Generalized Autoregressive Conditional Intensity Models with Long Range Dependence

Nikolaus Hautsch

Klausurtagung SFB 649 ”Ökonomisches Risiko”

Motzen

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1. Introduction

- Modelling of financial data at the transaction level is an ongoing topic in the area of financial econometrics.
- New area of research: ”High-Frequency Econometrics”
- Issues in high-frequency econometrics:
  - Market microstructure investigations
  - Volatility estimation
  - Liquidity estimation
  - Trading models
1. Introduction

Statistical Properties of Financial Transaction Data

- Irregular Spacing in Time
- Discreteness of price changes
- Bid ask bounce
- Strong intraday seasonalities
- Clustering of the trading intensity
The Irregularity in Trade Arrivals
The Irregularity in Quote Arrivals
A multivariate point process
1. Introduction

Using the time information...

- to explicitly account for the irregular spacing of the data
- to analyze market microstructure relationships and order book dynamics: trade/quote intensities, buy/sell intensities, order aggressiveness
- for volatility measurement: time between cumulative absolute price changes
- for liquidity measurement:
  - time until a certain (buy or sell) buy volume is traded
  - time until a certain net order flow is observed
- to study financial markets at different time scales ("intrinsic time", Dacorogna et. al, 2001)
Financial durations - statistical properties

- positive serial dependence
- high persistence
- duration distributions imply non-monotonic hazard functions
- price durations reveal overdispersion and volume durations reveal underdispersion
- strong impact of intraday seasonality
Two Ways to Model Financial Durations

- Modelling inter-event waiting times (durations) $x_i = t_i - t_{i-1}$
  - Autoregressive Conditional Duration (ACD) (Engle/Russell, 1998)
  - Arises from GARCH literature
  - Accelerated Failure Time (AFT) model
  - not suitable for multivariate specifications and time-varying covariates

- Modelling the (stochastic) intensity

$$\lambda(t; \mathcal{F}_t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} \mathbb{E} \left[ N(t + \Delta) - N(t) | \mathcal{F}_t \right]$$

- Autoregressive Conditional Intensity (ACI) (Russell, 1999)
- Arises from point process literature or duration literature resp.
- Proportional Intensity (PI) model
- Continuous-time specification
Focus of this Paper

- Bringing together ACD and ACI models in a unified generalized ACI framework
- Relaxing the assumption of a proportional intensity model and allowing for generalized accelerated failure time specifications.
- Allowing for long range dependence in the intensity process
- Allowing for more distributional flexibility by proposing a semiparametric specification of the baseline intensity component.

⇒ The resulting model is called *Generalized Long Memory Autoregressive Conditional Intensity (GLMACI) Model.*
Outline of the Talk

1. Introduction
2. Long Range Dependence in Financial Durations
3. Autoregressive Conditional Duration
4. Univariate Generalized Long Memory ACI Models
5. Multivariate Generalized Long Memory ACI Models
6. Statistical Inference
7. Empirical Evidence
8. Conclusions
**Definition**: Let $X_t$ be a (weakly) stationary process with autocorrelation function of order $k$, $\rho(k)$. Then, $X_t$ implies long range dependence if

$$\lim_{k \to \infty} \frac{\rho(k)}{ck^{-\alpha}} = 1$$

with $\alpha \in (0, 1)$ and $c > 0$.

- in terms of the Hurst (self-similarity) parameter: $H = 1 - \alpha/2$
- for $\alpha \in (0, 1)$ resp. $H \in (0.5, 1)$: $\sum_{k=-\infty}^{\infty} \rho(k) = \infty$
- $\text{V} [\bar{x}_n] \approx cn^{-\alpha}$ with $c > 0$
2. Long Range Dependence in Financial Durations

\[ \log |\rho(k)| \text{ against } \log k, \text{ intertrade durations, AOL, NYSE} \]
2. Long Range Dependence in Financial Durations

\[ \log|\rho(k)| \] against \( \log k \), intertrade durations, Allianz, XETRA

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Allianz trade durations, XETRA

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Seasonal adjusted Allianz trade durations, XETRA

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2. Long Range Dependence in Financial Durations

$log |\rho(k)|$ against $\log k$, intertrade durations, BHP, ASX

![Graphs showing log(|\rho(k)|) against log(k) for BHP and ASX durations, with slopes indicated.](image)
log $|\rho(k)|$ against log $k$, $0.1$ price durations, IBM, NYSE
$V[\bar{X}_n]$ against $n$, intertrade durations, AOL, NYSE
$V[\bar{X}_n]$ against $n$, intertrade durations, Allianz, XETRA
\[ V[\tilde{X}_n] \] against \( n \), intertrade durations, BHP, ASX
$V[\tilde{X}_n]$ against $n$, $0.1$ price durations, IBM, NYSE
3. Autoregressive Conditional Duration

- **Inter-event duration**: \( x_i := t_i - t_{i-1} \).

- **Principle of the ACD model** (Engle/Russell, 1998)

\[
\varepsilon_i = \frac{x_i}{\Psi_i} \sim \text{i.i.d.},
\]

where \( \Psi_i := \Psi_i(\theta) = E[x_i | \mathcal{F}_{t_{i-1}}; \theta] \)

- **Basic specification**: \( \Psi_i = \omega + \sum_{j=1}^{P} \alpha_j x_{i-j} + \sum_{j=1}^{Q} \beta_j \Psi_{i-j} \),

where \( \omega > 0, \alpha \geq 0, \beta \geq 0 \).

- **Exponential ACD (EACD) model**: \( \varepsilon_i \sim \text{Exp}(1) \)
ARMA representation:

\[ x_i = \omega + \sum_{j=1}^{\max(P,Q)} (\alpha_j + \beta_j)x_{i-j} - \sum_{j=1}^{Q} \beta_j \eta_{i-j} + \eta_i, \]

where \( \eta_i := x_i - \Psi_i. \)

Intensity representation:

\[ \lambda(t; F_t) = \tilde{\lambda}_\varepsilon \left( \frac{x(t)}{\Psi \tilde{N}(t)+1} \right) \frac{1}{\Psi \tilde{N}(t)+1}, \]

where \( \tilde{\lambda}_\varepsilon(s) \) denotes the hazard function of \( \varepsilon_i \), \( \tilde{N}(t) \) denotes the left-continuous counting function and \( x(t) := t - t_{\tilde{N}(t)}. \)

\( \Rightarrow \) Accelerated Failure Time Model.
Specifications of $\Psi_i$

- Linear Model: $\Psi_i = \omega + \sum_{j=1}^{P} \alpha_j x_{i-j} + \sum_{j=1}^{Q} \beta_j \Psi_{i-j}$,

- Log ACD based on ACD innovations (Bauwens/Giot, 01):
  $\ln \Psi_i = \omega + \sum_{j=1}^{P} \alpha_j \varepsilon_{i-j} + \sum_{j=1}^{Q} \beta_j \ln \Psi_{i-j}$,

- Alternative:
  \[ \tilde{\varepsilon}_i = \Lambda(t_{i-1}, t_i) := \int_{t_{i-1}}^{t_i} \lambda(s) ds = \frac{1}{\Psi_i} \int_{t_{i-1}}^{t_i} \tilde{\lambda}_\varepsilon \left( \frac{x(s)}{\Psi_i} \right) \]

- Brown/Nair (88): $\Lambda(t_{i-1}, t_i) := \int_{t_{i-1}}^{t_i} \lambda(s) ds$ corresponds to the increment of a Poisson process.
The basic structure of the univariate GLMACI model is given by

\[ \lambda(t) = \lambda_0(\eta(t)) \Phi_{\tilde{N}(t)+1}s(t), \]

\[ \eta(t) := x(t) \cdot \left[ \Phi_{\tilde{N}(t)+1}s(t) \right]^\delta. \]

- \( \Phi_{\tilde{N}(t)} \) is a function capturing the model dynamics as well as possible covariates.
- \( s(t) \) is a (deterministic) function of time capturing possible seasonality effects.
- \( \lambda_0(\cdot) \) denotes a baseline intensity function in terms of \( \eta(t) \).
- \( \eta(t) \) corresponds to the backward recurrence time \( x(t) := t - t_{\tilde{N}(t)} \) scaled by \( \Phi \) and \( s(t) \).
Parameterization of $\Phi_i$: A Fractionally Integrated Process

- $\Phi_i = \exp \left( \tilde{\Phi}_i + z'_{i-1} \gamma \right)$.
- Innovation term:

$$\varepsilon_i := 1 - \Lambda(t_{i-1}, t_i)$$

with $\Lambda(t_{i-1}, t_i) \sim \text{i.i.d. } \text{Exp}(1)$.

- Infinite series representation (Koulikov 03, Hosking, 1981):

$$\tilde{\Phi}_i = \omega + \alpha (1 - \beta L)^{-1} (1 - L)^{-d} \varepsilon_{i-1} \quad \text{with } |\beta| < 1$$

$$\alpha (1 - \beta L)^{-1} (1 - L)^{-d} \varepsilon_{i-1} = \alpha \sum_{j=1}^{\infty} \theta_{j-1} \varepsilon_{i-j},$$

$$\theta_j := \sum_{k=0}^{j} \alpha \beta^k \theta^*_j - k, \quad \theta^*_j := \frac{\Gamma(d + j)}{\Gamma(d) \Gamma(1 + j)} \quad \forall j \geq 0$$
Stationarity Properties of $\tilde{\Phi}_i$

Adapting the results by Koulikov (2003) for MD-ARCH($\infty$) processes:

- **Sufficient condition for covariance stationarity:**
  \[
  \sum_{j=0}^{\infty} \theta_j^2 < 1,
  \]

- **Necessary conditions:**
  - $\alpha < 1$, $\beta < 1$
  - $d < 0.5$.

- **Non-summable autocorrelation functions for $d \in (0, 1)$**.
4. Univariate GLMACI Models

Special Cases

- $d = 0$ and $\delta = 0$

- Basic ACI(1,1) specification (Russell, 99):

$$\tilde{\Phi}_i = \omega + \alpha (1 - \beta L)^{-1} \varepsilon_{i-1}$$

$$= \omega + \alpha \varepsilon_{i-1} + \beta (\tilde{\Phi}_{i-1} - \omega)$$
Special Cases - cntd.

- If $\lambda_0(\cdot)$ is non-specified, and $\delta = 1$, $\omega = \alpha = \beta = 0$: AFT model (Kalbfleisch/Prentice, 1980) with
  \[
  \lambda(t) = \lambda_0 \left[ x(t) \exp(z_{\tilde{N}}(t)'\gamma) \right] \exp(z_{\tilde{N}}(t)'\gamma).
  \]
  Then: \[\ln x_i = -z_{i-1}'\gamma + \xi_i\]

- If $\lambda_0(\cdot)$ is non-specified, and $\delta = \omega = \alpha = \beta = 0$: Semiparametric PI model (Cox, 1972) with
  \[
  \lambda(t) = \lambda_0 [x(t)] \exp(z_{\tilde{N}}(t)'\gamma).
  \]
4. Univariate GLMACI Models

Special Cases - cntd.

- If $\lambda_0(\cdot) = 1$: Long memory Log-ACD model with
  \[
  \Phi_i = E[x_i|\mathcal{F}_{t_{i-1}}]^{-1} := \gamma_i^{-1}
  \]
  and
  \[
  \gamma_i = \exp \left( \tilde{\gamma}_i - z_{i-1}' \gamma \right)
  \]
  \[
  \tilde{\gamma}_i = -\omega + \alpha (1 - \beta L)^{-1} (1 - L)^{-d} (x_{i-1}/\gamma_{i-1} - 1),
  \]

- If $\lambda_0(\cdot) = 1$ and $d = 0$: Log-ACD model with
  \[
  \tilde{\gamma}_i = -\omega + \alpha (x_{i-1}/\gamma_{i-1} - 1) + \beta (\tilde{\gamma}_{i-1} - \omega)
  \]
4. Univariate GLMACI Models

Special Cases - cntd.

- $\lambda_0(\eta(t)) = p \cdot \eta(t)^p - 1 (1 + \kappa \eta(t)^p)^{-1}$ and $\delta = 1$:
  Burr type (long memory) ACD model with

  $\Phi_i = E[x_i | \mathcal{F}_{t-1}]^{-1} \left[ \frac{\kappa^{1+1/p} \Gamma(1 + 1/\kappa)}{\Gamma(1 + 1/p) \Gamma(\kappa^{-1} - 1/a)} \right] := \gamma_i,$

  where $\tilde{\gamma}_i = -\omega - \alpha (1 - \beta L)^{-1} (1 - L)^{-d} \varepsilon_{i-1}.$

  $\Rightarrow$ If $d = 0$: Burr-ACD model with

  $\tilde{\gamma}_i = -\omega - \alpha \varepsilon_{i-1} + \beta (\tilde{\gamma}_{i-1} - \omega),$

  where $\varepsilon_i = 1 - \Lambda(t_{i-1}, t_i)$ equals the integrated intensity.
Extensions of the Basic Univariate GLMACI Model

Component-Specific Acceleration Effects

Motivation: Accounting for the multiplicative nature of the model. Acceleration effects might be driven by individual components.

We allow for component-specific acceleration effects by re-formulating the basic model as

\[ \eta(t) := x(t) \cdot \Phi^{\delta_\Phi} N(t+1) s(t)^{\delta_s}, \]

where \( \delta_\Phi \) and \( \delta_s \) are specific acceleration parameters affecting the individual components separately.

Acceleration/deceleration effects might be driven individually by intraday dynamics and seasonality effects.

Basic model for \( \delta_\Phi = \delta_s = \delta \).
Flexible Baseline Intensities

Motivation: Burr distribution is good starting point for financial duration processes (Grammig/Maurer, 00) but often not sufficient (Bauwens et al, 04, Bauwens/Hautsch, 06).

Define $\nu(t) := 1 - \exp(-x(t))$ with $\nu(t) \in [0; 1]$.

Idea: Augmentation of a Burr parameterization by a flexible Fourier form. Then,

$$\lambda_0(\eta(t)) = \frac{p \cdot \eta(t)^{p-1}}{1 + \kappa \eta(t)^p}$$

$$\times \exp \left[ \sum_{m=1}^{M} p_{m,s} \sin(2m\pi\nu(t)) + p_{m,c} \cos(2m\pi\nu(t)) \right],$$

where $M$ denotes the order of the process and $p_{m,s}$ and $p_{m,c}$ are coefficients.
Asymmetric News Response

Motivation: Recent literature on financial duration models illustrates presence of nonlinear news impact functions (Dufour/Engle, 01, Fernandes/Grammig, 05, or Hautsch, 06.)

Following Nelson (91) we can re-specify the process $\tilde{\Phi}_i$ as

$$
\tilde{\Phi}_i = \omega + \alpha (1 - \beta L)^{-1} (1 - L)^{-d} \varepsilon_{i-1} + \varsigma (1 - \beta L)^{-1} (1 - L)^{-d} \bar{\varepsilon}_{i-1},
$$

where $\bar{\varepsilon}_i := |\varepsilon_i| - E[|\varepsilon_i|]$ and $\varsigma$ denotes the asymmetry parameter.
5. Multivariate GLMACI Models

- $K$-variate marked point process with arrival times $\{t_i^k\}_{i \in \{1, \ldots, n_k\}}$, and $N_k(t) := \sum_{i \geq 1} \mathbb{1}_{\{t_i^k \leq t\}}$, $k = 1, \ldots, K$.

- $k$-type intensity: $E[N_k(s) - N_k(t) | \mathcal{F}_t] = E[\int_t^s \lambda_k(u) du | \mathcal{F}_t]$ with compensator $\tilde{\Lambda}_k(t) := \int_0^t \lambda_k(u) du$.

- The processes of compensators $\left(\tilde{\Lambda}_1(t), \tilde{\Lambda}_2(t), \ldots, \tilde{\Lambda}_K(t)\right)$ are independent Poisson processes with unit intensity (Brown/Nair, 1988).

- The $k$-type integrated intensities $\Lambda_k(t_i^k, t_{i-1}^k) := \int_{t_{i-1}^k}^{t_i^k} \lambda_k(s) ds$ $\forall k = 1, \ldots, K$ are independently standard exponentially distributed.
The multivariate GLMACI model is given by

$$\lambda^k(t) = \lambda_0^k \left[ \eta^1(t), \ldots, \eta^K(t) \right] \Phi_{\tilde{N}(t)+1}^k s^k(t),$$

where

$$\eta^k(t) := \begin{cases} \eta^k(t_{\tilde{N}(t)}) + x(t) \cdot \Xi(t) & \text{if } t_{\tilde{N}(t)} \text{ was of type } r \neq k \\ x(t) \cdot \Xi(t) & \text{if } t_{\tilde{N}(t)} \text{ was of type } k \end{cases}$$

and

$$\Xi(t) := \prod_{r=1}^{K} \left( \Phi_{\tilde{N}(t)+1}^r \right)^{\delta^k_{r,\Phi}} (s^r(t))^{\delta^k_{r,s}}.$$

Hence, the acceleration/deceleration effects act piecewise on the backward recurrence times.
5. Multivariate GLMACI Models

Specification of $\Phi_i^k$

$$\Phi_i^k = \exp \left( \tilde{\Phi}_i^k + z_{i-1}^T \gamma^k \right)$$

$$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K} \left\{ \alpha_j^k (1 - \beta_j^k L)^{-1} (1 - L)^{-d_k} \varepsilon_{i-1} \right\} y_{i-1}^j,$$

$$\varepsilon_i = \sum_{k=1}^{K} \left( 1 - \Lambda_k^k (t_{i-1}^k, t_i^k) \right) y_i^k$$

Then:  $$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K} \left\{ \alpha_j^k \sum_{s=1}^{\infty} \theta_{s-1,i}^k \varepsilon_{i-s} \right\} y_{i-1}^j,$$

$$\theta_{s,i}^k := \begin{cases} \theta_{s}^{k*} & \text{if } s = 1 \\ \theta_{s}^{k*} + \sum_{m=1}^{s} \theta_{s-m}^{k*} \prod_{r=0}^{m-1} \sum_{j=1}^{K} \beta_{j}^k y_{i-r}^j & \text{if } s > 1. \end{cases}$$
Stationarity Properties of $\tilde{\Phi}^k_i$

- **Sufficient conditions for covariance stationarity:**
  \[
  \sum_{j=0}^{\infty} \left( \theta_{j,i}^k \right)^2 < 1, \quad i = 1, 2, \ldots
  \]

- **Necessary conditions:**
  \[\alpha^k < 1, \quad d^k \leq 0.5\]

- **Sufficient condition for $\beta^k_j$:**
  $\beta^k_j < 1$
5. Multivariate GLMACI Models

An Alternative Specification for $\tilde{\Phi}_i^k$

- **Idea:** Exploiting the fact that $N(t) - \int_0^t \lambda(s)ds$ is a MD.

- Then: $(N^k(t_i) - N^k(t_{i-1})) - \int_{t_{i-1}}^{t_i} \lambda^k(s)ds$ is also a MD

- Alternative specification:

$$\tilde{\Phi}_i^k = \omega^k + \sum_{j=1}^{K} \alpha_j^k (1 - \beta_j^k L)^{-1} (1 - L)^{-d_j^k} \varepsilon_i^j, \quad \varepsilon_i^j := (N^j(t_i) - N^j(t_{i-1})) - \Lambda^j(t_{i-1}, t_i)$$
Specification of the Baseline Intensity

Baseline intensity function $\lambda^k_0 [\eta^1(t), \ldots, \eta^K(t)]:$

$$\lambda^k_0 [\cdot] = \prod_{r=1}^{K} \frac{p^k_r \eta^r(t) p^k_r - 1}{1 + \kappa^k_r \eta^r(t) p^k_r} \exp \left[ \sum_{m=1}^{M} p^k_{m,r,s} \sin(2 \cdot m \pi \nu^r(t)) \right] \times \exp \left[ \sum_{m=1}^{M} p^k_{m,r,c} \cos(2 \cdot m \pi \nu^r(t)) \right], \quad p^k_r > 0, \kappa^k_r \geq 0,$$

where $\nu^k(t) := 1 - \exp(-x^k(t))$. 
Log Likelihood (Karr, 1991):

\[
\ln \mathcal{L}(\theta; \{N(t)\}_{t \in (0, T]}) = \sum_{k=1}^{K} \left[ \int_{0}^{T} (1 - \lambda^k(s)) ds + \int_{(0, T]} \ln \lambda^k(s) dN^k(s) \right],
\]

\[
= \sum_{i=1}^{n} \sum_{k=1}^{K} (1 - \hat{\Lambda}^k(t_{i-1}, t_i)) + \ln \left[ \lambda^k(t_i) \right] y_i^k + KT.
\]

Diagnostics based on three different types of model residuals:

\[
e_{i,1} := \hat{\Lambda}^k(t_{i-1}, t_i),
\]

\[
e_{i,2} := \sum_{k=1}^{K} \left( 1 - \hat{\Lambda}^k(t_{i-1}, t_i) \right) y_i^k
\]

\[
e_{i,3} := 1 - \hat{\Lambda}(t_{i-1}, t_i) = 1 - \sum_{k=1}^{K} \hat{\Lambda}^k(t_{i-1}, t_i).
\]
7. Empirical Evidence

- Transaction data of the GE stock traded at the NYSE, 01-02/2003.

- Financial durations:
  - Trade durations: Time between consecutive transactions.
  - Price durations: Time between absolute cumulative midquote changes.
  - Volume durations: Time until a cumulative volume is traded.
  - Net volume durations: Time until a cumulative net volume is traded.

- Overnight spells as well as all trades before 9:45 and after 16:00 are removed.

- Seasonality function: 
  
  \[ s(t) = 1 + \sum_{j=1}^{6} \nu_j (t - \tau_j) \cdot 1_{\{t > \tau_j\}} \]

  where \( \tau_j \), \( j = 1 \ldots 6 \), denote the 6 nodes within a trading day and \( \nu_j \) the corresponding parameters.
Estimates of GLMACI models for price durations, GE stock, $\Delta dp \in \{0.01, 0.05\}$

<table>
<thead>
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<th>$\Delta dp = 0.01$</th>
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<th>$\Delta dp = 0.05$</th>
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<tr>
<td>$\chi^2_{20}$ PIT</td>
<td>69.569</td>
<td>0.000</td>
<td>28.075</td>
<td>0.082</td>
<td>84.383</td>
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<tr>
<td>LB(20) PIT</td>
<td>77.041</td>
<td>0.000</td>
<td>20.660</td>
<td>0.417</td>
<td>48.002</td>
<td>0.000</td>
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</table>
QQ-plot and baseline intensity, ACI model, price durations, $\Delta dp = 0.01$. 
QQ-plot and baseline intensity, GLMACI model, price durations, $\Delta dp = 0.01$. 

![QQ-plot of integrated intensities](image1)

![Estimated baseline intensity function](image2)
QQ-plot and baseline intensity, ACI model, price durations, \( \Delta dp = 0.05 \).
QQ-plots and baseline intensity, GLMACI model, price durations, $\Delta dp = 0.05$
Estimates of GLMACI models for volume durations, GE stock $\Delta v \in \{5000, 10000\}$

<table>
<thead>
<tr>
<th></th>
<th>$\Delta v = 5,000$</th>
<th></th>
<th>$\Delta v = 10,000$</th>
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<tbody>
<tr>
<td>$\omega$</td>
<td>0.577 0.000</td>
<td>4.494 0.688</td>
<td>0.459 0.000</td>
<td>2.546 0.000</td>
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</tr>
<tr>
<td>$p$</td>
<td>1.699 0.000</td>
<td>4.012 0.244</td>
<td>1.658 0.000</td>
<td>3.033 0.000</td>
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<tr>
<td>$\kappa$</td>
<td>0.233 0.000</td>
<td>43.024 34.775</td>
<td>0.076 0.000</td>
<td>4.347 0.253</td>
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<tr>
<td>$p_{1,s}$</td>
<td>-0.198 0.190</td>
<td>0.221 0.042</td>
<td>0.040 0.862</td>
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</tr>
<tr>
<td>$p_{2,s}$</td>
<td>0.221 0.042</td>
<td>0.104 0.015</td>
<td>0.135 0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{3,s}$</td>
<td>1.002 0.031</td>
<td>0.254 0.071</td>
<td>0.056 0.000</td>
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<td></td>
</tr>
<tr>
<td>$p_{1,c}$</td>
<td>0.104 0.015</td>
<td>0.221 0.042</td>
<td>0.135 0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{2,c}$</td>
<td>1.002 0.031</td>
<td>0.027 0.021</td>
<td>0.056 0.000</td>
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<tr>
<td>$p_{3,c}$</td>
<td>0.221 0.042</td>
<td>0.027 0.021</td>
<td>0.135 0.000</td>
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<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.127 0.050</td>
<td>0.048 0.012</td>
<td>0.126 0.000</td>
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<tr>
<td>$\alpha$</td>
<td>0.044 0.000</td>
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<td>0.069 0.000</td>
<td>0.126 0.000</td>
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<tr>
<td>$\zeta$</td>
<td>0.010 0.005</td>
<td>0.048 0.012</td>
<td>0.021 0.000</td>
<td>0.061 0.000</td>
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<tr>
<td>$\beta$</td>
<td>0.993 0.000</td>
<td>0.101 0.072</td>
<td>0.990 0.000</td>
<td>-0.061 0.456</td>
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<tr>
<td>$d$</td>
<td>0.648 0.000</td>
<td>0.648 0.000</td>
<td>0.675 0.000</td>
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<tr>
<td>$n$</td>
<td>25303</td>
<td>25303</td>
<td>16538</td>
<td>16538</td>
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<tr>
<td>LL</td>
<td>-21465</td>
<td>-21135</td>
<td>-13492</td>
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<tr>
<td>BIC</td>
<td>-21496</td>
<td>-21206</td>
<td>-13521</td>
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<tr>
<td>Mean $e_i$</td>
<td>1.001</td>
<td>1.002</td>
<td>1.001</td>
<td>0.998</td>
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<tr>
<td>S.D. $e_i$</td>
<td>1.000</td>
<td>0.998</td>
<td>1.015</td>
<td>0.997</td>
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<tr>
<td>LB(20) $e_i$</td>
<td>110.559 0.000</td>
<td>16.728 0.670</td>
<td>105.922 0.000</td>
<td>25.681 0.176</td>
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<tr>
<td>Exc. Disp. $e_i$</td>
<td>0.065 0.947</td>
<td>0.178 0.858</td>
<td>1.383 0.166</td>
<td>0.260 0.794</td>
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<tr>
<td>$\chi^2_{20}$</td>
<td>227.959 0.000</td>
<td>15.601 0.683</td>
<td>63.272 0.000</td>
<td>15.322 0.701</td>
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<tr>
<td>LB(20) PIT</td>
<td>160.589 0.000</td>
<td>25.178 0.194</td>
<td>170.206 0.000</td>
<td>41.263 0.003</td>
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</table>
QQ-plot and baseline intensity, ACI model, volume durations, $\Delta v = 5,000$. 
QQ-plot and baseline intensity, GLMACI model, volume durations, $\Delta v = 5,000$. 
QQ-plot and baseline intensity, ACI model, volume durations, $\Delta \nu = 10,000$. 
QQ-plot and baseline intensity, GLMACI model, volume durations, $\Delta v = 10,000$. 
QQ-plots and baseline intensity, ACI model, net volume durations, $\Delta v = 5,000$
7. Empirical Evidence

QQ-plots and baseline intensity, GLMACI model, net volume durations, $\Delta v = 5,000$
Simultaneous Modelling of Price Intensities and Net Volume Intensities

- Relationship between speed of absolute price changes and one-sided volume absorption.
- Studying the relationship between intraday volatility and one-sided liquidity demand.
  - Testing for Granger causalities.
  - Dynamic properties?
- Measuring realized market depth: $\lambda(t)^{dp} - \lambda(t)^{nv}$
  - Net volume per price change.
  - Dynamic properties?
  - Predictability?
Estimates of Bivariate GLMACI models for price durations, $\Delta dp = 0.02$, and net volume durations $\Delta v = 10,000$

<table>
<thead>
<tr>
<th></th>
<th>ACI</th>
<th>GLMACI</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>p-v.</td>
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<tr>
<td>$\delta_1$</td>
<td>0.811</td>
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<tr>
<td>$\delta_2$</td>
<td>0.346</td>
<td>0.000</td>
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<tr>
<td>$\delta_1^*$</td>
<td>0.024</td>
<td>0.804</td>
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<tr>
<td>$\delta_2^*$</td>
<td>0.216</td>
<td>0.000</td>
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<tr>
<td>$\alpha_1$</td>
<td>0.053</td>
<td>0.000</td>
</tr>
<tr>
<td>$\alpha_2^*$</td>
<td>0.004</td>
<td>0.876</td>
</tr>
<tr>
<td>$\alpha_1^*$</td>
<td>0.015</td>
<td>0.005</td>
</tr>
<tr>
<td>$\alpha_2^*$</td>
<td>0.079</td>
<td>0.000</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.989</td>
<td>0.000</td>
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<tr>
<td>$\beta_{12}$</td>
<td>-0.007</td>
<td>0.074</td>
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<tr>
<td>$\beta_{21}$</td>
<td>-0.002</td>
<td>0.465</td>
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<td>$\beta_{22}$</td>
<td>0.956</td>
<td>0.000</td>
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<tr>
<td>$\delta_1^<em>^</em>$</td>
<td>0.541</td>
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<tr>
<td>$\delta_2^<em>^</em>$</td>
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<td>0.000</td>
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<tr>
<td>$n$</td>
<td>20259</td>
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<tr>
<td>LL</td>
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<td></td>
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<tr>
<td>BIC</td>
<td>-31918</td>
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<tr>
<td>LB(20) $e_i$</td>
<td>38.334</td>
<td>15.733</td>
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<tr>
<td>Exc. Disp. $e_i$</td>
<td>0.351</td>
<td>0.085</td>
</tr>
<tr>
<td>$\chi^2_{20}$ PIT</td>
<td>76.345</td>
<td>371.149</td>
</tr>
<tr>
<td>LB(20) PIT</td>
<td>104.650</td>
<td>53.482</td>
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</tbody>
</table>

Mean

S.D.
QQ-plots, bivariate basic ACI model
QQ-plots, bivariate GLMACI model
Estimated intraday seasonalities

Seasonality price intensities

Seasonality net volume intensities
QQ-plots, Estimated realized market depth, $\lambda(t)^{dp} - \lambda(t)^{nv}$
QQ-plots, Estimated realized market depth, $\lambda(t)^{dp} - \lambda(t)^{nv}$
8. Conclusions

- Basic ACI model is not flexible enough.
- Accounting for long range dependence significantly improves the dynamical properties.
  - However, often $d \in (0.5, 0.6)$
  - No square-summability of coefficients $\theta_j$.
- Semiparametric specification of baseline intensity significantly improves the goodness-of-fit.
- Clear evidence for acceleration effects. Cross-effects in bivariate model.
  - Proportional intensity specification rejected.