Generalized Autoregressive Conditional Intensity Models with Long Range Dependence

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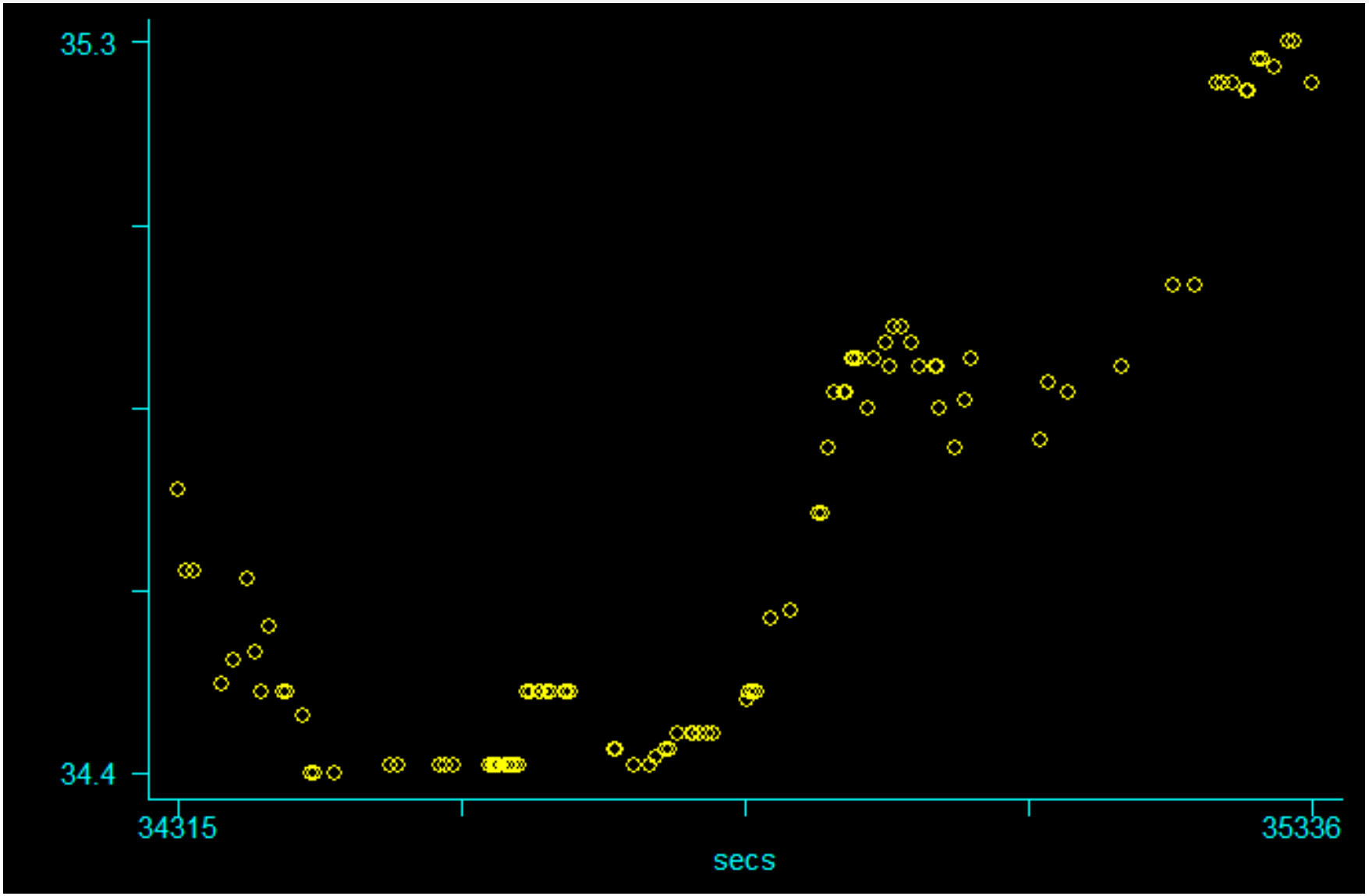
1. Introduction

- Modelling of financial data at the transaction level is an ongoing topic in the area of financial econometrics.
- Seminal papers: Hasbrouck (1991), Engle and Russell (1998), Engle (2000), Russell and Engle (2005)
- New area of research: "High-Frequency Econometrics"
- Issues in high-frequency econometrics:
  - Market microstructure investigations
  - Volatility estimation
  - Liquidity estimation
  - Trading models

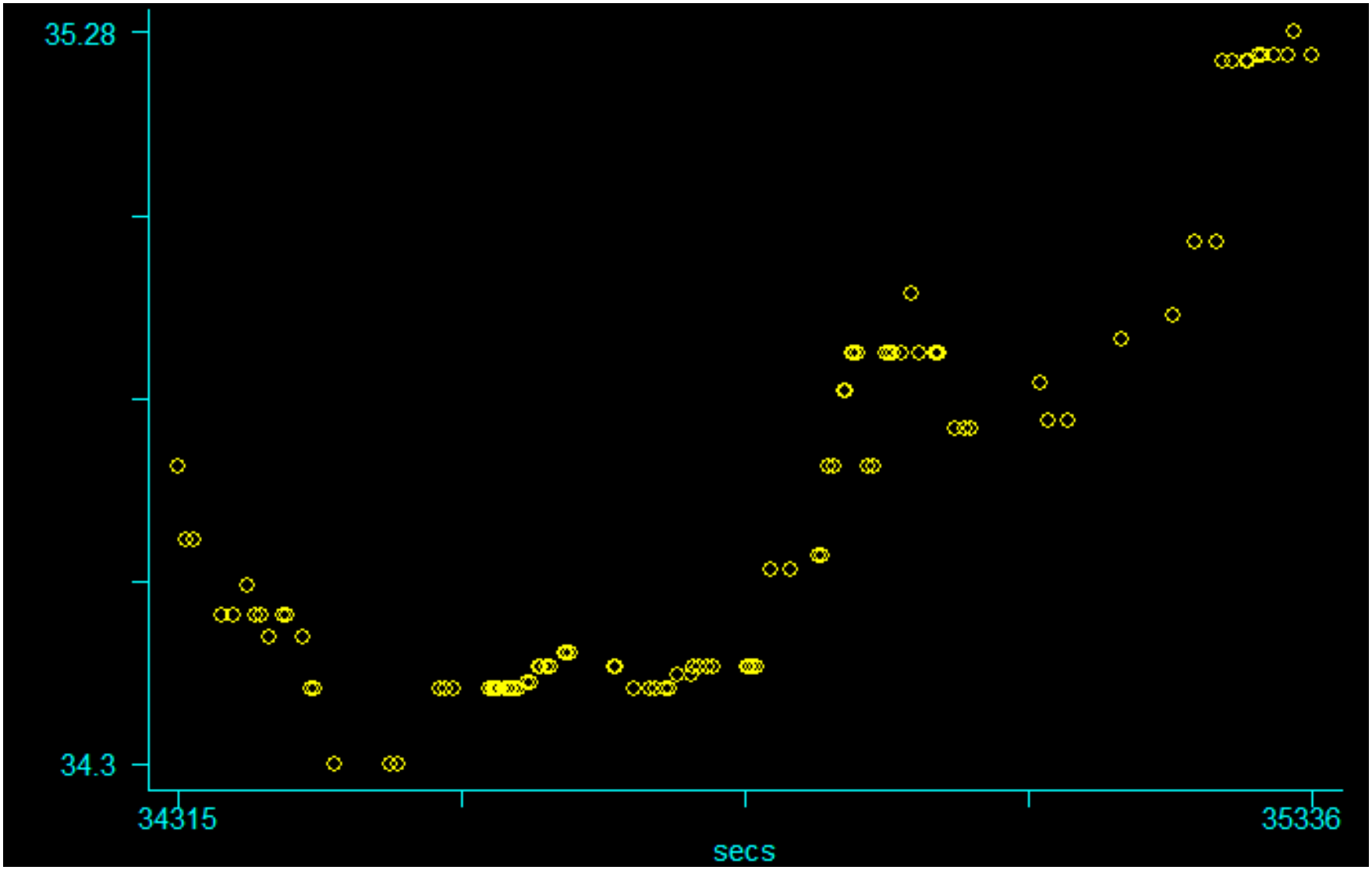
Statistical Properties of Financial Transaction Data

- Irregular Spacing in Time
- Discreteness of price changes
- Bid ask bounce
- Strong intraday seasonalities
- Clustering of the trading intensity

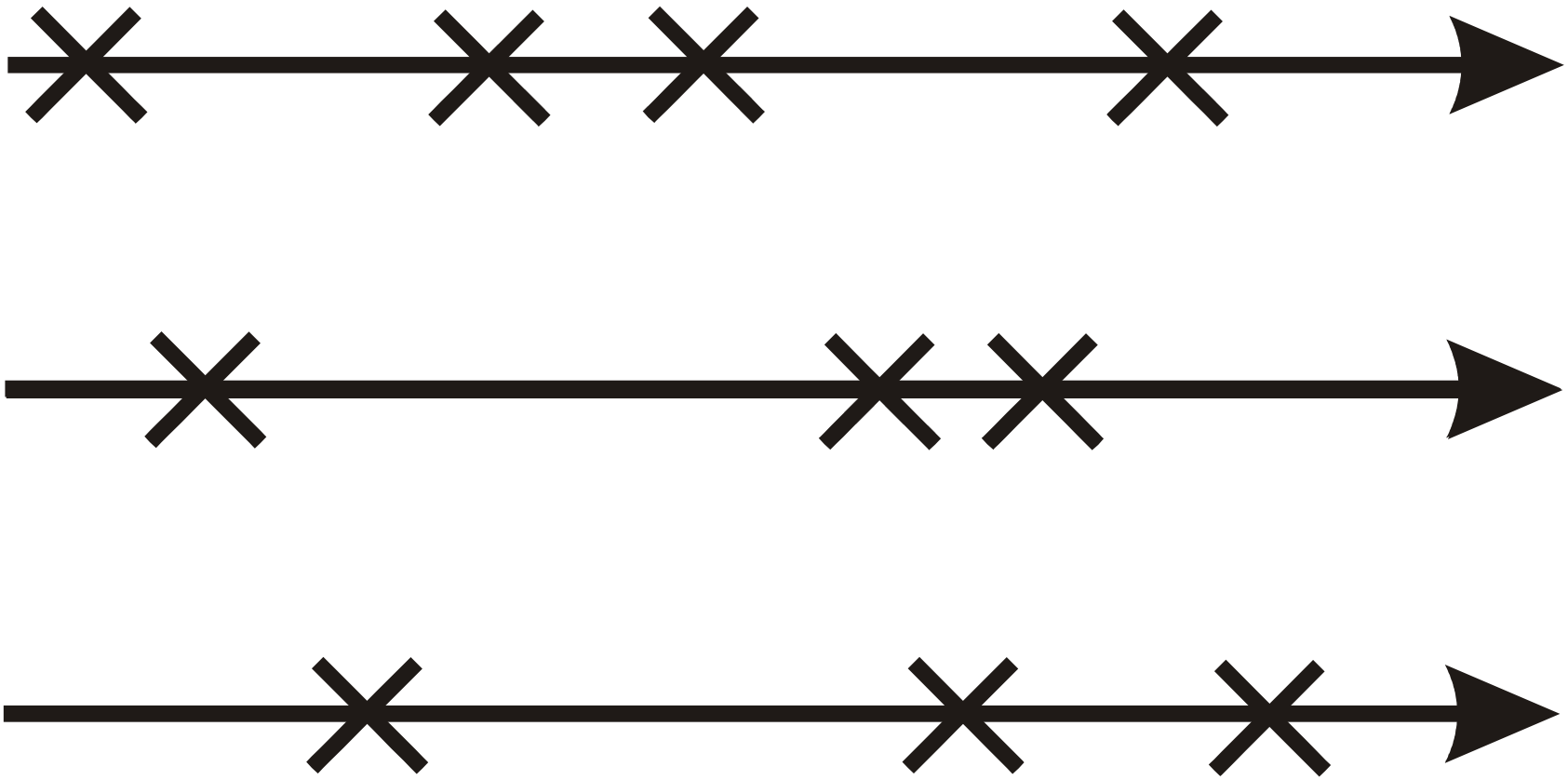
The Irregularity in Trade Arrivals



The Irregularity in Quote Arrivals



A multivariate point process



Using the time information...

- to explicitly account for the irregular spacing of the data
- to analyze market microstructure relationships and order book dynamics: trade/quote intensities, buy/sell intensities, order aggressiveness
- for volatility measurement: time between cumulative absolute price changes
- for liquidity measurement:
  - time until a certain (buy or sell) buy volume is traded
  - time until a certain net order flow is observed
- to study financial markets at different time scales ("intrinsic time", Dacorogna et. al, 2001)

Financial durations - statistical properties

- positive serial dependence
- high persistence
- duration distributions imply non-monotonic hazard functions
- price durations reveal overdispersion and volume durations reveal underdispersion
- strong impact of intraday seasonality

Two Ways to Model Financial Durations

- Modelling inter-event waiting times (durations) $x_i = t_i - t_{i-1}$
  - Autoregressive Conditional Duration (ACD) (Engle/Russell, 1998)
  - Arises from GARCH literature
  - Accelerated Failure Time (AFT) model
  - not suitable for multivariate specifications and time-varying covariates

- Modelling the (stochastic) intensity

$$\lambda(t; \mathcal{F}_t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} \mathrm{E}\left[N(t+\Delta) - N(t) | \mathcal{F}_t \right]$$

  - Autoregressive Conditional Intensity (ACI) (Russell, 1999)
  - Arises from point process literature or duration literature resp.
  - Proportional Intensity (PI) model
  - Continuous-time specification

Focus of this Paper

- Bringing together ACD and ACI models in a unified generalized ACI framework
- Relaxing the assumption of a proportional intensity model and allowing for generalized accelerated failure time specifications.
- Allowing for long range dependence in the intensity process
- Allowing for more distributional flexibility by proposing a semiparametric specification of the baseline intensity component.

$\Rightarrow$ The resulting model is called *Generalized Long Memory Autoregressive Conditional Intensity (GLMACI) Model*.

Outline of the Talk

2. Long Range Dependence in Financial Durations
Formal Definition

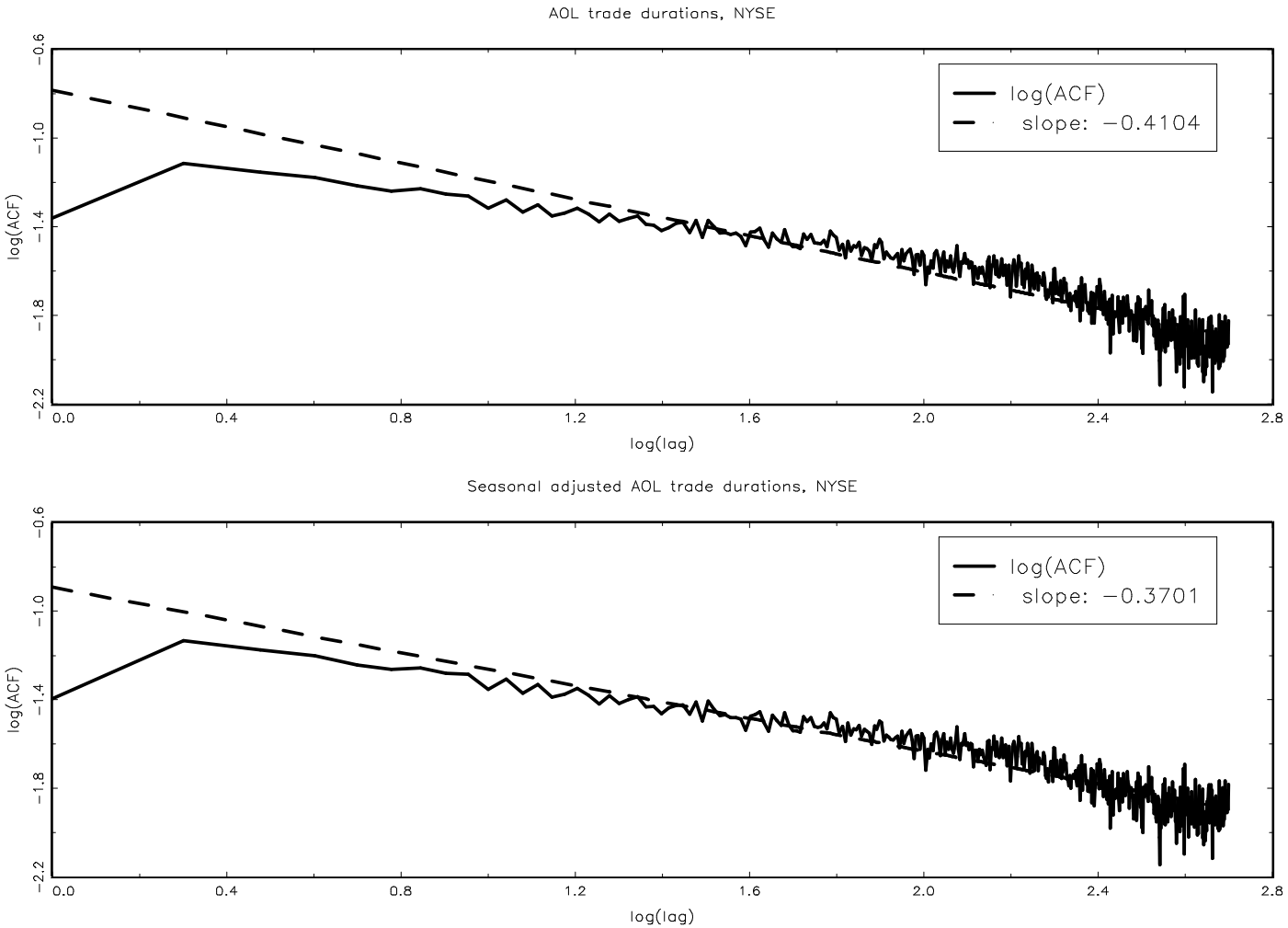
- Definition: Let X_t be a (weakly) stationary process with autocorrelation function of order k, $\rho(k)$. Then, X_t implies long range dependence if

$$\lim_{k \to \infty} \rho(k)/[ck^{-\alpha}] = 1$$

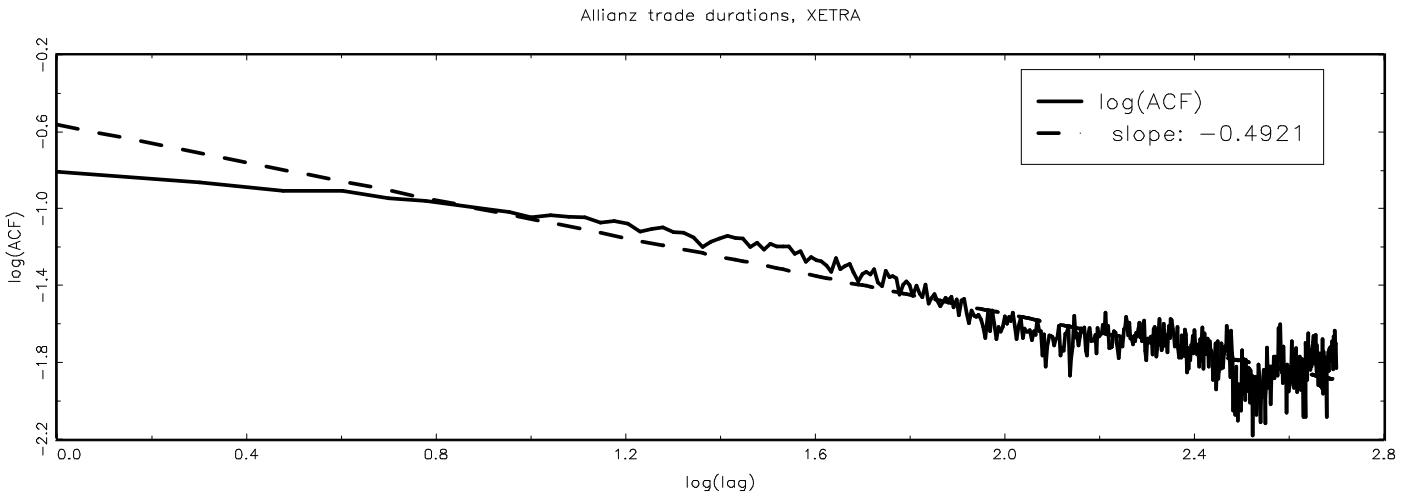
with $\alpha \in (0,1)$ and $c > 0$.

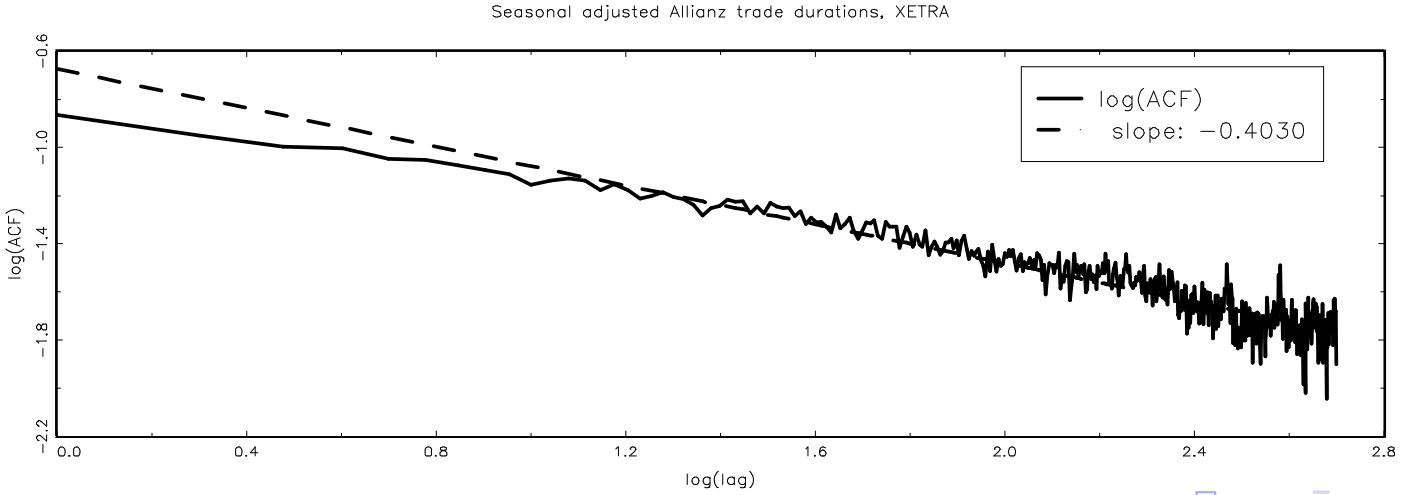
- in terms of the Hurst (self-similarity) parameter: $H = 1 - \alpha/2$
- for $\alpha \in (0,1)$ resp. $H \in (0.5,1)$: $\sum_{k=-\infty}^{\infty} \rho(k) = \infty$
- $\mathrm{V}[\bar{x}_n] \approx cn^{-\alpha}$ with $c > 0$

$\log|\rho(k)|$ against $\log k$, intertrade durations, AOL, NYSE

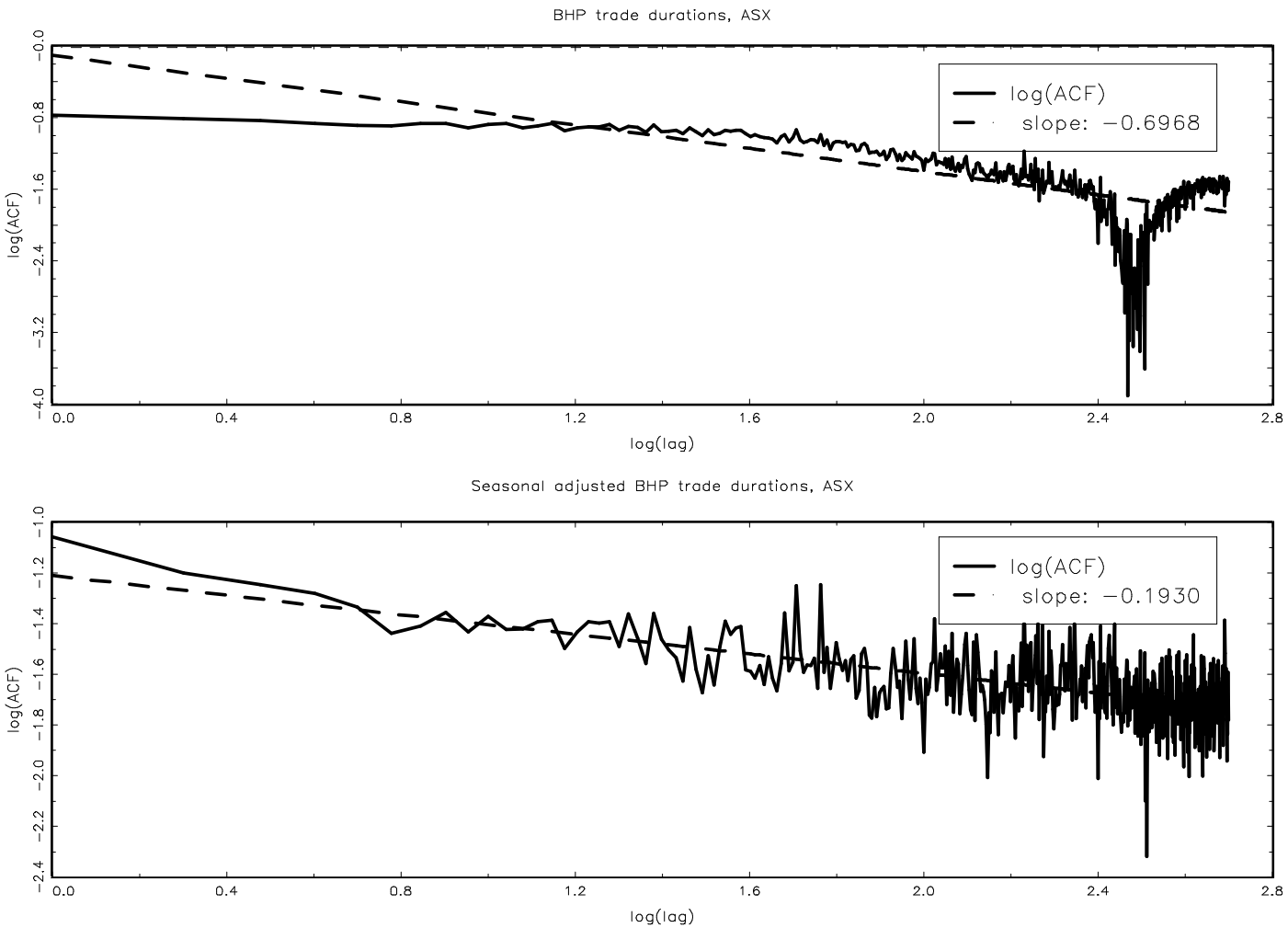


$\log|\rho(k)|$ against $\log k$, intertrade durations, Allianz, XETRA

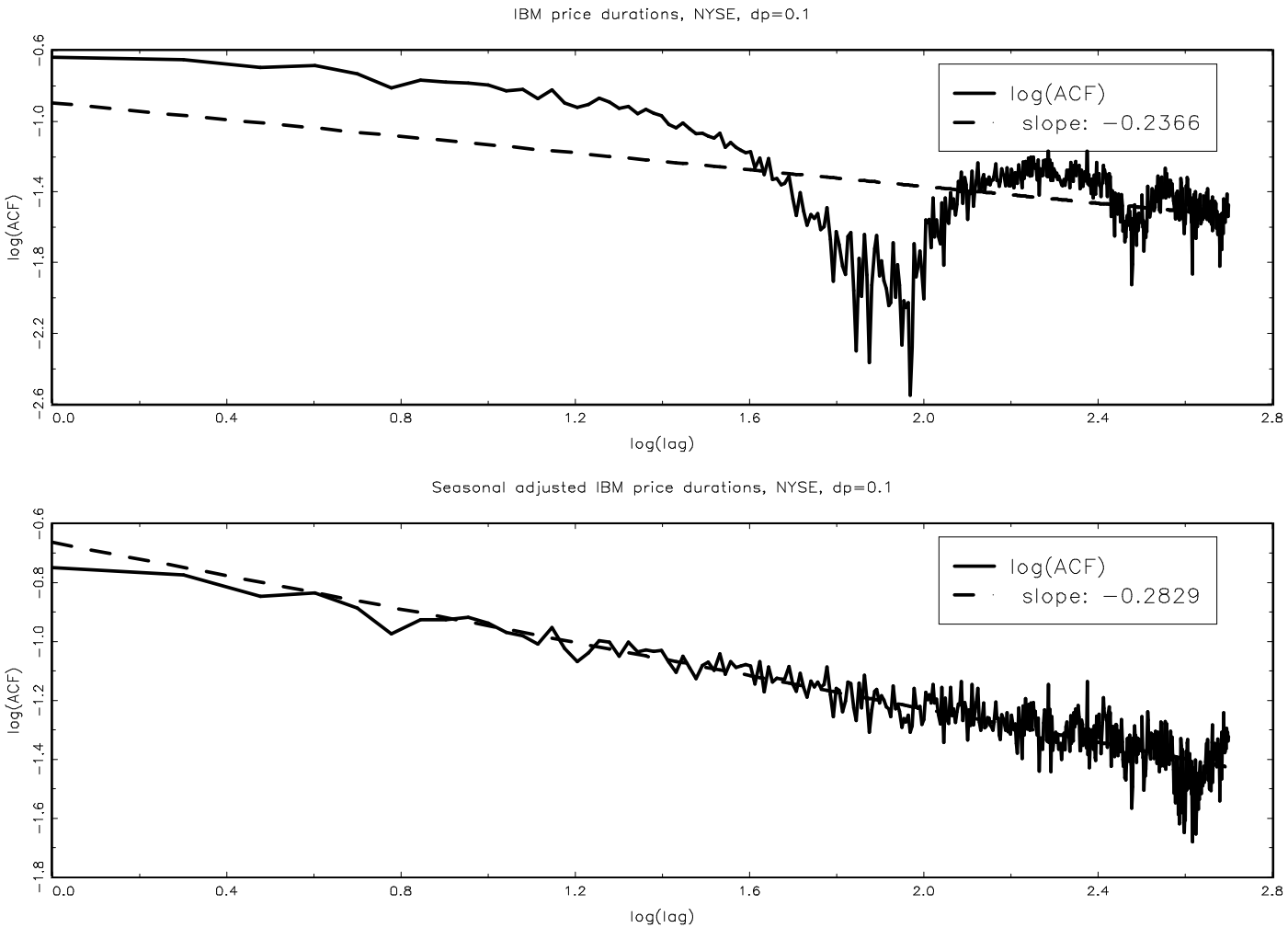




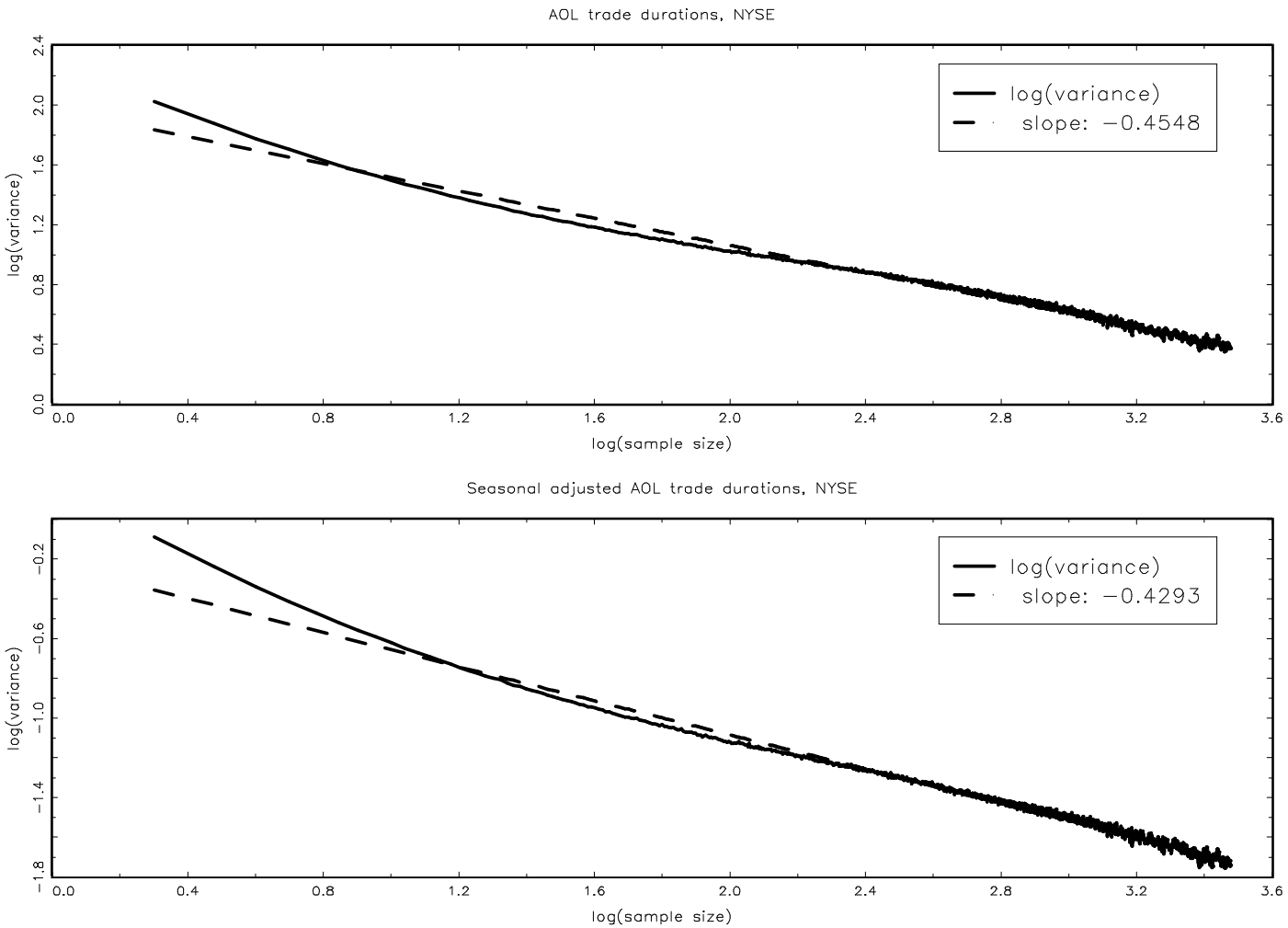
$\log|\rho(k)|$ against $\log k$, intertrade durations, BHP, ASX



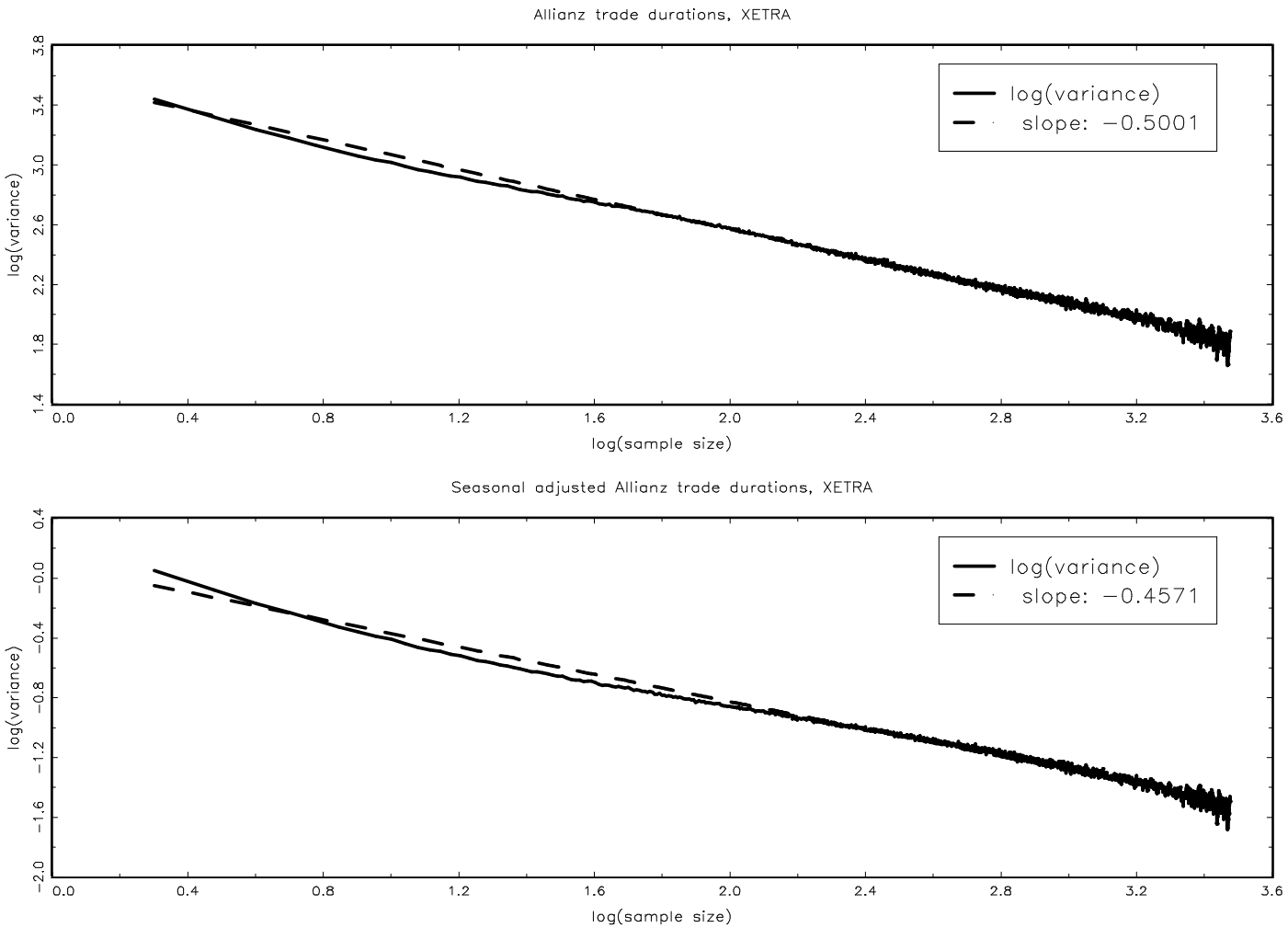
$\log|\rho(k)|$ against $\log k$, \$0.1 price durations, IBM, NYSE



$\mathrm{V}[\bar{X}_n]$ against n, intertrade durations, AOL, NYSE



$\mathrm{V}[\bar{X}_n]$ against n, intertrade durations, Allianz, XETRA



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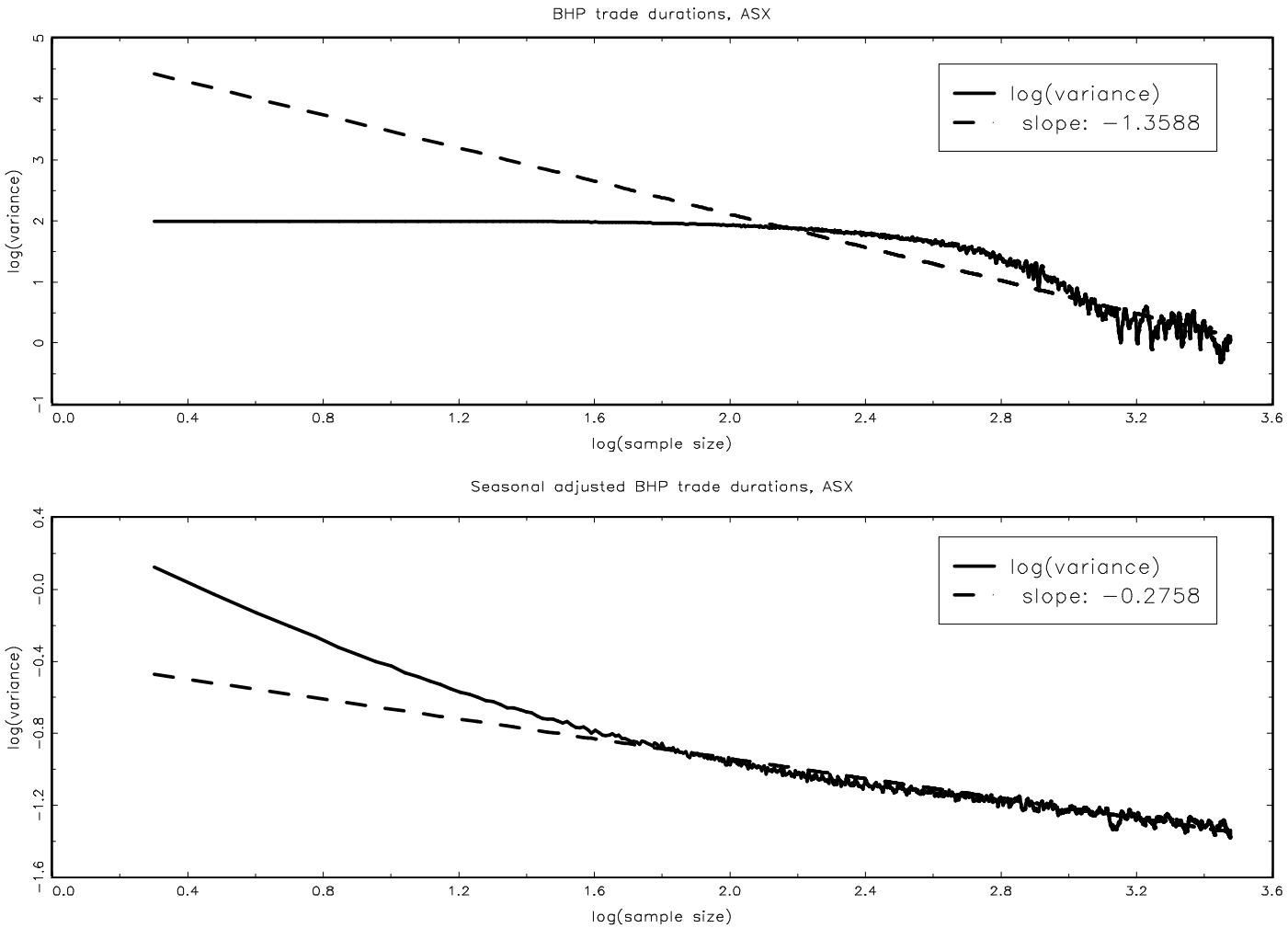

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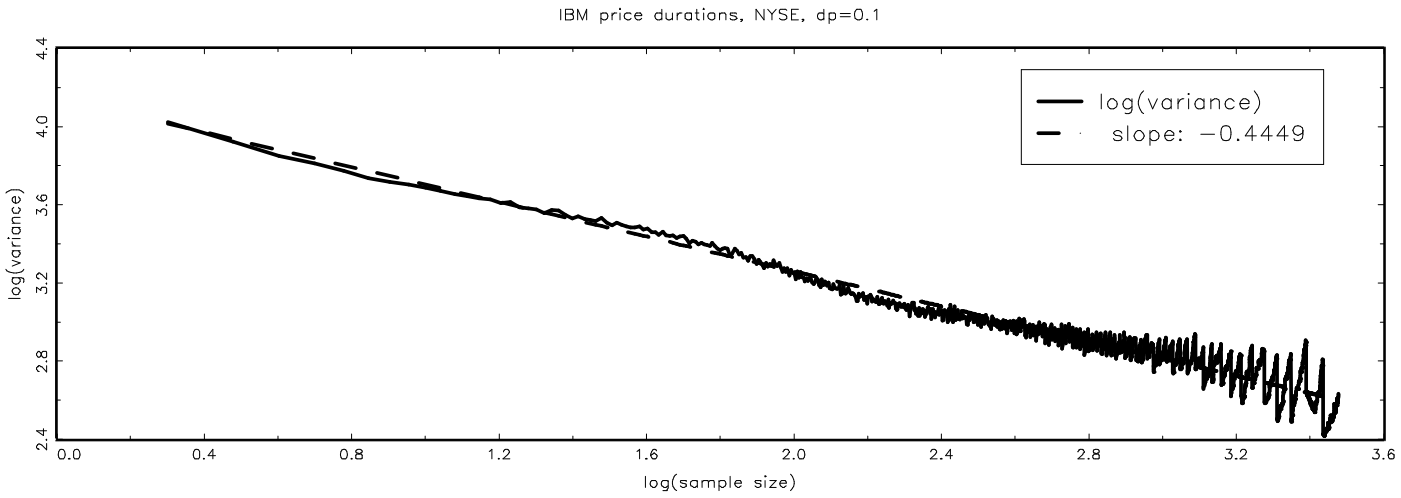
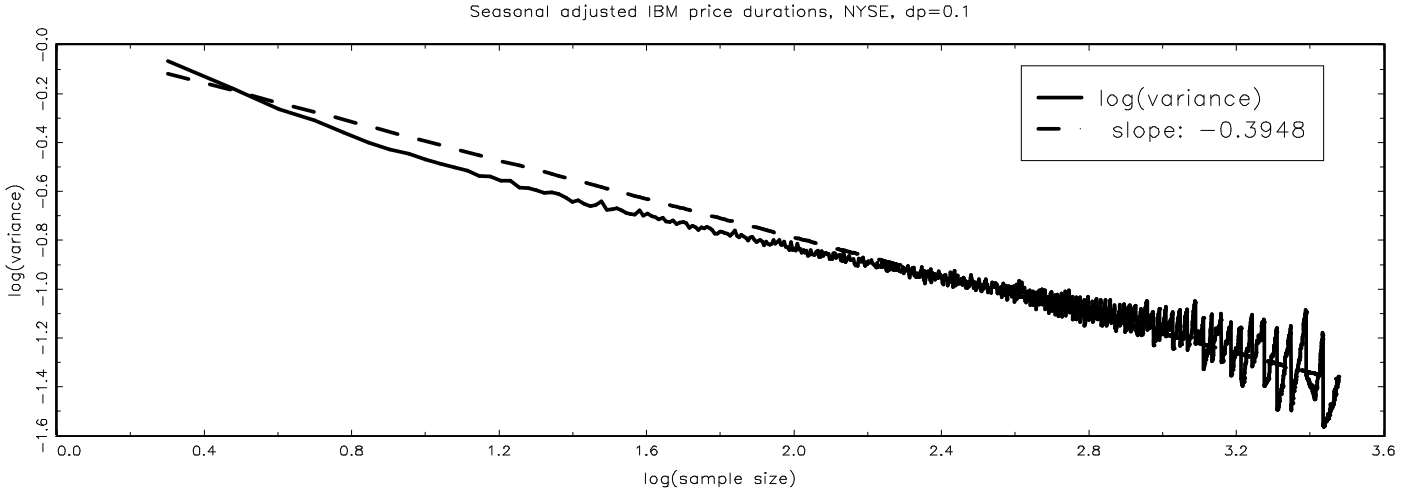




3. Autoregressive Conditional Duration

- Inter-event duration: $x_i := t_i - t_{i-1}$.
- Principle of the ACD model (Engle/Russell, 1998)

$$\varepsilon_i = \frac{x_i}{\Psi_i} \sim \text{i.i.d.},$$

where $\Psi_i := \Psi_i(\theta) = \mathrm{E}[x_i | \mathcal{F}_{t_{i-1}}; \theta]$

- Basic specification: $\Psi_i = \omega + \sum_{j=1}^{P} \alpha_j x_{i-j} + \sum_{j=1}^{Q} \beta_j \Psi_{i-j}$,

  where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$.
- Exponential ACD (EACD) model: $\varepsilon_i \sim Exp(1)$

- ARMA representation:

$$x_i = \omega + \sum_{j=1}^{max(P,Q)} (\alpha_j + \beta_j) x_{i-j} - \sum_{j=1}^{Q} \beta_j \eta_{i-j} + \eta_i,$$

where $\eta_i := x_i - \Psi_i$.

- Intensity representation:

$$\lambda(t; \mathcal{F}_t) = \tilde{\lambda}_\varepsilon \left(\frac{x(t)}{\Psi_{\tilde{N}(t)+1}} \right) \frac{1}{\Psi_{\tilde{N}(t)+1}},$$

where $\tilde{\lambda}_\varepsilon(s)$ denotes the hazard function of ε_i, $\tilde{N}(t)$ denotes the left-continuous counting function and $x(t) := t - t_{\tilde{N}(t)}$.

$\Rightarrow$ Accelerated Failure Time Model.

Specifications of Ψ_i

- Linear Model: $\Psi_i = \omega + \sum_{j=1}^{P} \alpha_j x_{i-j} + \sum_{j=1}^{Q} \beta_j \Psi_{i-j}$,
- Log ACD based on ACD innovations (Bauwens/Giot, 01):
  $\ln \Psi_i = \omega + \sum_{j=1}^{P} \alpha_j \varepsilon_{i-j} + \sum_{j=1}^{Q} \beta_j \ln \Psi_{i-j}$,

- <u>Alternative</u>:

$$\tilde{\varepsilon}_i = \Lambda(t_{i-1}, t_i) := \int_{t_{i-1}}^{t_i} \lambda(s) ds = \frac{1}{\Psi_i} \int_{t_{i-1}}^{t_i} \tilde{\lambda}_\varepsilon \left(\frac{x(s)}{\Psi_i} \right)$$

- Brown/Nair (88): $\Lambda(t_{i-1}, t_i) := \int_{t_{i-1}}^{t_i} \lambda(s) ds$ corresponds to the increment of a Poisson process.

4. Univariate GLMACI Models

- The basic structure of the univariate GLMACI model is given by

$$\lambda(t) = \lambda_0\left(\eta(t)\right)\Phi_{\check{N}(t)+1}s(t),$$
$$\eta(t) := x(t)\cdot\left[\Phi_{\check{N}(t)+1}s(t)\right]^{\delta}.$$

- $\Phi_{\check{N}(t)}$ is a function capturing the model dynamics as well as possible covariates.
- $s(t)$ is a (deterministic) function of time capturing possible seasonality effects.
- $\lambda_0(\cdot)$ denotes a baseline intensity function in terms of $\eta(t)$.
- $\eta(t)$ corresponds to the backward recurrence time $x(t) := t - t_{\check{N}(t)}$ scaled by Φ_i and $s(t)$.

Parameterization of Φ_i: A Fractionally Integrated Process

- $\Phi_i = \exp\left(\tilde{\Phi}_i + z'_{i-1}\gamma\right)$.
- Innovation term:

$$\varepsilon_i := 1 - \Lambda(t_{i-1}, t_i)$$

with $\Lambda(t_{i-1}, t_i) \sim$ i.i.d. $Exp(1)$.

- Infinite series representation (Koulikov 03, Hosking, 1981):

$$\tilde{\Phi}_i = \omega + \alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1} \quad \text{with } |\beta| < 1$$

$$\alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1} = \alpha \sum_{j=1}^{\infty} \theta_{j-1}\varepsilon_{i-j},$$

$$\theta_j := \sum_{k=0}^{j} \alpha\beta^k \theta^*_{j-k}, \qquad \theta^*_j := \frac{\Gamma(d+j)}{\Gamma(d)\Gamma(1+j)} \quad \forall j \geq 0$$

Stationarity Properties of $\tilde{\Phi}_i$

Adapting the results by Koulikov (2003) for MD-ARCH(∞) processes:

- Sufficient condition for covariance stationarity:

$$\sum_{j=0}^{\infty} \theta_j^2 < 1,$$

- Necessary conditions:
  - $\alpha < 1$, $\beta < 1$
  - $d < 0.5$.
- Non-summable autocorrelation functions functions for $d \in (0, 1)$.

Special Cases

- $d = 0$ and $\delta = 0$
- Basic ACI(1,1) specification (Russell, 99):

$$\begin{aligned}\tilde{\Phi}_i &= \omega + \alpha(1 - \beta L)^{-1}\varepsilon_{i-1} \\ &= \omega + \alpha\varepsilon_{i-1} + \beta(\tilde{\Phi}_{i-1} - \omega)\end{aligned}$$

Special Cases - cntd.

- If $\lambda_0(\cdot)$ is non-specified, and $\delta = 1$, $\omega = \alpha = \beta = 0$:
  AFT model (Kalbfleisch/Prentice, 1980) with

$$\lambda(t) = \lambda_0 \left[x(t) \exp(z_{\check{N}}(t)'\gamma)\right] \exp(z_{\check{N}}(t)'\gamma).$$

$$\text{Then:} \qquad \ln x_i = -z'_{i-1}\gamma + \xi_i$$

- If $\lambda_0(\cdot)$ is non-specified, and $\delta = \omega = \alpha = \beta = 0$:
  Semiparametric PI model (Cox, 1972) with

$$\lambda(t) = \lambda_0[x(t)] \exp(z_{\check{N}}(t)'\gamma).$$

Special Cases - cntd.

- If $\lambda_0(\cdot) = 1$: Long memory Log-ACD model with

$$\Phi_i = \mathrm{E}[x_i|\mathcal{F}_{t_{i-1}}]^{-1} := \Upsilon_i^{-1}$$

and

$$\Upsilon_i = \exp\left(\tilde{\Upsilon}_i - z_{i-1}'\gamma\right)$$
$$\tilde{\Upsilon}_i = -\omega + \alpha(1-\beta L)^{-1}(1-L)^{-d}(x_{i-1}/\Upsilon_{i-1} - 1),$$

- If $\lambda_0(\cdot) = 1$ and $d = 0$: Log-ACD model with

$$\tilde{\Upsilon}_i = -\omega + \alpha(x_{i-1}/\Upsilon_{i-1} - 1) + \beta(\tilde{\Upsilon}_{i-1} - \omega)$$

Special Cases - cntd.

- $\lambda_0(\eta(t)) = p \cdot \eta(t)^{p-1}(1+\kappa\eta(t)^p)^{-1}$ and $\delta = 1$:
  Burr type (long memory) ACD model with

$$\Phi_i = \mathrm{E}[x_i|\mathcal{F}_{t_{i-1}}]^{-1}\left[\frac{\kappa^{1+1/p}\Gamma(1+1/\kappa)}{\Gamma(1+1/p)\Gamma(\kappa^{-1}-1/a)}\right] := \Upsilon_i,$$

  where $\tilde{\Upsilon}_i = -\omega - \alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1}$.

$\Rightarrow$ If $d = 0$: Burr-ACD model with

$$\tilde{\Upsilon}_i = -\omega - \alpha\varepsilon_{i-1} + \beta(\tilde{\Upsilon}_{i-1} - \omega),$$

  where $\varepsilon_i = 1 - \Lambda(t_{i-1}, t_i)$ equals the integrated intensity.

Extensions of the Basic Univariate GLMACI Model
Component-Specific Acceleration Effects

- Motivation: Accounting for the multiplicative nature of the model. Acceleration effects might be driven by individual components.
- We allow for component-specific acceleration effects by re-formulating the basic model as

$$\eta(t) := x(t) \cdot \Phi_{\check{N}(t)+1}^{\delta_\Phi} s(t)^{\delta_s},$$

  where δ_Φ and δ_s are specific acceleration parameters affecting the individual components separately.
- Acceleration/decelaration effects might be driven individually by intraday dynamics and seasonality effects.
- Basic model for $\delta_\Phi = \delta_s = \delta$.

Flexible Baseline Intensities

- *Motivation*: Burr distribution is good starting point for financial duration processes (Grammig/Maurer, 00) but often not sufficient (Bauwens et al, 04, Bauwens/Hautsch, 06).
- Define $\nu(t) := 1 - \exp(-x(t))$ with $\nu(t) \in [0; 1]$.
- *Idea*: Augmentation of a Burr parameterization by a flexible Fourier form. Then,

$$\lambda_0(\eta(t)) = \frac{p \cdot \eta(t)^{p-1}}{1 + \kappa\eta(t)^p} \times \exp\left[\sum_{m=1}^{M} p_{m,s} \sin(2m\pi\nu(t)) + p_{m,c} \cos(2m\pi\nu(t))\right],$$

where M denotes the order of the process and $p_{m,s}$ and $p_{m,c}$ are coefficients.

Asymmetric News Response

- Motivation: Recent literature on financial duration models illustrates presence of nonlinear news impact functions (Dufour/Engle, 01, Fernandes/Grammig, 05, or Hautsch, 06.)

- Following Nelson (91) we can re-specify the process $\tilde{\Phi}_i$ as

$$\tilde{\Phi}_i = \omega + \alpha(1-\beta L)^{-1}(1-L)^{-d}\varepsilon_{i-1} + \varsigma(1-\beta L)^{-1}(1-L)^{-d}\bar{\varepsilon}_{i-1},$$

where $\bar{\varepsilon}_i := |\varepsilon_i| - \mathrm{E}[|\varepsilon_i|]$ and ς denotes the asymmetry parameter.

5. Multivariate GLMACI Models

- K-variate *marked* point process with arrival times $\{t_i^k\}_{i\in\{1,\ldots,n^k\}}$, and $N^k(t) := \sum_{i\geq 1} \mathbb{1}_{\{t_i^k \leq t\}}$, $k = 1, \ldots K$.
- k-type intensity: $\mathrm{E}[N^k(s) - N^k(t)|\mathcal{F}_t] = \mathrm{E}\left[\int_t^s \lambda^k(u)du|\mathcal{F}_t\right]$ with compensator $\tilde{\Lambda}^k(t) := \int_0^t \lambda^k(u)du$.
- The processes of compensators $\left(\tilde{\Lambda}^1(t), \tilde{\Lambda}^2(t), \ldots, \tilde{\Lambda}^K(t)\right)$ are independent Poisson processes with unit intensity (Brown/Nair, 1988).
- The k-type integrated intensities $\Lambda^k(t_{i-1}^k, t_i^k) := \int_{t_{i-1}^k}^{t_i^k} \lambda^k(s)ds$ $\forall\, k = 1, \ldots, K$ are independently standard exponentially distributed.

Basic Model

- The multivariate GLMACI model is given by

$$\lambda^k(t) = \lambda_0^k \left[\eta^1(t), \ldots, \eta^K(t)\right] \Phi^k_{\check{N}(t)+1} s^k(t),$$

where

$$\eta^k(t) := \begin{cases} \eta^k(t_{\check{N}(t)}) + x(t) \cdot \Xi(t) & \text{if } t_{\check{N}(t)} \text{ was of type } r \neq k \\ x(t) \cdot \Xi(t) & \text{if } t_{\check{N}(t)} \text{ was of type } k \end{cases}$$

and

$$\Xi(t) := \prod_{r=1}^{K} \left(\Phi^r_{\check{N}(t)+1}\right)^{\delta^k_{r,\Phi}} \left(s^r(t)\right)^{\delta^k_{r,s}}.$$

- Hence, the acceleration/deceleration effects act piecewise on the backward recurrence times.

Specification of Φ_i^k

$$\Phi_i^k = \exp\left(\tilde{\Phi}_i^k + z_{i-1}'\gamma^k\right)$$

$$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K}\left\{\alpha_j^k(1-\beta_j^k L)^{-1}(1-L)^{-d^k}\varepsilon_{i-1}\right\} y_{i-1}^j,$$

$$\varepsilon_i = \sum_{k=1}^{K}\left(1-\Lambda^k(t_{i-1}^k, t_i^k)\right) y_i^k$$

Then: $$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K}\left\{\alpha_j^k \sum_{s=1}^{\infty}\theta_{s-1,i}^k \varepsilon_{i-s}\right\} y_{i-1}^j,$$

$$\theta_{s,i}^k := \begin{cases} \theta_s^{k*} & \text{if } s = 1 \\ \theta_s^{k*} + \sum_{m=1}^{s}\theta_{s-m}^{k*}\prod_{r=0}^{m-1}\sum_{j=1}^{K}\beta_j^k y_{i-r}^j & \text{if } s > 1. \end{cases}$$

Stationarity Properties of $\tilde{\Phi}_i^k$

- Sufficient conditions for covariance stationarity:

$$\sum_{j=0}^{\infty} \left(\theta_{j,i}^k\right)^2 < 1, \qquad i = 1, 2, \ldots$$

- Necessary conditions:
  $\alpha^k < 1$, $d^k \leq 0.5$
- Sufficient condition for β_j^k: $\beta_j^k < 1$

An Alternative Specification for $\tilde{\Phi}_i^k$

- Idea: Exploiting the fact that $N(t) - \int_0^t \lambda(s)ds$ is a MD.
- Then: $(N^k(t_i) - N^k(t_{i-1})) - \int_{t_{i-1}}^{t_i} \lambda^k(s)ds$ is also a MD
- Alternative specification:

$$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K} \alpha_j^k (1 - \beta_j^k L)^{-1} (1 - L)^{-d^k} \varepsilon_{i-1}^j,$$

$$\varepsilon_i^j := (N^j(t_i) - N^j(t_{i-1})) - \Lambda^j(t_{i-1}, t_i)$$

Specification of the Baseline Intensity

- Baseline intensity function $\lambda_0^k\left[\eta^1(t),\ldots,\eta^K(t)\right]$:

$$\lambda_0^k[\cdot] = \prod_{r=1}^{K} \frac{p_r^k \eta^r(t)^{p_r^k-1}}{1+\kappa_r^k \eta^r(t)^{p_r^k}} \exp\left[\sum_{m=1}^{M} p_{m,r,s}^k \sin(2\cdot m\pi\nu^r(t))\right]$$
$$\times \exp\left[\sum_{m=1}^{M} p_{m,r,c}^k \cos(2\cdot m\pi\nu^r(t))\right], \quad p_r^k > 0,\ \kappa_r^k \geq 0,$$

where $\nu^k(t) := 1-\exp(-x^k(t))$.

6. Statistical Inference

Log Likelihood (Karr, 1991):

$$\begin{aligned}
&\ln \mathcal{L}(\theta; \{N(t)\}_{t\in(0,T]}) \\
&= \sum_{k=1}^{K} \left[\int_0^T (1-\lambda^k(s))ds + \int_{(0,T]} \ln \lambda^k(s) dN^k(s) \right], \\
&= \sum_{i=1}^{n} \sum_{k=1}^{K} (-\Lambda^k(t_{i-1}, t_i)) + \ln \left[\lambda^k(t_i) \right] y_i^k + KT.
\end{aligned}$$

Diagnostics based on three different types of model residuals:

$$\begin{aligned}
e_{i,1}^k &:= \hat{\Lambda}^k(t_{i-1}^k, t_i^k), \\
e_{i,2} &:= \sum_{k=1}^{K} \left(1 - \hat{\Lambda}^k(t_{i-1}^k, t_i^k)\right) y_i^k \\
e_{i,3} &:= 1 - \hat{\Lambda}(t_{i-1}, t_i) = 1 - \sum_{k=1}^{K} \hat{\Lambda}^k(t_{i-1}, t_i).
\end{aligned}$$

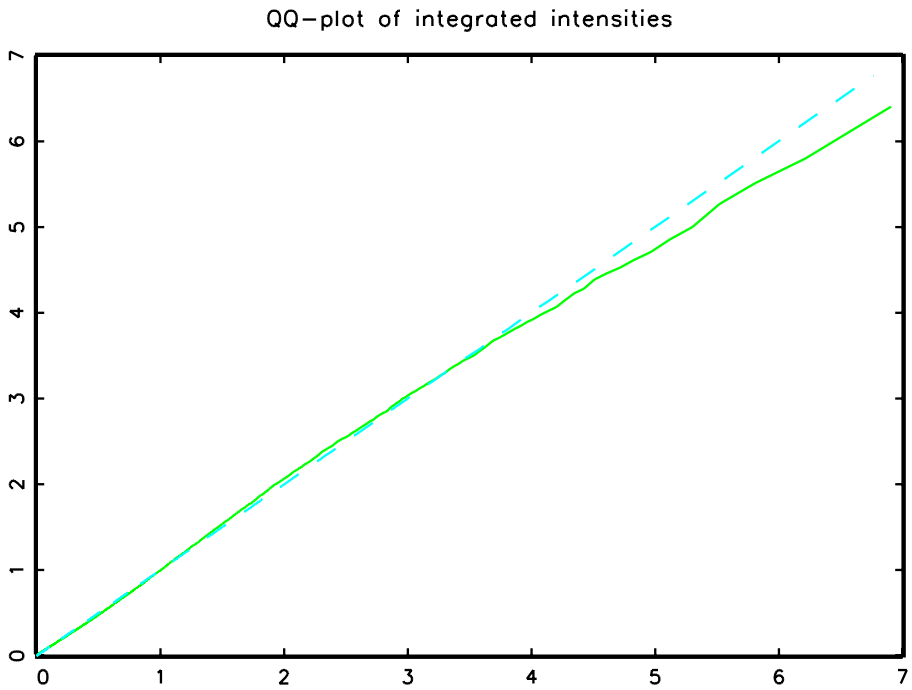
7. Empirical Evidence

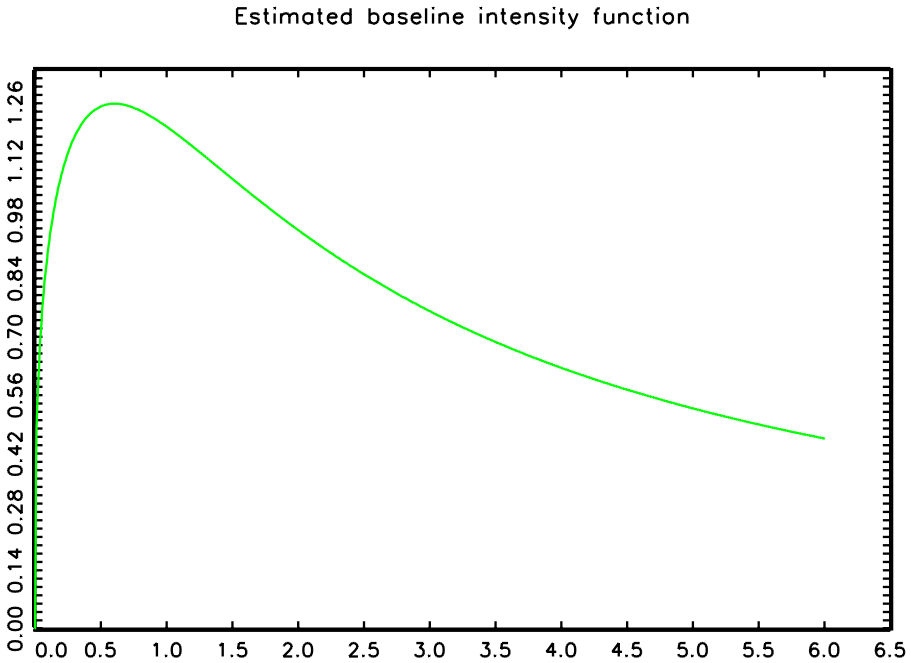
- Transaction data of the GE stock traded at the NYSE, 01-02/2003.
- Financial durations:
  - Trade durations: Time between consecutive transactions.
  - Price durations: Time between absolute cumulative midquote changes.
  - Volume durations: Time until a cumlative volume is traded.
  - Net volume durations: Time untile a cumulative net volume is traded.
- Overnight spells as well as all trades before 9:45 and after 16:00 are removed.
- Seasonality function: $s(t) = 1 + \sum_{j=1}^{6} \nu_j (t - \tau_j) \cdot \mathbb{1}_{\{t > \tau_j\}}$, where τ_j, $j = 1 \ldots, 6$, denote the 6 nodes within a trading day and ν_j the corresponding parameters.

Estimates of GLMACI models for price durations, GE stock, $\Delta dp \in \{0.01, 0.05\}$

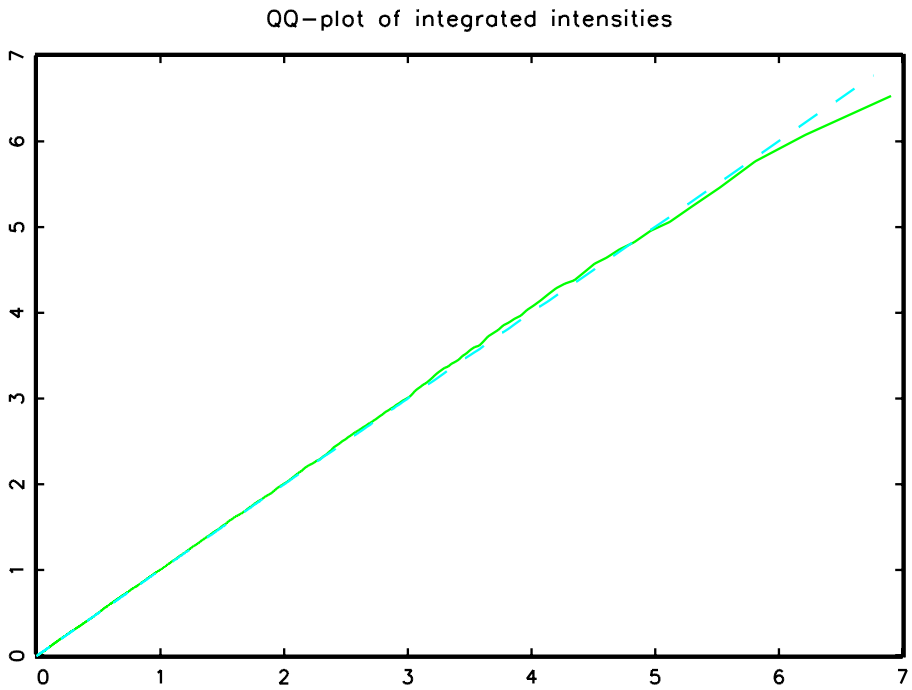
	$\Delta dp = 0.01$				$\Delta dp = 0.05$			
	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.
ω	0.665	0.000	2.215	0.0000	0.823	0.000	1.709	0.000
p	1.316	0.000	2.014	0.0000	1.376	0.000	1.917	0.000
κ	0.616	0.000	5.765	0.0309	0.727	0.000	2.716	0.011
$p_{1,s}$			-0.140	0.1462			0.065	0.445
$p_{2,s}$			-0.000	0.9950			0.074	0.005
$p_{3,s}$			0.024	0.1790			0.028	0.129
$p_{1,c}$			0.397	0.0000			0.315	0.000
$p_{2,c}$			0.158	0.0000			0.069	0.015
$p_{3,c}$			0.075	0.0000			0.008	0.666
δ			0.647	0.0000			0.752	0.000
α	0.057	0.000	0.111	0.0000	0.245	0.000	0.252	0.000
ς	0.010	0.063	0.031	0.0533	0.000	0.000	0.128	0.000
β	0.977	0.000	0.147	0.1295	0.866	0.000	0.717	0.000
d			0.558	0.0000			0.186	0.000
n	16122		16122		9504		9504	
LL	-15042		-14880		-8246		-8102	
BIC	-15071		-14948		-8274		-8166	
Mean e_i	1.000		1.006		1.002		1.002	
S.D. e_i	0.994		0.999		1.017		0.996	
LB(20) e_i	68.682	0.000	11.408	0.934	17.450	0.623	24.080	0.238
Exc. Disp. e_i	0.486	0.626	0.020	0.983	1.213	0.224	0.2213	0.8249
χ^2_{20} PIT	69.569	0.000	28.075	0.082	84.383	0.000	8.718	0.977
LB(20) PIT	77.041	0.000	20.660	0.417	48.002	0.000	26.018	0.165

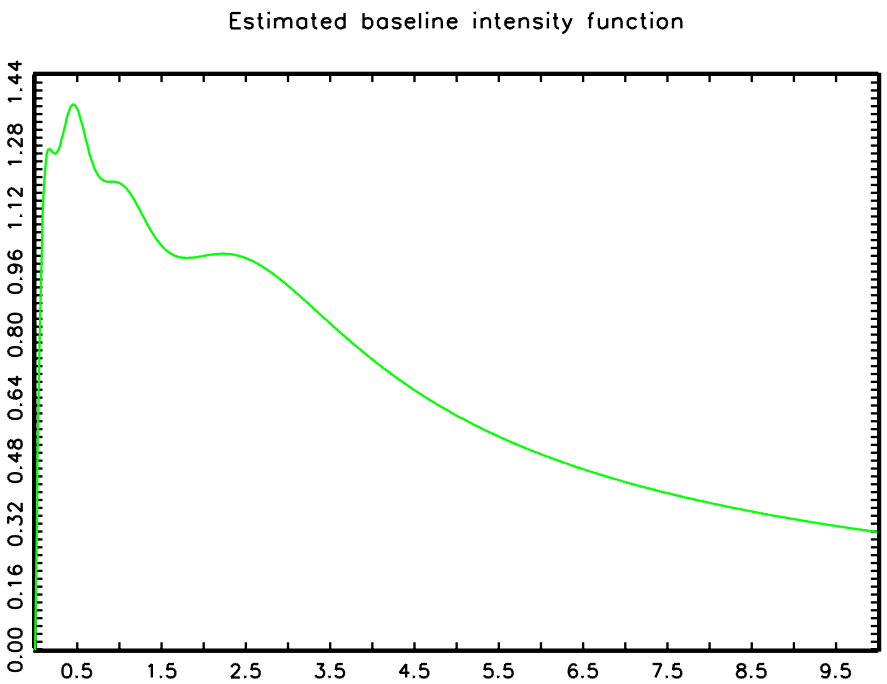
QQ-plot and baseline intensity, ACI model, price durations, $\Delta dp = 0.01$.



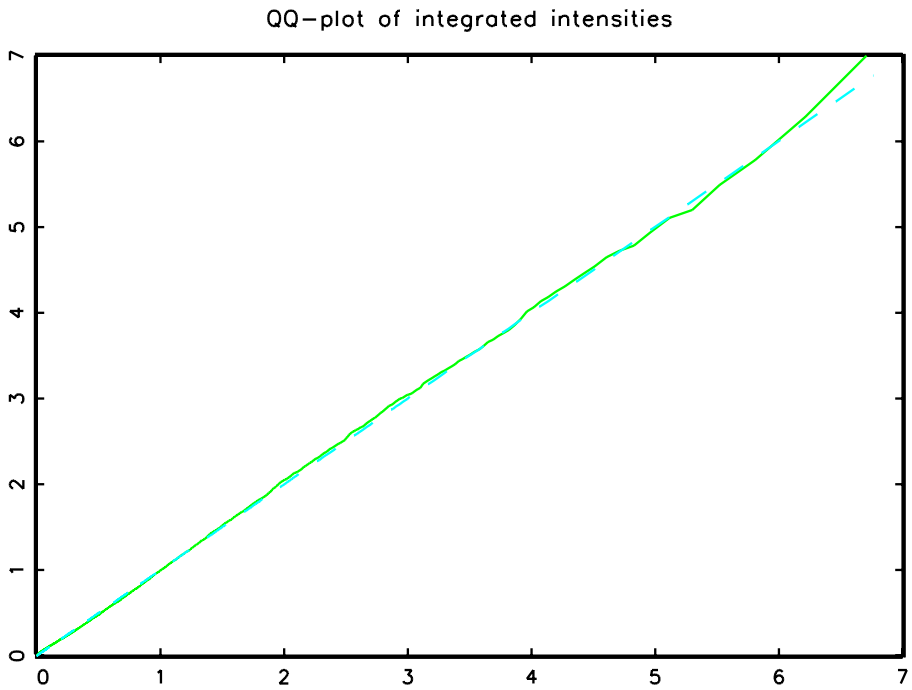


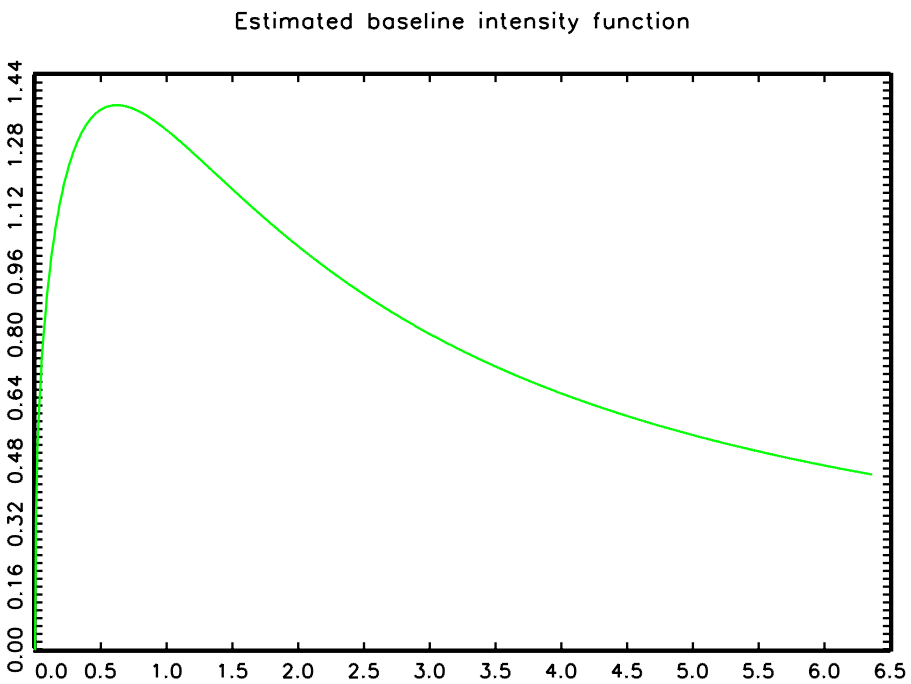
QQ-plot and baseline intensity, GLMACI model, price durations, $\Delta dp = 0.01$.



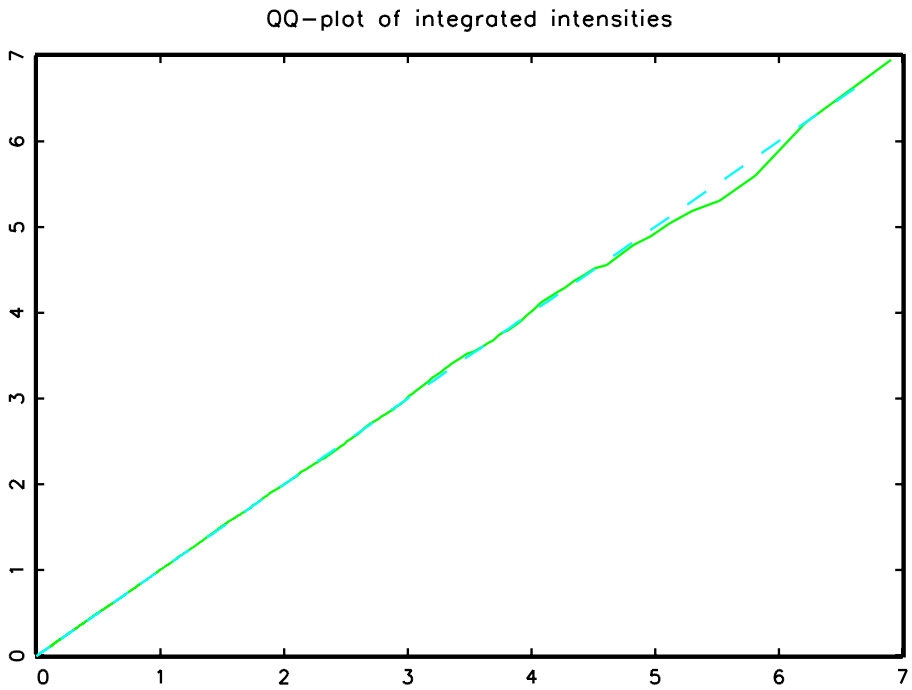


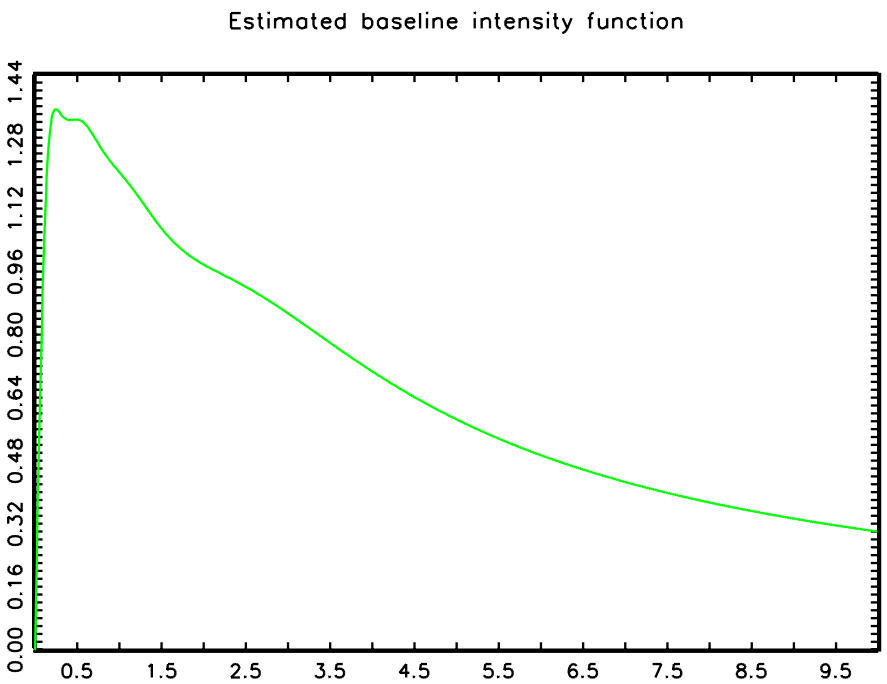
QQ-plot and baseline intensity, ACI model, price durations, $\Delta dp = 0.05$.





QQ-plots and baseline intensity, GLMACI model, price durations, $\Delta dp = 0.05$

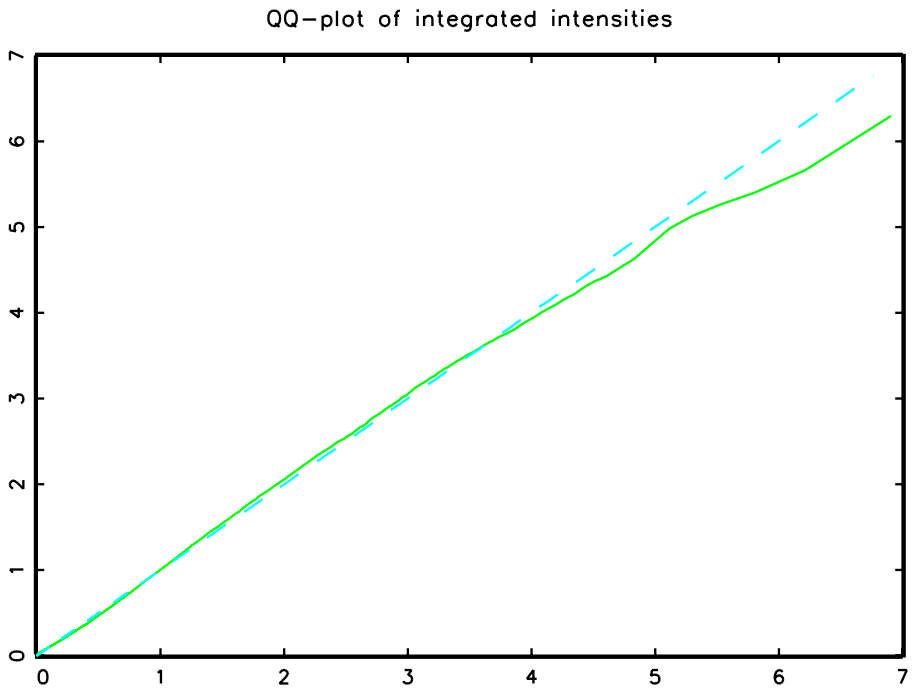


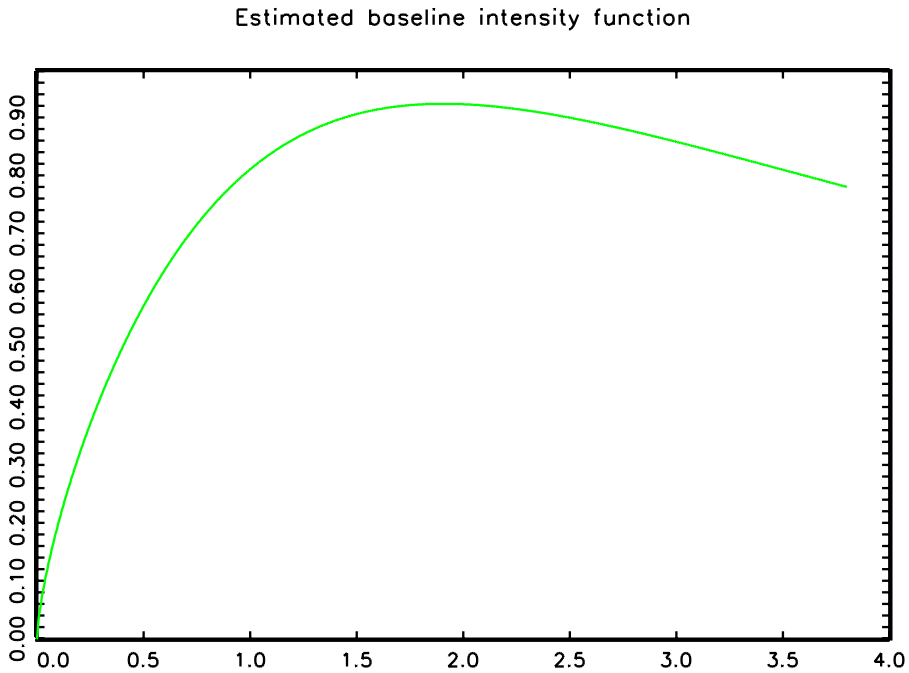


Estimates of GLMACI models for volume durations, GE stock $\Delta v \in \{5000, 10000\}$

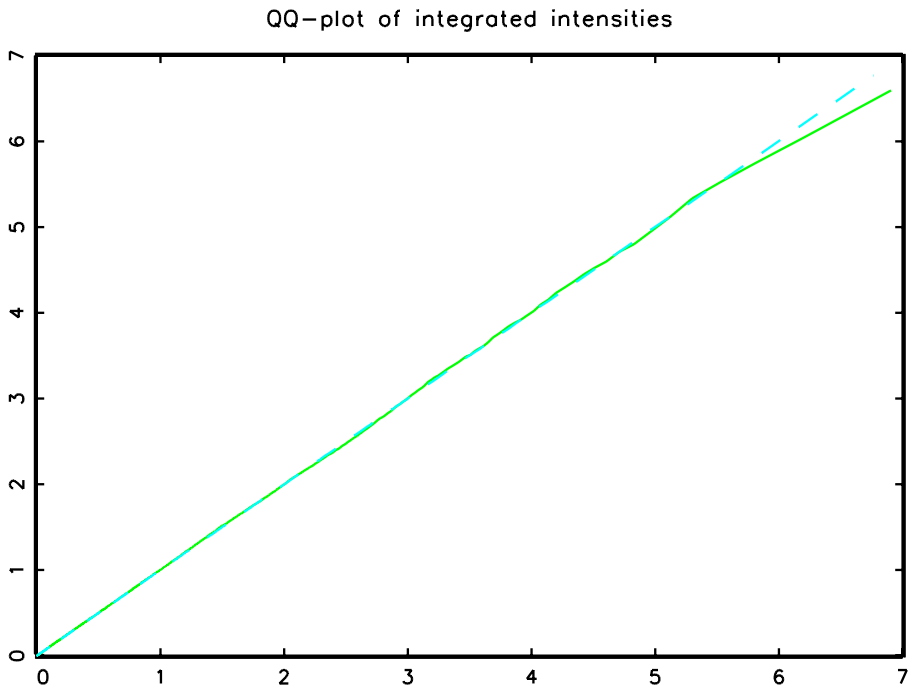
	$\Delta v = 5,000$				$\Delta v = 10,000$			
	Est.	p-v.	Est.	p-v.	Est.	p-v.	Est.	p-v.
ω	0.577	0.000	4.494	0.688	0.459	0.000	2.546	0.000
p	1.699	0.000	4.012	0.244	1.658	0.000	3.033	0.000
κ	0.233	0.000	43.024	34.775	0.076	0.000	4.347	0.253
$p_{1,s}$			-0.198	0.190			0.040	0.862
$p_{2,s}$			0.221	0.042			0.135	0.000
$p_{3,s}$			0.104	0.015			0.056	0.000
$p_{1,c}$			1.002	0.031			0.886	0.000
$p_{2,c}$			0.254	0.071			0.188	0.000
$p_{3,c}$			0.027	0.021			0.028	0.126
δ			0.127	0.050			0.304	0.002
α	0.044	0.000	0.112	0.008	0.069	0.000	0.126	0.000
ς	0.010	0.005	0.048	0.012	0.021	0.000	0.061	0.000
β	0.993	0.000	0.101	0.072	0.990	0.000	-0.061	0.456
d			0.648	0.000			0.675	0.000
n	25303		25303		16538		16538	
LL	-21465		-21135		-13492		-13352	
BIC	-21496		-21206		-13521		-13420	
Mean e_i	1.001		1.002		1.001		0.998	
S.D. e_i	1.000		0.998		1.015		0.997	
LB(20) e_i	110.559	0.000	16.728	0.670	105.922	0.000	25.681	0.176
Exc. Disp. e_i	0.065	0.947	0.178	0.858	1.383	0.166	0.260	0.794
χ^2_{20} PIT	227.959	0.000	15.601	0.683	63.272	0.000	15.322	0.701
LB(20) PIT	160.589	0.000	25.178	0.194	170.206	0.000	41.263	0.003

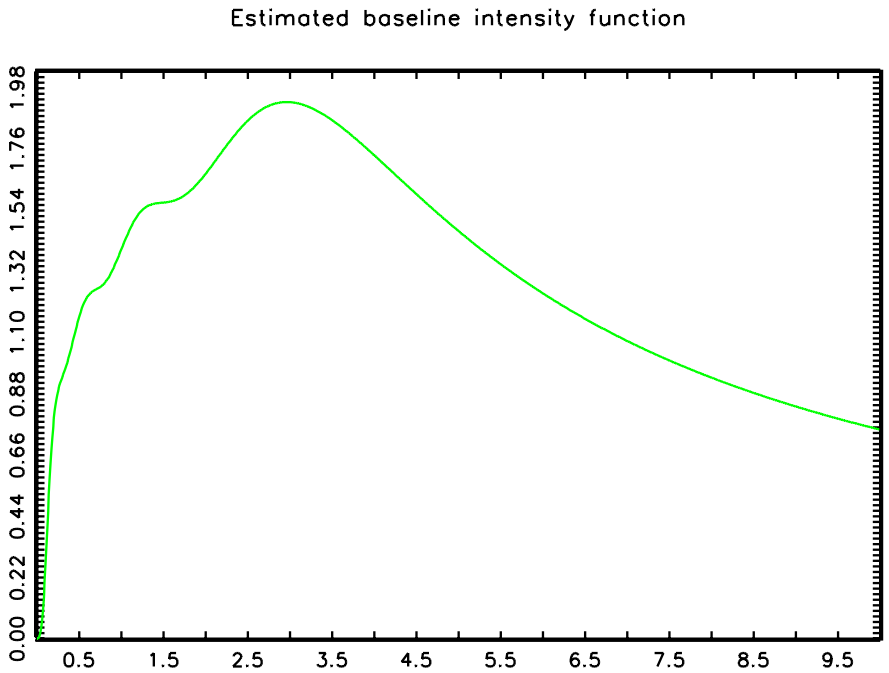
QQ-plot and baseline intensity, ACI model, volume durations, $\Delta v = 5,000$.



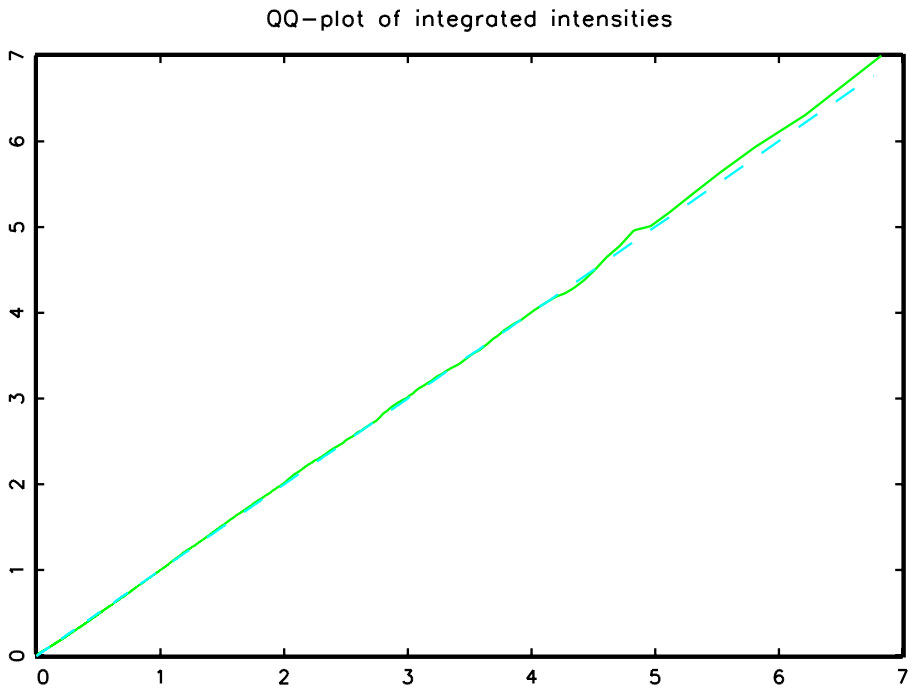


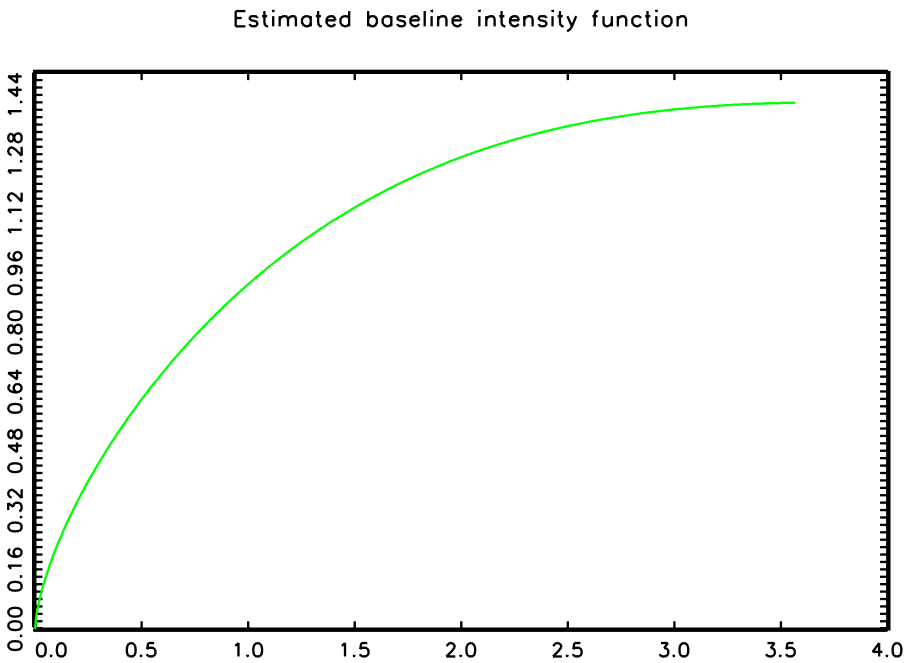
QQ-plot and baseline intensity, GLMACI model, volume durations, $\Delta v = 5,000$.



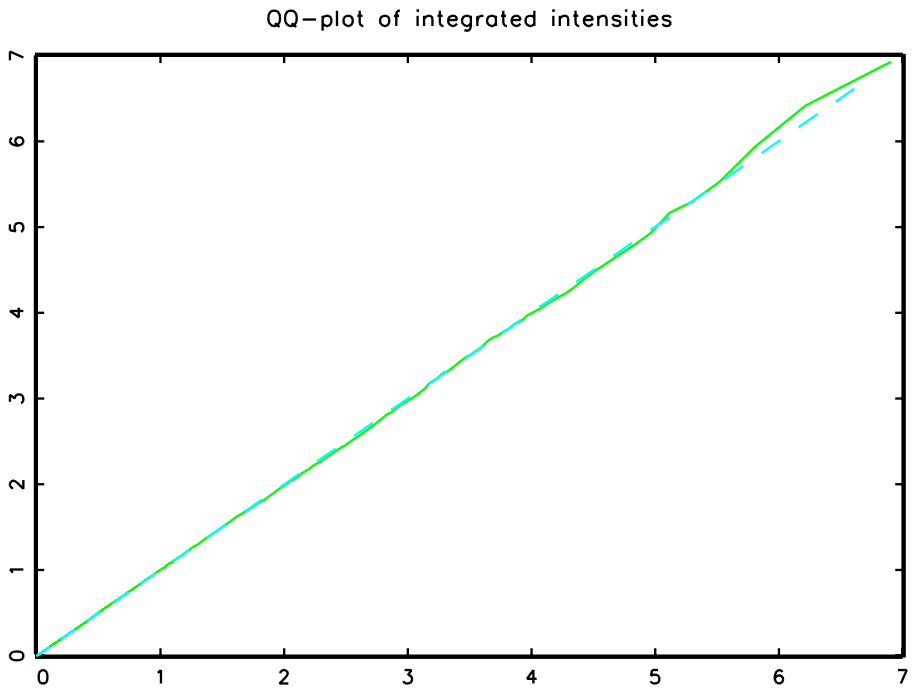


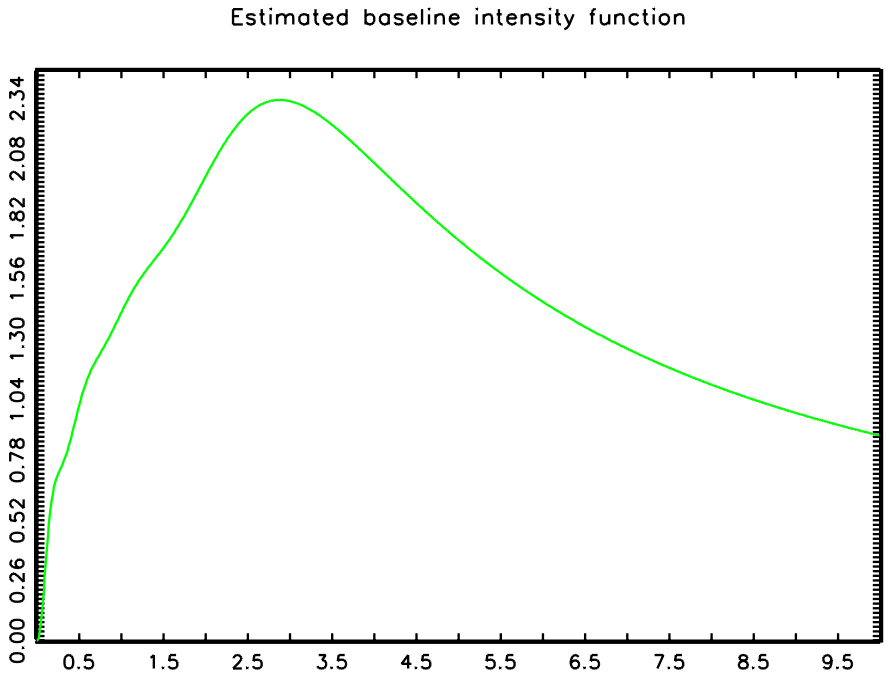
QQ-plot and baseline intensity, ACI model, volume durations, $\Delta v = 10,000$.



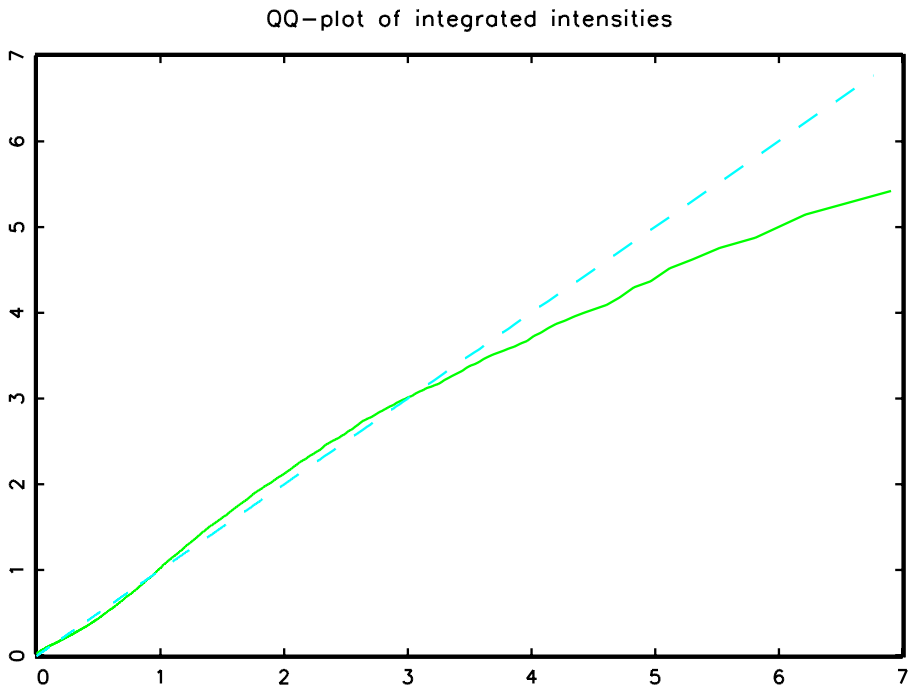


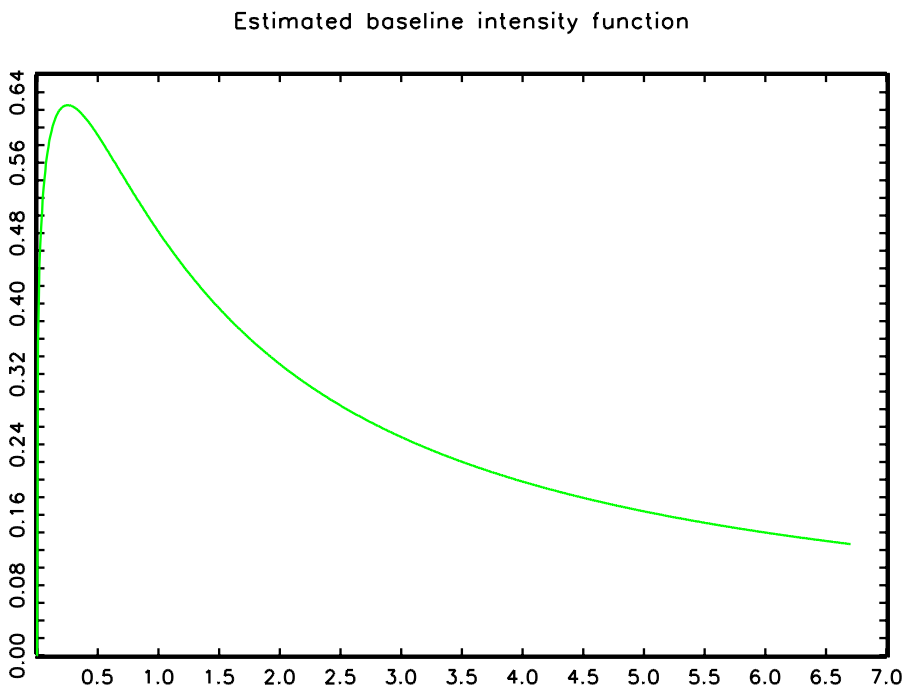
QQ-plot and baseline intensity, GLMACI model, volume durations, $\Delta v = 10,000$.



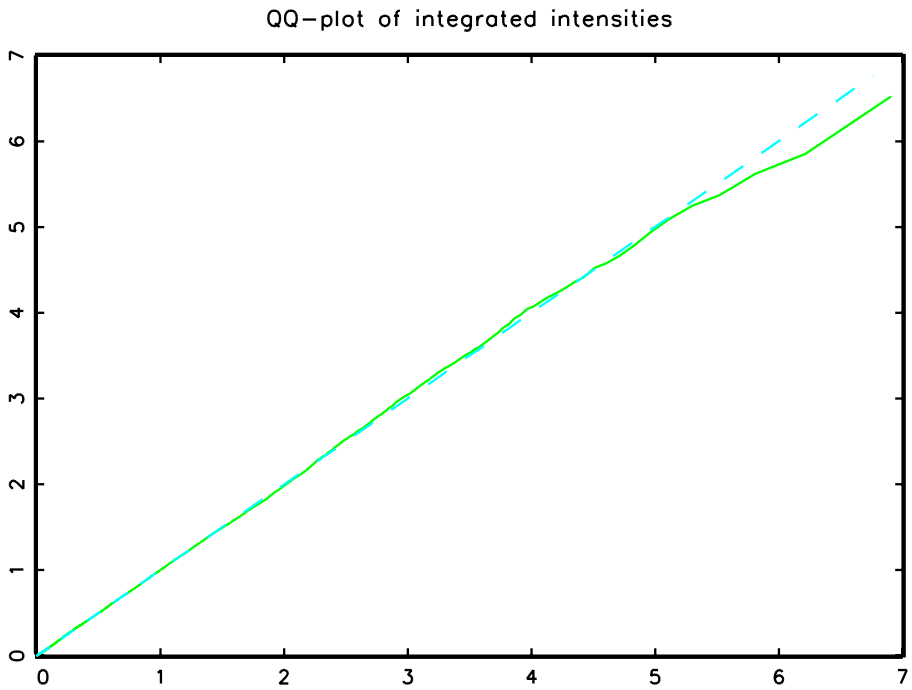


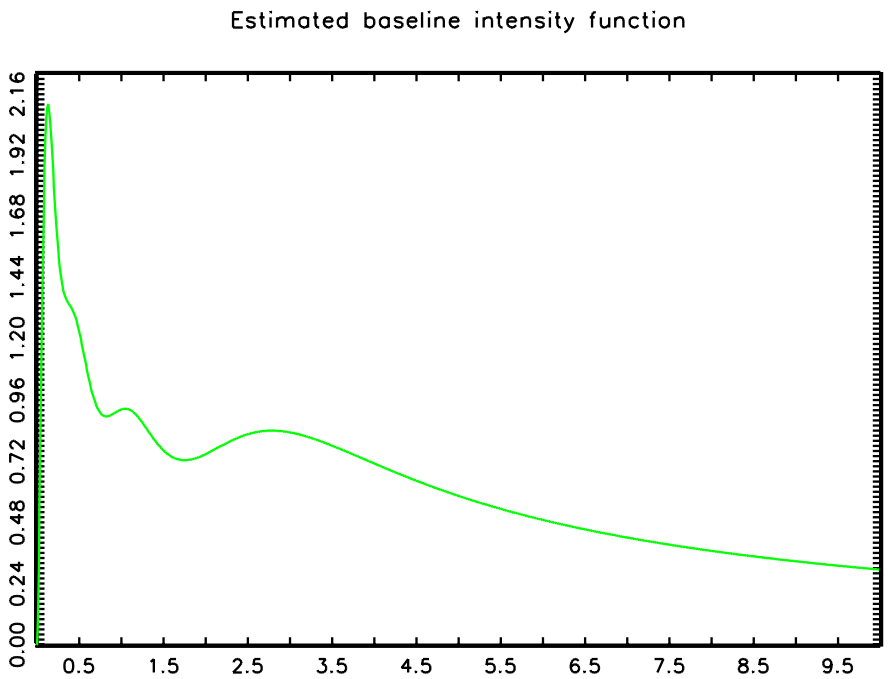
QQ-plots and baseline intensity, ACI model, net volume durations, $\Delta v = 5,000$





QQ-plots and baseline intensity, GLMACI model, net volume durations, $\Delta v = 5,000$





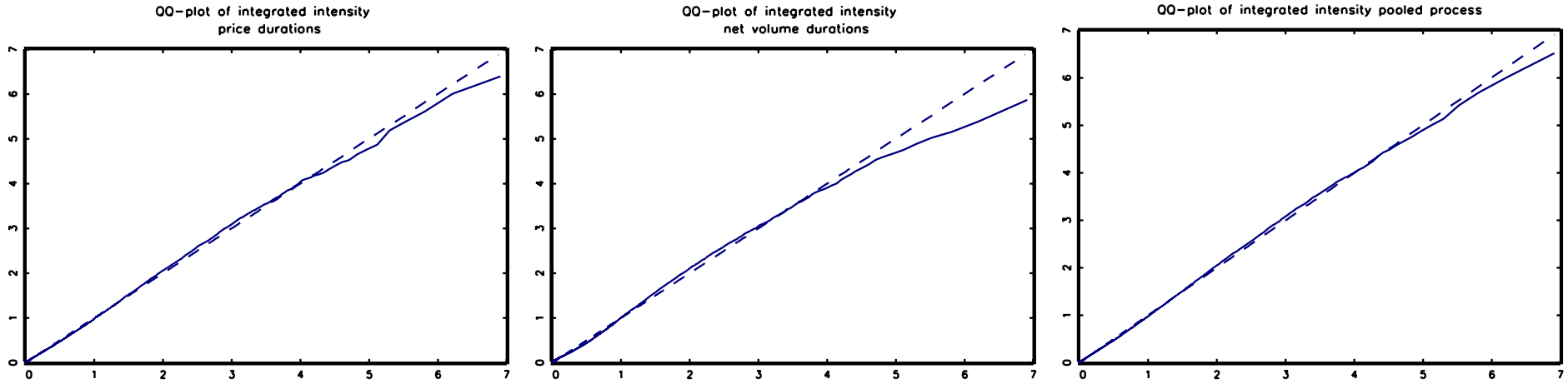
Simultaneous Modelling of Price Intensities and Net Volume Intensities

- Relationship between speed of absolute price changes and one-sided volume absorption.
- Studying the relationship between intraday volatility and one-sided liquidity demand.
  - Testing for Granger causalities.
  - Dynamic properties?
- Measuring realized market depth: $\lambda(t)^{dp}$-$\lambda(t)^{nv}$
  - Net volume per price change.
  - Extension of VNET measure by Engle and Lange (2001).
  - Dynamic properties?
  - Predictability?

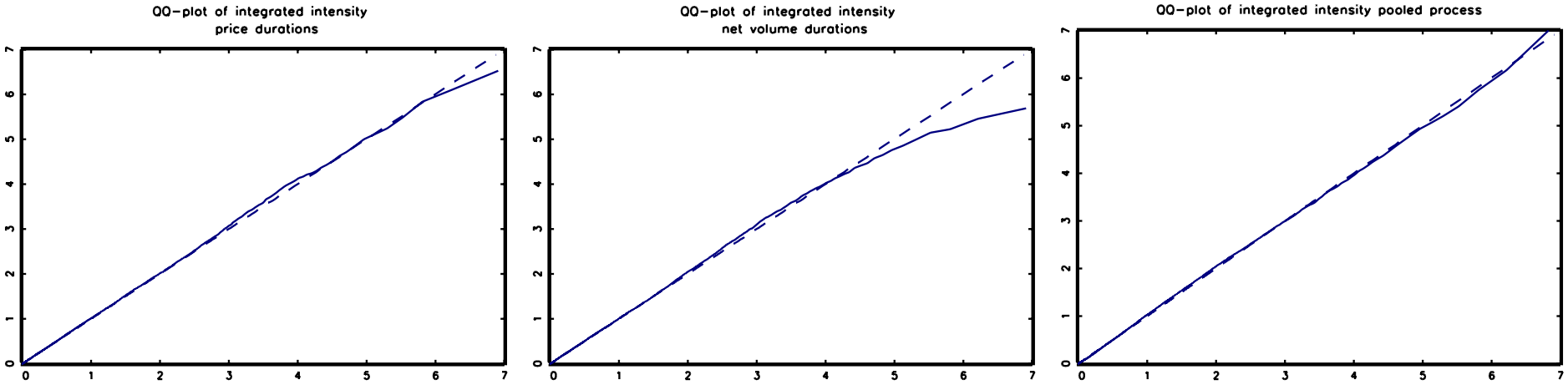
Estimates of Bivariate GLMACI models for price durations, $\Delta dp = 0.02$, and net volume durations $\Delta v = 10,000$

	ACI				GLMACI			
	Est.	p-v.			Est.	p-v.		
δ_1^1					0.811	0.000		
δ_2^2					0.346	0.000		
δ_2^1					0.024	0.804		
δ_1^2					0.216	0.000		
α_1^1	0.053	0.000			0.104	0.000		
α_1^2	0.000	0.876			0.022	0.042		
α_2^1	0.015	0.005			0.097	0.000		
α_2^2	0.079	0.000			0.118	0.000		
β_{11}	0.989	0.000			0.458	0.000		
β_{12}	-0.007	0.074			-0.478	0.000		
β_{21}	-0.002	0.465			-0.113	0.027		
β_{22}	0.956	0.000			0.430	0.000		
d^1					0.541	0.000		
d^2					0.496	0.000		
n	20259				20259			
LL	-31769				-31319			
BIC	-31918				-31557			
	$e_{i,1}^1$	$e_{i,1}^2$	$e_{i,2}$	$e_{i,3}$	$e_{i,1}^1$	$e_{i,1}^2$	$e_{i,2}$	$e_{i,3}$
Mean	1.000	0.999	1.000	1.000	1.010	1.007	1.008	1.008
S.D.	1.005	1.001	1.003	1.009	1.006	0.987	0.996	1.000
LB(20) e_i	38.334	15.733	33.892	51.836	16.786	14.120	19.332	42.596
Exc. Disp. e_i	0.351	0.085	0.303	0.964	0.4252	0.902	0.355	0.049
χ^2_{20} PIT	76.345	371.149	350.128	350.128	14.810	36.763	31.501	31.501
LB(20) PIT	104.650	53.482	108.090	112.615	41.665	45.459	46.772	46.209

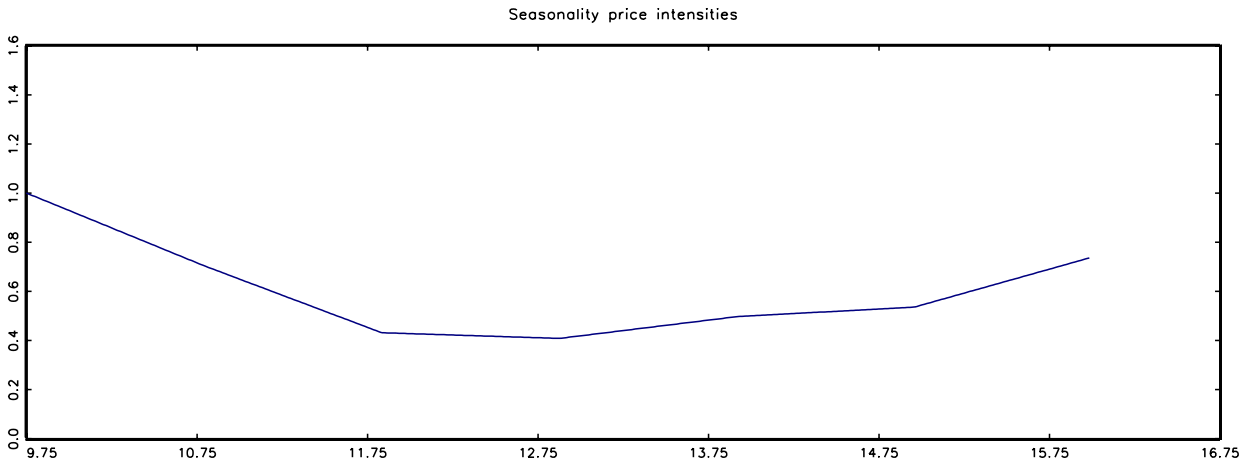
QQ-plots, bivariate basic ACI model

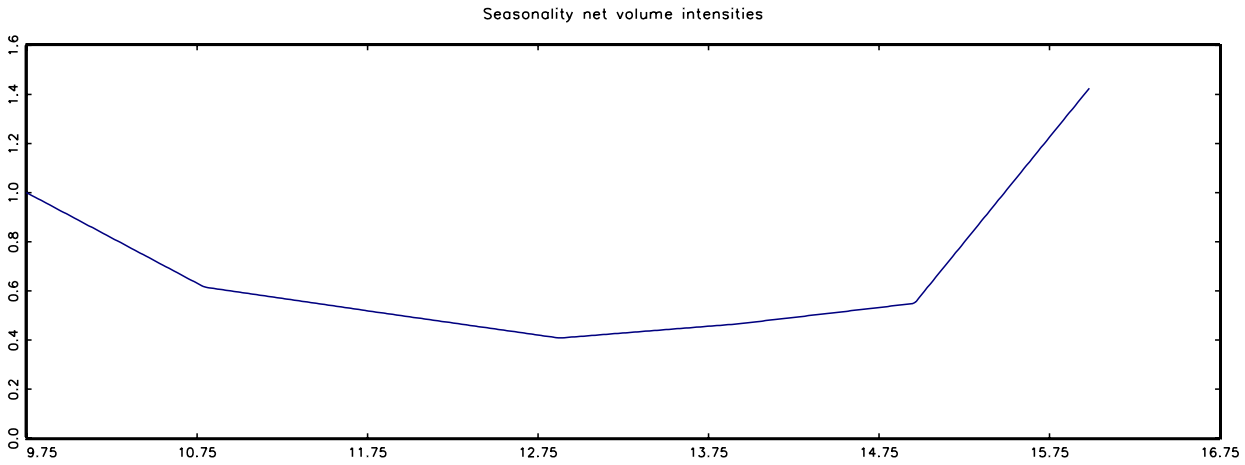


QQ-plots, bivariate GLMACI model

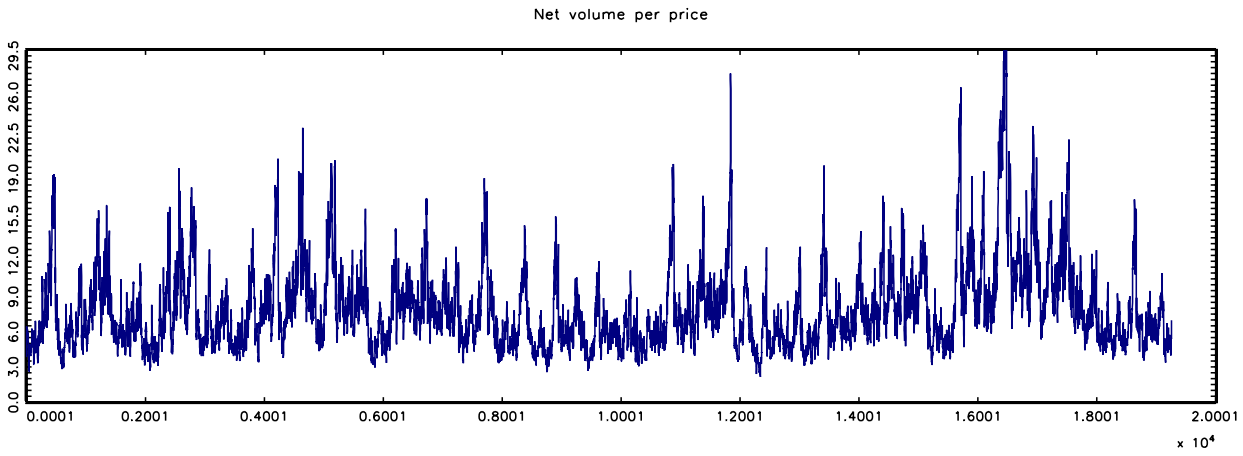


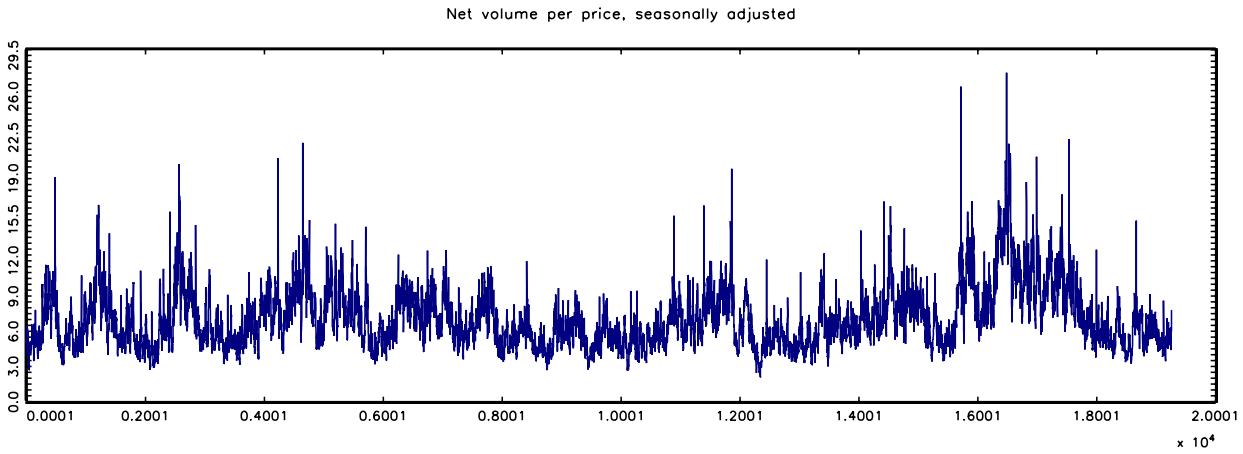
Estimated intraday seasonalities



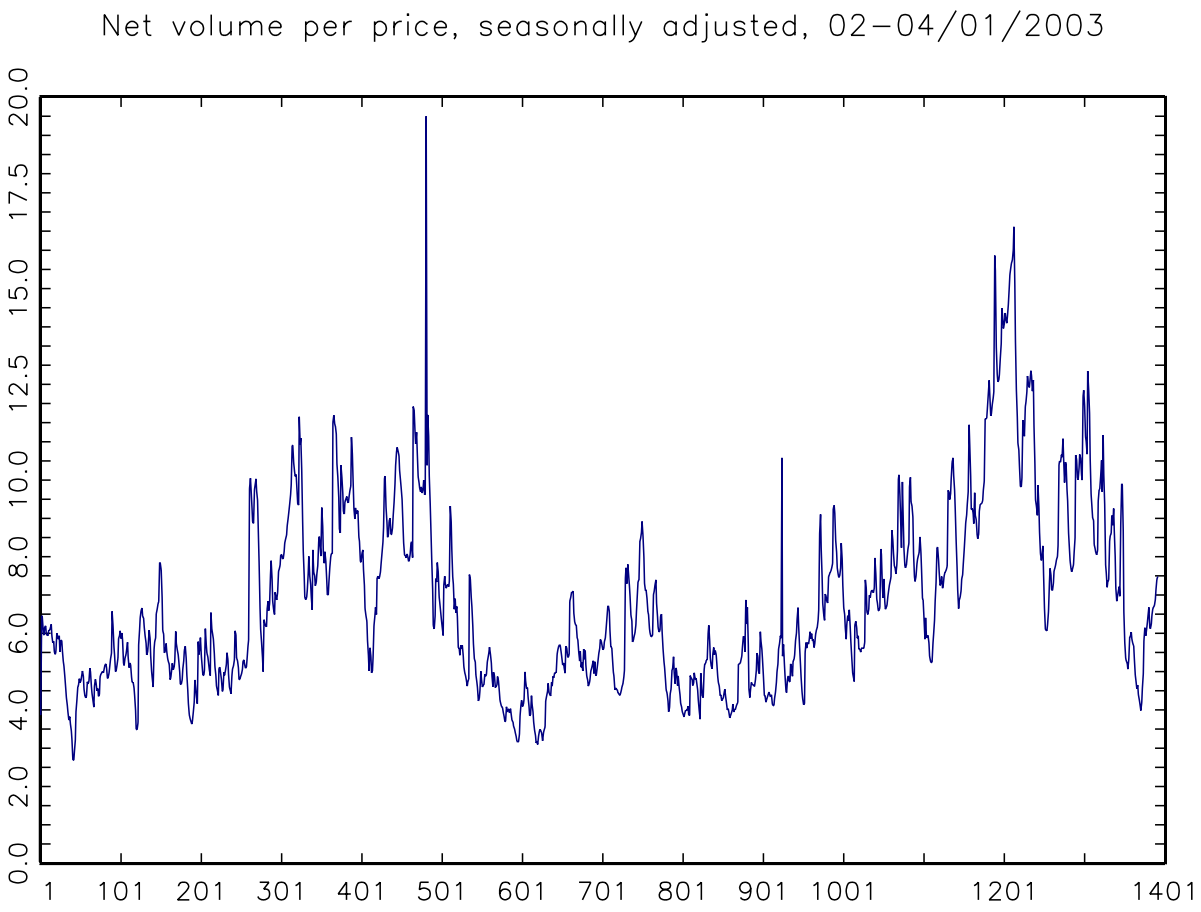


QQ-plots, Estimated realized market depth, $\lambda(t)^{dp}$-$\lambda(t)^{nv}$





QQ-plots, Estimated realized market depth, $\lambda(t)^{dp}$-$\lambda(t)^{nv}$



8. Conclusions

- Basic ACI model is not flexible enough.
- Accounting for long range dependence significantly improves the dynamical properties.
  - $\longrightarrow$ However, often $d \in (0.5, 0.6)$
  - $\Rightarrow$ No square-summability of coefficients θ_j.
- Semiparametric specification of baseline intensity significantly improves the goodness-of-fit.
- Clear evidence for acceleration effects. Cross-effects in bivariate model.
  - $\longrightarrow$ Proportional intensity specification rejected.