating CAT Bonds for Mexican Earthquakes
Capital market intensity: $\lambda_2$

- CAT bond with coupons every 3 months and payment of the principal $P$ at $T$
- Coupon bonds pay a fixed spread $z=230$ bp. over LIBOR
- Coupons equal to $C = \left( \frac{r+z}{4} \right) \cdot P = 3.1055$ mio
- In case of an event: investor sacrifices principal $P$ & coupons
- Let $G$ be the investors’ gain
A discounted *fair bond price* at time $t = 0$ is given by:

$$P = E \left[ G \left( \frac{1}{1 + r_{\tau}} \right)^T \right]$$

$$= E \left[ \sum_{t=1}^{12} C \cdot 1(\tau > \frac{t}{4}) \left( \frac{1}{1 + r_{t}} \right)^{\frac{t}{4}} + P \cdot 1(\tau > 3) \left( \frac{1}{1 + r_{t}} \right)^{3} \right]$$

Then,

$$160 = \sum_{t=1}^{12} 3.1055 \left( \frac{e^{-\lambda_{2}}}{1.0541} \right)^{\frac{t}{4}} + \frac{160e^{-3\lambda_{2}}}{(1.0541)^{3}} \quad (2)$$

Hence, $\lambda_{2} = 0.0241$. The capital market estimates a probability of occurrence of an event in 3 years equal to 0.0644, equivalently to 2.4 events in 100 years.
## Historical Intensity: $\lambda_3$

<table>
<thead>
<tr>
<th></th>
<th>time(t)</th>
<th>magnitude(Mw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>1900</td>
<td>6.50</td>
</tr>
<tr>
<td>Maximum</td>
<td>2003</td>
<td>8.20</td>
</tr>
<tr>
<td>Mean</td>
<td>-</td>
<td>6.90</td>
</tr>
<tr>
<td>Median</td>
<td>-</td>
<td>6.90</td>
</tr>
<tr>
<td>Sdt. Error</td>
<td>-</td>
<td>0.37</td>
</tr>
<tr>
<td>25% Quantile</td>
<td>-</td>
<td>6.60</td>
</tr>
<tr>
<td>75% Quantile</td>
<td>-</td>
<td>7.10</td>
</tr>
<tr>
<td>Excess</td>
<td>-</td>
<td>0.25</td>
</tr>
<tr>
<td>Nr. obs.</td>
<td>192</td>
<td>192.00</td>
</tr>
<tr>
<td>Distinct obs.</td>
<td>82</td>
<td>18.00</td>
</tr>
</tbody>
</table>

Table 3: Descriptive statistics of EQ data from 1900 to 2003 (SSN)
Intensity model

- Let $Y_i$ be i.i.d rvs. indicating $Mw$ of the $i^{th}$ EQ at time $t$
- Let $\varepsilon_i = 1(Y_i \geq \bar{u})$ characterizing EQ with $Mw$ higher than a defined threshold for a specific location
- $N_t$ is a HPP with intensity $\lambda > 0$

A new process $B_t$ defines the trigger event process:

$$B_t = \sum_{i=1}^{N_t} 1(\varepsilon_i > 0)$$  \hspace{1cm} (3)

- Data contains only 3 events: the calibration of the intensity of $B_t$ is based on 2 waiting times $\tau_i$
Figure 6: Mw of trigger events (filled circles), EQs in zone 1 (black circles), EQs in zone 2 (green circles), EQs in zone 5 (magenta circles), EQs out of insured zones (blue circles) eq65thMexcase.xpl
Consider $B_t$ and define $p$ as the probability of occurrence of a trigger event conditional on the occurrence of the earthquake. The probability of no event up to time $t$:

$$P(B_t = 0) = \sum_{k=0}^{\infty} P(N_t = k)(1 - p)^k$$

$$= \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t}(1 - p)^k$$

$$= e^{-\lambda pt} = e^{-\lambda_3 t}$$

(4)

The annual historical intensity rate for a trigger event is equal to $\lambda_3 = \lambda p = 1.8504 \left(\frac{3}{192}\right) = 0.0289$
Calibration of intensity rates

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity ($10^{-2}$)</td>
<td>2.15</td>
<td>2.41</td>
<td>2.89</td>
</tr>
<tr>
<td>Prob. of event in 1 year ($10^{-2}$)</td>
<td>2.12</td>
<td>2.19</td>
<td>2.84</td>
</tr>
<tr>
<td>Prob. of event in 3 year ($10^{-2}$)</td>
<td>6.24</td>
<td>6.44</td>
<td>8.30</td>
</tr>
<tr>
<td>No. expected events in 100 years</td>
<td>2.15</td>
<td>2.41</td>
<td>2.89</td>
</tr>
</tbody>
</table>

Table 4: Intensity rates

$\lambda_1 \neq \lambda_2$:
- Absence of the public & liquid market of EQ risk in the reinsurance market: limited information is available
- Contracts in the capital market are more expensive than in the reinsurance market: cost of risk capital & risk of default
\( \lambda_1 \neq \lambda_2 \neq \lambda_3 \):

- \( \lambda_3 \) is based on the time period of the historical data
- If \( \lambda_3 \) would be the "real" intensity rate:
  - The Mexican government paid total premiums of $26 mio. that is 0.75 times the real actuarially fair one:
    \[
    \int_0^3 450\lambda_3 e^{-t(r_t+\lambda_3)} dt = 34.49
    \]
  - Savings of $8.492 mio.? NO
  - Probability of defaults of the reinsurer over the 3 three years \( \approx \) the price discount that the Government gets in the risk transfer of EQ risk
  - The mix of the reinsurance contract and the CAT bond: 35% of the total seismic risk to the investors
Modeled-Index CAT bond for earthquakes

- Minimization of basis risk borne by the sponsor, while remaining non-indemnity based.
- Other variables can affect the value of losses: magnitude ($M_w$), depth ($DE$), location, impact on Mexico city $IMP(0, 1)$.
  
  - Losses are $\propto M_w \&$ time $t$ \& inversely $\propto DE$ of EQ
- We built loss data from EQs during 1900-2003
- Losses $\{X_k\}_{k=1}^{\infty}$ adjusted to population, inflation, exchange rate
- Missing loss data treatment: Expectation-Maximum (EM) algorithm
Losses of EQ in 100 years

Figure 7: Adjusted Losses - Richter Scale

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Compound Doubly Stochastic Poisson Pricing Model

Baryshnikov et al. (2001):

- A doubly stochastic Poisson process $N_s$, i.e. a Poisson process conditional on a stochastic intensity process $\lambda_s$ with $s \in [0, T]$, describing the flow of a particular catastrophic natural event in a specified region.
  - HPP with an intensity $\lambda = 1.8504$
  - NHPP with intensity $\lambda^1_s = 1.8167$
Figure 8: The accumulated number of EQs (solid blue line) and mean value functions $E(N_t)$ of the HPP with intensity $\lambda_s = 1.8504$ (solid black line) and the $\lambda^1_s = 1.8167$ (dashed red line). CMXrisk03.xpl
The countinuous and predictable aggregate loss process is:

\[ L_t = \sum_{i=1}^{N_t} X_i \]  

(5)

where \( \{X_k\}_{k=1}^{\infty} \) at \( t \) are i.i.d with \( F(x) = P(X_i < x) \)

The threshold level \( D \) and a continuously compounded discount interest rate \( r \):

\[ e^{-R(s,t)} = e^{\int_s^t r(\xi) d\xi} \]

A threshold time event \( \tau = \inf \{ t : L_t \geq D \} \), defined it as a point of a doubly stochastic Poisson process \( M_t = 1(L_t > D) \) with a stochastic intensity:

\[ \Lambda_s = \lambda_s \{1 - F(D - L_s)\} \mathbf{1}(L_s < D) \]  

(6)
Modeled loss:
\[
\log(X) = -27.99 + 2.10Mw + 4.44DE - 0.15\text{IMP}(0, 1) - 1.11 \log(Mw) \cdot DE
\]

Figure 9: Historical and modeled losses of EQs from 1900-2003 (left panel), without the outlier of the EQ in 1985 (middle panel), without outliers of EQ in 1985 and 1999 (right panel) [CMXmyEMalgorithm.xpl](#)
Fitting & testing the loss distribution:

<table>
<thead>
<tr>
<th>Distrib.</th>
<th>Log-normal</th>
<th>Pareto</th>
<th>Burr</th>
<th>Exponential</th>
<th>Gamma</th>
<th>Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>$\mu = 1.387$</td>
<td>$\alpha = 2.394$</td>
<td>$\alpha = 3.323$</td>
<td>$\beta = 0.143$</td>
<td>$\alpha = 0.143$</td>
<td>$\beta = 0.220$</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 1.644$</td>
<td>$\lambda = 12.92$</td>
<td>$\lambda = 16.67$</td>
<td>$\beta = -0.007$</td>
<td>$\beta = 0.220$</td>
<td>$\tau = 0.764$</td>
</tr>
<tr>
<td>Kolmogorov Sminorv</td>
<td>0.173</td>
<td>0.131</td>
<td>0.137</td>
<td>0.135</td>
<td>0.295</td>
<td>0.145</td>
</tr>
<tr>
<td>(D test)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
</tr>
<tr>
<td>Kuiper</td>
<td>0.296</td>
<td>0.248</td>
<td>0.260</td>
<td>0.222</td>
<td>0.569</td>
<td>0.282</td>
</tr>
<tr>
<td>(V test)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
</tr>
<tr>
<td>Cramér-von Mises</td>
<td>1.358</td>
<td>0.803</td>
<td>0.884</td>
<td>0.790</td>
<td>7.068</td>
<td>1.051</td>
</tr>
<tr>
<td>(W$^2$ test)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
</tr>
<tr>
<td>Anderson Darling</td>
<td>10.022</td>
<td>5.635</td>
<td>5.563</td>
<td>9.429</td>
<td>36.076</td>
<td>5.963</td>
</tr>
<tr>
<td>(A$^2$ test)</td>
<td>(&lt; 0.005)</td>
<td>(0.005)</td>
<td>(0.01)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
<td>(&lt; 0.005)</td>
</tr>
</tbody>
</table>

Table 5: Parameter estimates by $A^2$ minimization procedure and test statistics. In parenthesis, the related $p$-values based on 1000 simulations.
Zero Coupon CAT bonds (ZCCB)

- Pays \( P \) at \( T \) conditional on \( \tau > T \)
- The payment at maturity is independent from the occurrence and timing of \( D \)
- In case of a trigger event \( P \) is fully lost

The non arbitrage price of the ZCCB \( V_t^1 \), Burnecki and Kukla (2003):

\[
V_t^1 = \mathbb{E} \left[ P e^{-R(t,T)} (1 - M_T) | \mathcal{F}_t \right] \\
= \mathbb{E} \left[ P e^{-R(t,T)} \left\{ 1 - \int_t^T \lambda_s \{1 - F(D - L_s)\} \mathbf{1}(L_s < D) \, ds \right\} | \mathcal{F}_t \right]
\]
Coupon CAT bonds (CCB)

- Pays $P$ at $T$ & gives coupons $C_s$ until $\tau$
- The payment at maturity is independent from the occurrence and timing of $D$
- Pays a fixed spread $z$ (bp.+LIBOR)
- In case of a trigger event $P$ is fully lost

The non-arbitrage price of the CCB $V_t^2$, Burnecki and Kukla (2003):

$$V_t^2 = \mathbb{E} \left[ P e^{-R(t,T)} (1 - M_T) + \int_t^T e^{-R(t,s)} C_s (1 - M_s) \, ds \, | \mathcal{F}_t \right]$$

$$= \mathbb{E} \left[ P e^{-R(t,T)} + \int_t^T e^{-R(t,s)} \left\{ C_s \left( 1 - \int_t^s \lambda_s \left\{ 1 - F(D - L_s) \right\} \right) 1(L_s < D) \right. \right.$$

$$\left. - P e^{-R(s,T)} \lambda_s \left\{ 1 - F(D - L_s) \right\} 1(L_s < D) \right\} \, ds \, | \mathcal{F}_t \]$$
Calibration

- $r$ equal to the LIBOR ($r = \log(1.0541)$)
- $P = $160 mio.
- $T \in [0.25, 3]$ years
- $D \in [$100, $135]$ mio. (0.7 & 0.8-quantiles of 3 yearly acc.losses)
- $z = 235$ bp. over LIBOR
- Quarterly $C_t = \left(\frac{\text{LIBOR} + 235\text{bp}}{4}\right)$ $160 = $3.1055 mio.
- $N_t$ is an HPP with intensity $\lambda_s = 1.8504$
- 1000 Monte Carlo simulations
Figure 10: The CCB price (vertical axis) with respect to $T$ (horizontal left axis) & $D$ (horizontal right axis) in the Burr-HPP (left panel) & Pareto-HPP (right panel) for the modeled loss CMX07e.xpl.
Burr - Pareto differences in CAT Bond Prices

Figure 11: The difference in ZCCB price (left panel) & CCB prices (right panel) in the vertical axis left panel between the Burr & Pareto distributions under a HPP, with respect to the $T$ (horizontal left axis) & $D$ (horizontal right axis)

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Calibrating a Modeled-Index CAT bond

<table>
<thead>
<tr>
<th></th>
<th>Min. (% Principal)</th>
<th>Max. (% Principal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diff. ZCB Burr-Pareto</td>
<td>-2.640</td>
<td>0.614</td>
</tr>
<tr>
<td>Diff. CB Burr-Pareto</td>
<td>-1.552</td>
<td>0.809</td>
</tr>
<tr>
<td>Diff. ZCB-CB Burr</td>
<td>-6.228</td>
<td>-0.178</td>
</tr>
<tr>
<td>Diff. ZCB-CB Pareto</td>
<td>-5.738</td>
<td>-0.375</td>
</tr>
</tbody>
</table>

Table 6: Min. & max. of the diff. in the ZCCB-CCB prices in % of $P$ for the Burr-Pareto distributions of the modeled loss

- $V_t^1$ & $V_t^2$ decreases as $T$ increases
- $V_t^1$ & $V_t^2$ increases as $D$ increases
- $F(x)$ influences the price of the CAT bond
- Modeled loss: no significant impact on ZCCB-CCB prices, but more important than the loss distribution
Conclusion

- Seismic risk can be transferred with CAT bonds
- CAT bonds: No credit risk, high returns and better performance of the portfolio
- Calibration of a Mexican CAT bond:
  1. Parametric trigger (physical parameters): the intensity rates of EQ in ≠ parts of the contract vs. real historical
    - $N_t$ is a HPP with intensity $\lambda$
  2. Modeled-Index loss trigger considers several variables: the intensity rate of EQ and the level of accumulated losses $L_s$
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