The Natural Rate Hypothesis (NRH)

"There is always a temporary trade-off between inflation and employment; **there is no permanent trade-off**." (Friedman, 1968)

"One cannot call for demand to be higher than average on average." (Lucas, 1972)

"On average, output should be equal to potential output, **for any monetary policy.**" (McCallum, 1998)

Money and Inflation

"There is perhaps no empirical regularity among economic phenomena that is based on so much evidence for so wide a range of circumstances as the connection between substantial changes in the stock of money and in the level of prices." (Friedman 1958)

"[M]onetary restraint is a necessary and sufficient condition for controlling inflation." (Milton Friedman, quoted in Nelson and Schwartz, 2008)

A Natural Rate Perspective on Equilibrium Selection and Monetary Policy

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June 3, 2010
Motzen, Jahrestagung des SFB 649

Preview: Result I

I show the determinacy bounds on monetary policy to be

- identical for all supply equations that satisfy Lucas's (1972) NRH
- identical to those for nom. determinacy in the assoc. frictionless model
- independent of parameters outside MP with standard dyn. IS demand

As a consequence:

Questions the correctness of the determinacy literature.

- No specific knowledge of supply side beyond NRH is necessary to assess whether a particular monetary policy will ensure determinacy

Current (NK) determinacy analyses dubious as their recommendations

- are inextricably linked to their failure to fulfill NRH!

▸ Example: Expected Inflation Targeting

Preview: Result II

In the context of the class of models that satisfy the NRH, I

- confirm Cochrane's (2007) criticism of determinacy as arbitrary
- and provide a concrete reason for why: explosive nominal paths are valid as they are accommodated by explosive money growth!
- Commitment of MP to stable money growth essential.

As a consequence:

Questions the relevancy of the determinacy literature.

- Nominal interest rate is a misleading indicator of monetary stance:
  - "Taylor principle" is a MP commitment to wildly inflationary or deflationary money creation should all but one equil. come to pass
  - "Stabilizing" logic of interest rate rules "old-Keynesian" logic.
- Monetary policy must mention money
  - E.g. threat of "monetarist experiment" ensures only the stable nom. path is valid

Outline

Literature Overview

Linking the NRH, the Long Run, and Determinacy

Determinacy and the Long Run

Determinacy:

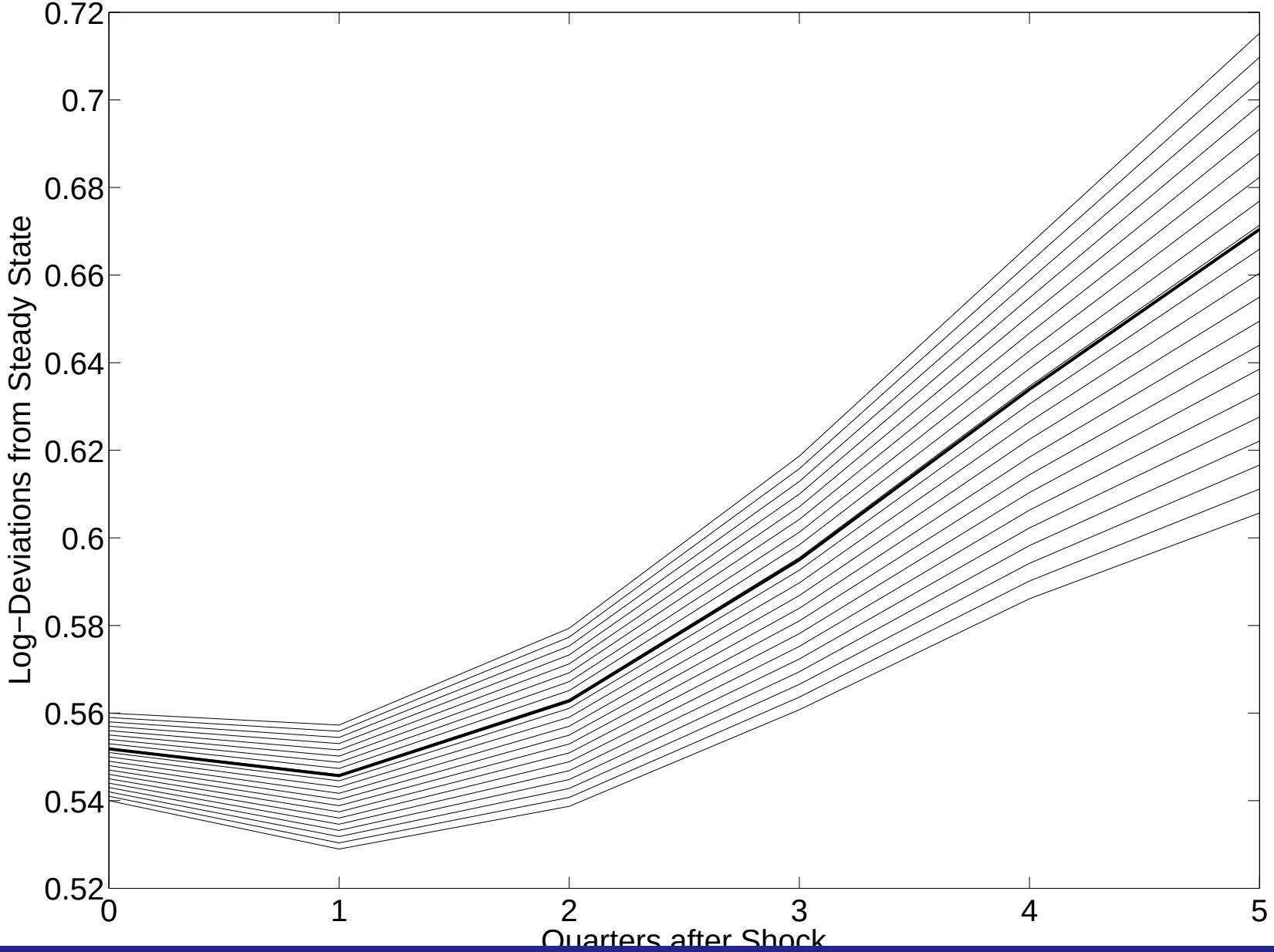
- all equilibrium paths but one diverge/explode/become unbounded.
- ascertained by Blanchard and Kahn's (1980) eigenvalue counting.

Roughly speaking, a model is brought into first-order form

$$E_t\left[G_{t+1}\right] = HG_t \tag{1}$$

where some elements of G_t might be predetermined,

- determinacy if the number of stable eigenvalues in H is exactly equal to the number of predetermined variables in G_t.
- Thus, the instantaneous reaction of G_t to some disturbance is sufficient to ascertain whether some equilibrium path will lead to explosive or stable behavior in G_t.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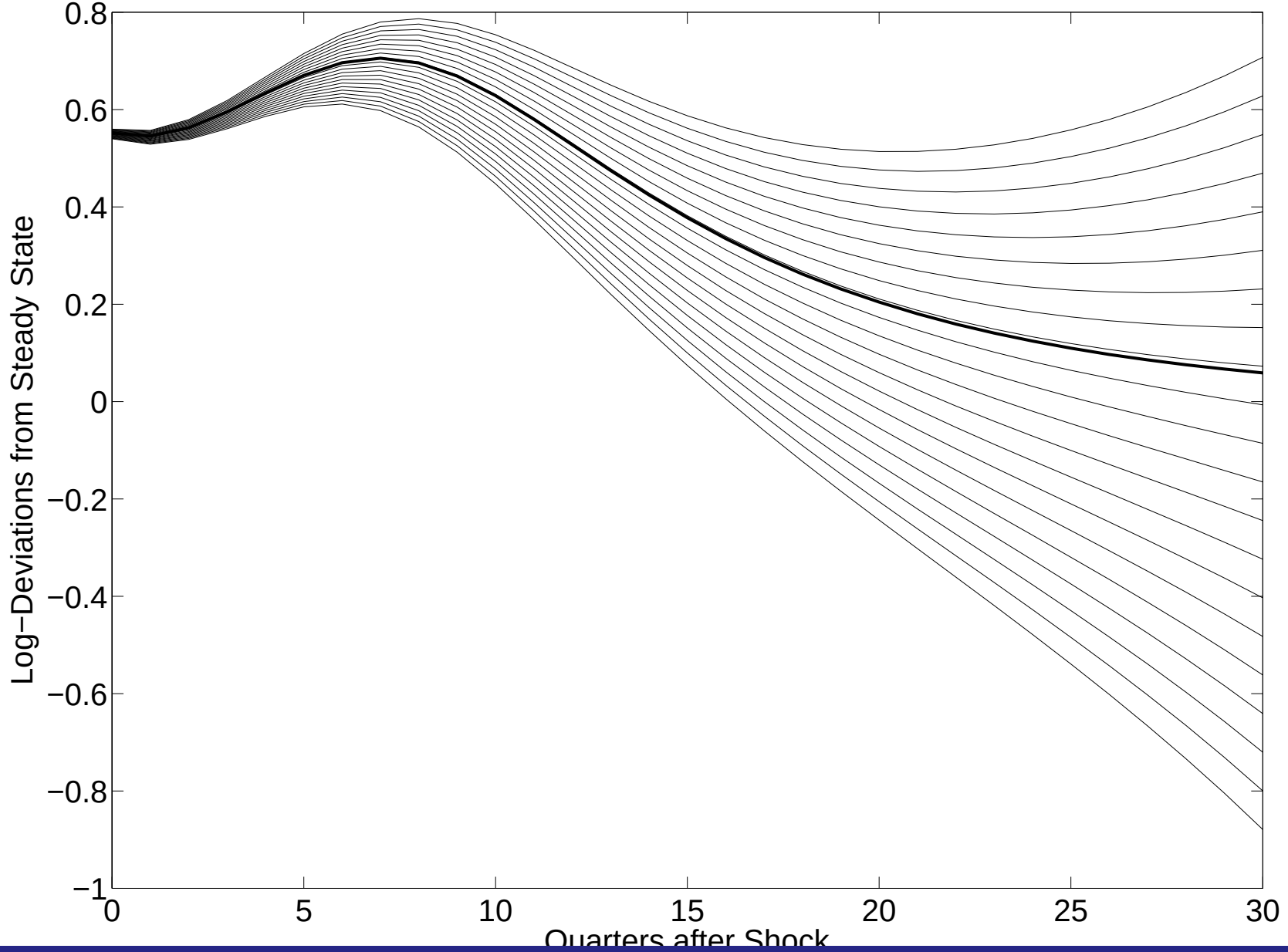
Quarters after Shock


0.8
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−0.6
−0.8
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Log−Deviations from Steady State
0
5
10
15
20
25
30
Quarters after Shock

Link between the NRH, the Long Run, and Determinacy

Determinacy means all equilibrium paths but one diverge/explode/become unbounded.

- Determinacy: but a path only actually explodes in the long-run
- and NRH: imposes only long-run restrictions
  - a stable short-run Phillips curve not contradictory

Hence, as

- It is the long-run behavior that counts for determinacy
- And all NRH supply-sides have same long-run characteristics

Determinacy should be the same for all NRH supply-sides.

The Natural Rate Hypothesis and Standard NK Models

Standard NK Phillips curve

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t \tag{2}$$

Even if $\beta = 1$, $E[y_t] \neq 0$ if π_t non-stationary.
But $E[y_t] = 0$ needs to hold "**for any monetary policy.**"

Indexation, either to steady-state inflation,

$$\pi_t - \bar{\pi} = \beta E_t [\pi_{t+1} - \bar{\pi}] + \kappa y_t \tag{3}$$

or past inflation

$$\pi_t = \frac{\gamma}{1+\gamma\beta}\pi_{t-1} + \frac{\beta}{1+\gamma\beta}E_t[\pi_{t+1}] + \frac{\kappa}{1+\gamma\beta}y_t \tag{4}$$

Still fails to satisfy NRH, due to possibility of non-stationary π_t.

▸ Detour: Misleading semantics

The Natural Rate Hypothesis and Standard NK Models

Standard NK Phillips curve

$$y_t = \frac{1}{\kappa}\left(\pi_t - \beta E_t\left[\pi_{t+1}\right]\right) \tag{2}$$

Even if $\beta = 1$, $E\left[y_t\right] \neq 0$ if π_t non-stationary.
But $E\left[y_t\right] = 0$ needs to hold "**for any monetary policy.**"

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$$y_t = \frac{1}{\kappa}\left(\pi_t - \bar{\pi} - \beta E_t\left[\pi_{t+1} - \bar{\pi}\right]\right) \tag{3}$$

or past inflation

$$y_t = \frac{1+\gamma\beta}{\kappa}\left(\pi_t - \frac{\gamma}{1+\gamma\beta}\pi_{t-1} - \frac{\beta}{1+\gamma\beta}E_t\left[\pi_{t+1}\right]\right) \tag{4}$$

Still fails to satisfy NRH, due to possibility of non-stationary π_t.

▸ Detour: Misleading semantics

A Class of NRH Models

From Carlstrom and Fuerst (2002), a NRH in finite time

$$E_{t-k}\left[y_t\right] = 0 \; \forall t \tag{5}$$

Can trivially express any supply function that fulfills this hypothesis as

$$y_t = \sum_{j=0}^{k-1} \left(E_{t-j}\left[y_t\right] - E_{t-j-1}\left[y_t\right]\right) \tag{6}$$

- Output gaps entirely innovations or forecast errors
  - Have not made any conjecture as to admissible solutions
- in the words of Friedman (1977), "[o]nly surprises matter."
- Effects of surprises can have a lasting—but not permanent—effect.
  - I.e., any stable short-run tradeoff btw. the output gap and inflation okay

Frictionless Counterpart Model

In the frictionless counterpart model, the output gap is always zero,

$$y_t = 0 \; \forall t \tag{7}$$

the special case of $k = 0$ in (5). In this case, standard IS reduces to

$$y_t = E_t \left[y_{t+1} \right] - a_1 R_t + a_1 E_t \left[\pi_{t+1} \right] \tag{8}$$

- Woodford's (2003) 'Fisher' eq. for nom. determinacy
- Cochrane (2007) and McCallum's (2009) simple supply eq., for examining appropriateness of determinacy

k periods after some disturbance, (5) and (7) identical.
$\longrightarrow$ any two models that satisfy (5) for some k are identical in the long-run (given a common remainder of the model)

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Determinacy in Natural Rate Models

Class of Models

I will analyze linear rational-expectations models of the following class:

$$0 = \sum_{i=0}^{p} \sum_{j=-m}^{n} Q(i,j) E_{t-i} X_{t+j}, \quad X_t = \begin{bmatrix} R_t \\ \pi_t \\ y_t \end{bmatrix}, \quad 0 \leq p, m, n < \infty \tag{9}$$

the $Q(i,j)$'s are matrices of dimensions 3×3.

- Model is composed of three structural equations determining the supply side, demand side, and monetary policy.
- The class encompasses all linear rational-expectations models in the three variables of interest that
  1. have a finite number of leads (given by n)
  2. have a finite number of lags (given by m)
  3. have expectations formed at horizons from t into finite past $t - p$.

Monetary Policy

Only restriction on MP is that it fits into the class defined in (9)
Let monetary policy be the third equation of (9), given by

$$0 = \sum_{i=0}^{p} \sum_{j=-m}^{n} Q_{3,.}(i,j) E_{t-i} \left[X_{t+j} \right] \tag{10}$$

This captures a wide range of interest rate rules found in the literature,

- including current and forward-looking inflation targeting, interest rate smoothing, and output-gap targeting as examined in Woodford (2003)
- and all the rules of Bullard and Mitra(2002)

Lemma

Lemma

For the system (9) to be determinate

1. *The perfect foresight version of the model must have a unique saddle-point stable solution.*
2. *The expectations structure must allow for a unique resolution of the forecasting errors.*

Formal representation

An Unsubstantiated Claim

Carlstrom and Fuerst (2002, p. 82) claim: "[I]n a model that satisfies the NRH, there is real determinacy if and only if there is nominal determinacy in the corresponding flexible-price economy."

Using the foregoing lemma

- necessity follows: perfect foresight versions of both identical. Details
- sufficiency fails: forecasting errors not always resolvable. Details

Fortunately, though, failures in the sufficiency direction are mundane singularities. Details

Standard Dynamic IS Demand

Corollary

Consider a model in (9) with

- *demand given by*

$$y_t = E_t\left[y_{t+1}\right] - a_1 R_t + a_1 E_t\left[\pi_{t+1}\right] \tag{11}$$

- *any supply equation satisfying (5)*
- *restricted to rule out the singularity of (26).*

Determinacy is a function solely of the parameters in the interest rate rule

1. *pertaining to inflation and the interest rate*
2. *and can be ascertained by the interest rate rule and the "Fisher" equation* $R_t = E_t\left[\pi_{t+1}\right]$

Determinacy Regions: A Comparison

Interest Rate Rule	Sticky Prices	NRH
	Non-Price-Related Feedback	
$R_t = 0$	$\emptyset$	$\emptyset$
$R_t = \phi_y y_t$	$\phi_y > \frac{\kappa}{1-\beta}$	$\emptyset$
	Contemporaneous Inflation Feedback	
$R_t = \phi_\pi \pi_t$	$\phi_\pi > 1$	$\phi_\pi > 1$
$R_t = \phi_R R_{t-1} + \phi_y y_t + \phi_\pi \pi_t$	$\phi_\pi > 1 - \phi_R - \frac{1-\beta}{\kappa}\phi_y$	$\phi_\pi > 1 - \phi_R$
	Inflation-Forecast Feedback	
$R_t = \phi_\pi E_t[\pi_{t+1}]$	$1 < \phi_\pi < 1 + 2\frac{1+\beta}{a_1 \kappa}$	$\emptyset$
$R_t = \phi_R R_{t-1} + \phi_y y_t + \phi_\pi E_t[\pi_{t+1}]$	$\phi_\pi > 1 - \phi_R - \frac{1-\beta}{\kappa}\phi_y$ $\phi_\pi < 1 + \phi_R + \frac{1+\beta}{\kappa}\left(\phi_y + 2\frac{1+\phi_R}{a_1}\right)$	$\phi_\pi > 1 - \phi_R$ $\phi_\pi < 1 + \phi_R$

Determinacy and Sticky Prices

For sticky-price determinacy regions to coincide with NRH regions,

- $\kappa \longrightarrow \infty$

But this means

- The sticky-price model must satisfy the NRH! NRH Intuition

Conclusion: Determinacy in standard New Keynesian models gives policy recommendations that are inextricably linked to a violation of the NRH!

Determinacy and Sticky Prices

For sticky-price determinacy regions to coincide with NRH regions,

- $\kappa \to \infty$

But this means

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \kappa y_t$$

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For sticky-price determinacy regions to coincide with NRH regions,

- $\kappa \to \infty$

But this means

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t \Rightarrow \; y_t = \frac{1}{\kappa} (\pi_t - \beta E_t [\pi_{t+1}])$$

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$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t \Rightarrow \; y_t = \frac{1}{\kappa} (\pi_t - \beta E_t [\pi_{t+1}]) \; \Rightarrow \; y_t = 0!$$

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- And as the tradeoff in the SP Phillips curve is the same at all horizons:

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- And as the tradeoff in the SP Phillips curve is the same at all horizons:

$$E_t [y_t] = \frac{1}{\kappa} (E_t [\pi_t] - \beta E_t [\pi_{t+1}])$$

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For sticky-price determinacy regions to coincide with NRH regions,

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- The sticky-price model must satisfy the NRH! NRH Intuition
- And as the tradeoff in the SP Phillips curve is the same at all horizons:

$$E_{t-1} [y_t] = \frac{1}{\kappa} (E_{t-1} [\pi_t] - \beta E_{t-1} [\pi_{t+1}])$$

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- And as the tradeoff in the SP Phillips curve is the same at all horizons:

$$E_{t-i} [y_t] = \frac{1}{\kappa} (E_{t-i} [\pi_t] - \beta E_{t-i} [\pi_{t+1}])$$

- It has to give up its short-run tradeoff to satisfy the NRH.

Conclusion: Determinacy in standard New Keynesian models gives policy recommendations that are inextricably linked to a violation of the NRH!

The Critique of Cochrane (2007) and the Validity of Determinacy

Cochrane's (2007) Critique of Determinacy

- "Nothing in economics rules out explosive [..] nominal paths."
- In determinate New Keynesian models, MP "commits to raise future inflation explosively in response to higher inflation today" Confirm
- These models "do not say that higher inflation causes the Fed to raise real interest rates, which in turn lowers 'demand' and reduces future inflation." "That's 'old- Keynesian' logic."

Cochrane criticizes current attempts to eliminate nominal explosions as resting on

- "government threats to blow up the world should any but one equilibrium occur."
- "[He] opines that this not a credible threat, or, more to the point, it is not likely to be a good characterization of people's expectations.

Determinacy and NRH

The model of discourse for Cochrane (2007 & 2009) and McCallum (2009a & 2009b): The fully flexible model of the previous section! ▸ Odd results with NK

- As determinacy occurs (saving for singularities) at the same parameter constellations
- Cochrane's (2007) critique applies at identical policy specifications for all NRH models

I will argue that

- these inflation explosions are legitimate because the central bank is accommodating them with explosive money supply growth rates,
- the credible, reasonable, and sufficient "threat" to rule out these nominal explosions is a commitment to long-run stability of a monetary aggregate (or its growth rate).

Origin of Nominal Explosions

Consider the frictionless model appended with a simplified standard money demand function (in first differences):

$$\mu_t - \pi_t = -\eta_R \Delta R_t \tag{12}$$

where μ_t is the money growth rate. Simplifying assumptions

Foregoing plus

- the "Fisher equation": $R_t = E_t [\pi_{t+1}]$,
- and a process for the money supply / nom. interest rate (MP)

complete specification for inflation, money growth, and the nominal interest rate.

Case of Money Supply Rules — Speculative Hyperinflation

This is standard macro from '70s and '80s.

With $\mu_t = 0$, the system reduces to

$$R_t = \eta_R E_t \left[\Delta R_{t+1}\right] = \frac{\eta_R}{1+\eta_R} E_t \left[R_{t+1}\right] \tag{13}$$

- One solution is $R_t = \pi_t = 0$, McCallum's (2001) "monetarist solution."
- Continuum of add. solutions with R_t and π_t diverging to $\pm\infty$.
  - Additional solutions are speculative (not supported by money supply)
  - and *can* be ruled out by economic theory: Brock (1975), Scheinkman (1980), Obstfeld and Rogoff (1983), Gray (1984) Sunspot example

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With explosive money growth rate $\mu_t = \lambda \mu_{t-1}$ (Hyperinflationary?)

$$R_t = \eta_R E_t \left[\Delta R_{t+1}\right] + E_t \left[\mu_{t+1}\right] = \frac{\eta_R}{1+\eta_R} E_t \left[R_{t+1}\right] + \frac{\lambda}{1+\eta_R} \mu_t \tag{14}$$

- One solution: R_t and π_t increasing at the same rate as μ_t
- Continuum of add. solutions about this monetarist solution

Determinacy and Interest Rate Rules

In repsonse to, e.g., Sargent and Wallace (1975) exogenous nom. interest rate $\rightarrow$ indeterminacy (multiple stable paths).

New Keynesian models use interest rate feedback rules like

$$R_t = \phi_\pi \pi_t \tag{15}$$

Combining this rule and the Fisher equation

$$E_t \left[\pi_{t+1}\right] = \phi_\pi \pi_t \tag{16}$$

one solution is $\pi_t = R_t = 0$, which implies through (12) that $\mu_t = 0$.

Fallacy of Taylor Principle

But a whole continuum of solution satisfying

$$\pi_t = \phi_\pi \pi_{t-1} \tag{17}$$

are also potential equilibria.

For determinacy, $\phi_\pi > 1$ (Taylor Principle),

- meaning alternate equilibria are explosive paths for inflation and the nominal interest rate.
- But this implies that the money supply growth rate is increasing proportionally with the inflation rate. I.e.,

$$\mu_t = \phi_\pi \mu_{t-1} \tag{18}$$

Cochrane (2007) is right: The explosive equilibria are fully legitimate.

- Monetary policy accommodates these hyperinflationary equilibrium
  - making this explosive path of inflation a consistent "moneterist solution" through extraordinary money supply growth.
- And this applies at identical policy specs in all NRH models.

Nominal Explosions and Interest Rate Rules

The frictionless model has is no liquidity effect, only the Fisher effect;
The liquidity effect dries up in a NRH model, leaving the same Fisher effect.

- Stabilizing logic of nominal interest rate control "old Keynesian" logic.

"[L]ow interest rates are a sign that monetary policy has been tight—in the sense that the quantity of money has grown slowly;
high interest rates are a sign that monetary policy has been easy–in the sense that the quantity of money has grown rapidly" Friedman (1968, p. 7)

- Raising the nom. rate in long run means easing monetary policy.

The Nominal Interest Rate

MP Not Fully Specified By the Nominal Interest Rate

Most notably the monetary analysis pillar of the ECB, but also

- Sec. 2a of the Federal Reserve Act: the Fed "shall maintain long run growth of the monetary and credit aggregates [...] so as to promote effectively [...] stable prices"
  - Chairman Bernanke (2008, pp. 317 & 319) emphasizes that monetary data is and will continue to be monitored by the Federal Reserve as a sensible part of the framework of monetary policy.
- Woodford (2006) agreed that "money matters under extreme circumstances[, and] the commitment of a central bank to undertake [...] drastic steps" is decisive to preventing such circumstances.
- Nelson (2008) reiterates that MP has no direct control over interest rate in the long run More
  - Steady state inflation only determined by MP through open market op's

Completing the Specification of MP

Let monetary policy be fully specified by adding an average growth rate for the money supply, leading to a steady-state inflation rate:

Proposition

Consider the NRH model of the foregoing section appended with (12). Specify monetary policy with an interest rate rule and an average money growth rate.
If the interest-rate rule is associated with a determinate equilibrium, this equilibrium is the unique equilibrium.

▸ Beware tempting equivalent measures!

Conclusion

"It is a dangerous illusion that you can always control the price level in an economy where the money stock however measured is left to vary in purely endogenous fashion." (Leijonhufvud 2009)

Determinacy is, at its core, a long-run consideration.

- If we impose the long-run restrictions of the NRH,
- we obtain strong restrictions on MP to avoid indeterminacy

Cochrane (2007) is right: on their face, explosive nominal paths admissible

- Appealing to determinacy to select a nominal equilibrium
- requires a(n implicit) commitment to monetary restraint
- to rule out accommodative reckless money growth

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Thank you for your attention!

Appendices

Appendices

Literature

- Large literature on determinacy in New Keynesian models focusing on "Taylor (1993)" rules
  - Bernanke and Woodford (1997), Clarida, Galí, and Gertler (1999)
  - Bullard and Mitra (2002) and Woodford (2003).
- Issues such as the Taylor (1999) Principle addressed and extended in, e.g., Woodford (2003)
- Carlstrom and Fuerst (2002) explore determinacy with pure inflation targeting in a stylized NRH model.
- McCallum (2004) criticizes stnd. New Keynesian model's violation of the NRH
- Nelson (2008) casts doubt on the nom. interest rate, reiterating the difference between the liquidity and the Fisher effect.

Literature

- Cochrane (2007) criticizes determinacy as arbitrary:
  - "Nothing in economics rules out explosive nominal paths [...,] nothing in economics generically allows us to insist on the unique 'locally-bounded" equilibrium of a new-Keynesian model."
- echoing earlier literature
  - "Speculative Hyperinflation"—e.g., Obstfeld and Rogoff (1983), Gray (1984)
  - Sargent and Wallace's (1975) criticism of a fixed interest rate
- Two attempts to answer Cochrane's (2007) critique
  - McCallum (2009) "eliminates" Cochrane's (2007) explosive equil. with LS Learning
  - Minford and Srinivasan (2009) let an "optimizing gov't set the inflation tax"

Back to presentation introduction

Expected Inflation Targeting—Forward Looking MP

An interest rate rule that targets expected inflation ($R_t = \phi_\pi E_t [\pi_{t+1}]$) is associated

- a lower bound (Taylor principle $\phi_\pi > 1$)
- and an upper bound ($\phi_\pi < 1 + 2\frac{1+\beta}{a_1 \kappa}$ "standard" values yields ≈ 25)

This upper bound would seem empirically irrelevant—a theoretical curiosity.

Back to presentation introduction
Back to presentation main text

Expected Inflation Targeting—Forward Looking MP

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- a lower bound (Taylor principle $\phi_\pi > 1$)
- and an upper bound ($\phi_\pi < 1 + 2\frac{1+\beta}{a_1 \kappa}$ "standard" values yields ≈ 25)

This upper bound would seem empirically irrelevant—a theoretical curiosity. But for a model that satisfies the NRH,

- the upper bound collapses to one
- for determinacy: $1 < \phi_\pi < 1$, a contradiction

Under NRH, relevant system is MP and Fisher ($R_t = E_t [\pi_{t+1}]$).
π_t wholly absent with forward looking MP.

Back to presentation introduction
Back to presentation main text

Which Natural Rate Hypothesis?

McCallum (2004) distinguishes between

1. The stronger or Lucas definition I have employed
2. The weaker or Friedman definition, by which
   - a higher, but constant, rate of inflation cannot permanently affect output.

At very least, the association with Friedman in the latter is unfortunate: "[S]ome substitute a stable relation between the acceleration of inflation and unemployment for a stable relationship between inflation and unemployment—aware of but not concerned about the possibility that the same logic that drove them to a second derivative will drive them to even higher derivatives." (Friedman 1977, p. 274) Back to presentation

Mundane Singularity?

Necessary but insufficient. A simple example:

$$aE_t\left[\theta_{t+1}\right] = b\theta_t + cE_{t-1}\left[\theta_t\right] \tag{19}$$

Highest lagged expectation:

$$aE_{t-1}\left[\theta_{t+1}\right] = (b+c)\,E_{t-1}\left[\theta_t\right] \tag{20}$$

Determinacy: $|\frac{b+c}{a}| > 1$. No shocks, so $E_{t-1}\left[\theta_t\right] = E_t\left[\theta_{t+1}\right] = 0$ Thus

$$0 = b\theta_t \tag{21}$$

But what if $b = 0$? Then θ_t undetermined. See more: Klein (2000) Back to presentation

Mundane Singularity: Example in Klein (2000) format

In Klein's (2000) format

$$\begin{bmatrix} a & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \theta_t^{dummy} \\ E_t\left[\theta_{t+1}\right] \end{bmatrix} = \begin{bmatrix} c & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_{t-1}^{dummy} \\ \theta_t \end{bmatrix} \tag{22}$$

Or

$$E_t\left[\begin{bmatrix} \omega_{t+1}^s \\ \omega_{t+1}^u \end{bmatrix}\right] = \begin{bmatrix} 0 & 0 \\ 0 & \frac{b+c}{a} \end{bmatrix} \begin{bmatrix} \omega_t^s \\ \omega_t^u \end{bmatrix} \tag{23}$$

where $E_t\left[\omega_{t+1}^s\right] = \omega_{t+1}^s$ and

$$\begin{bmatrix} \theta_{t-1}^{dummy} \\ \theta_t \end{bmatrix} = \frac{1}{b+c} \begin{bmatrix} b & c \\ -c & c \end{bmatrix} \begin{bmatrix} \omega_t^s \\ \omega_t^u \end{bmatrix} \tag{24}$$

The case $b = 0$ violates Klein's (2000) invertibility condition! ($Z_{11} = \frac{b}{b+c}$)

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Lemma

Lemma

For the system (9) to be determinate

1. *The model*

$$0 = \sum_{j=-m}^{n} \tilde{Q}_j X_{t+j}, \text{ where } \tilde{Q}_j = \sum_{i=0}^{p} Q(i,j) \tag{25}$$

   must have a unique saddle-point stable solution.

2. *The matrix*

$$\begin{bmatrix} \mathbf{Q} \\ \mathbf{B} \end{bmatrix} \tag{26}$$

   must be non-singular. ▸ Definitions: **Q** and **B**

Lemma con't

Q and **B** are block matrices of dimensions $3p \times 3(p+n)$ and $3n \times 3(p+n)$ respectively with blocks of dimension 3×3. The s'th block row of **Q** is given by

$$\begin{bmatrix} 0_{max(0,s-1-m)} & \tilde{Q}\left(s-1, -min(s-1,m), n\right) & 0_{p-s} \end{bmatrix} \tag{27}$$

where 0_i is a $3 \times 3i$ block vector of zeros and $\tilde{Q}\left(a,b,c\right) = \begin{bmatrix} \tilde{Q}(a,b) & \tilde{Q}(a,b+1) & \dots \tilde{Q}(a,c) \end{bmatrix}$ with $\tilde{Q}(a,b) = \sum_{i=0}^{min(p,a)} Q(i,b)$. The s'th block row of **B** is given by

$$\begin{bmatrix} 0_{max(0,s+p-m-1)} & -\tilde{B}\left(min(p+s-1,m)\right) & I & 0_{n-s} \end{bmatrix} \tag{28}$$

where I is a 3×3 identity matrix and $\tilde{B}\left(a\right)$ being the last $3 \times 3a$ elements of the $3 \times 3m$ matrix B that forms Anderson's (2010) convergent autoregressive solution to (25). Back to presentation

Necessity

Assume the opposite is true.

- Thus, the NRH model is determinate and the fully flexible model is not.
- From the former, according to lemma 3,
  1. the system (25) is saddle-point stable
  2. and (26) is invertible.
- But the system (25) is the same for both models and (26) is lower triangular for the fully flexible model.
- Thus, the fully flexible model that satisfies (7) is determinate,

a contradiction. Back to presentation

Sufficiency

If the fully flexible model is determinate,

1. the system (25) is saddle-point stable.
   - This system is the same for all NRH (5) models.
2. The second requirement (26) is lower triangular for the flexible model
   - But is left unrestricted for NRH models.

Thus,

- there exist NRH models with a singular (26) that are thusly indeterminate,
- even though the corresponding fully flexible model is determinate.

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Intuition for the Results under the NRH

With Lucas's (1972) version of the NRH

- no relationship between output and inflation in the long run

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Intuition for the Results under the NRH

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- real variables Picture converge independently of nominal paths Picture

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Intuition for the Results under the NRH

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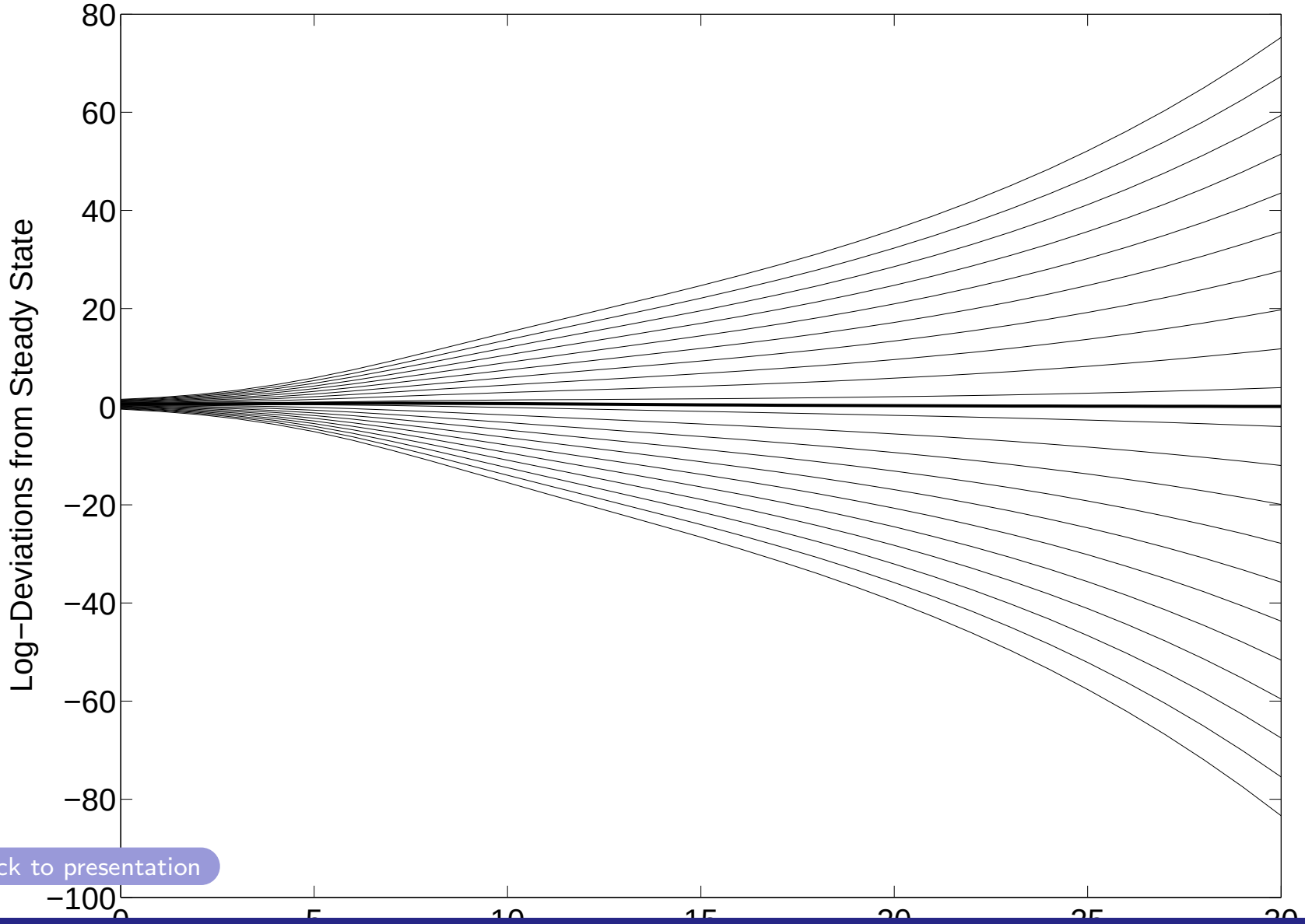
  - and the nominal side equivalent to flexible model.
- Thus determinacy of inflation in NRH obtains
- under the same conditions as for nominal determinacy in flexible model
  - a purely monetary phenomenon.
- But output depends on inflation at all finite horizons:
- the path of inflation determines the actual path of output.

  ▸ Example: Forward-looking MP

▸ Back to presentation



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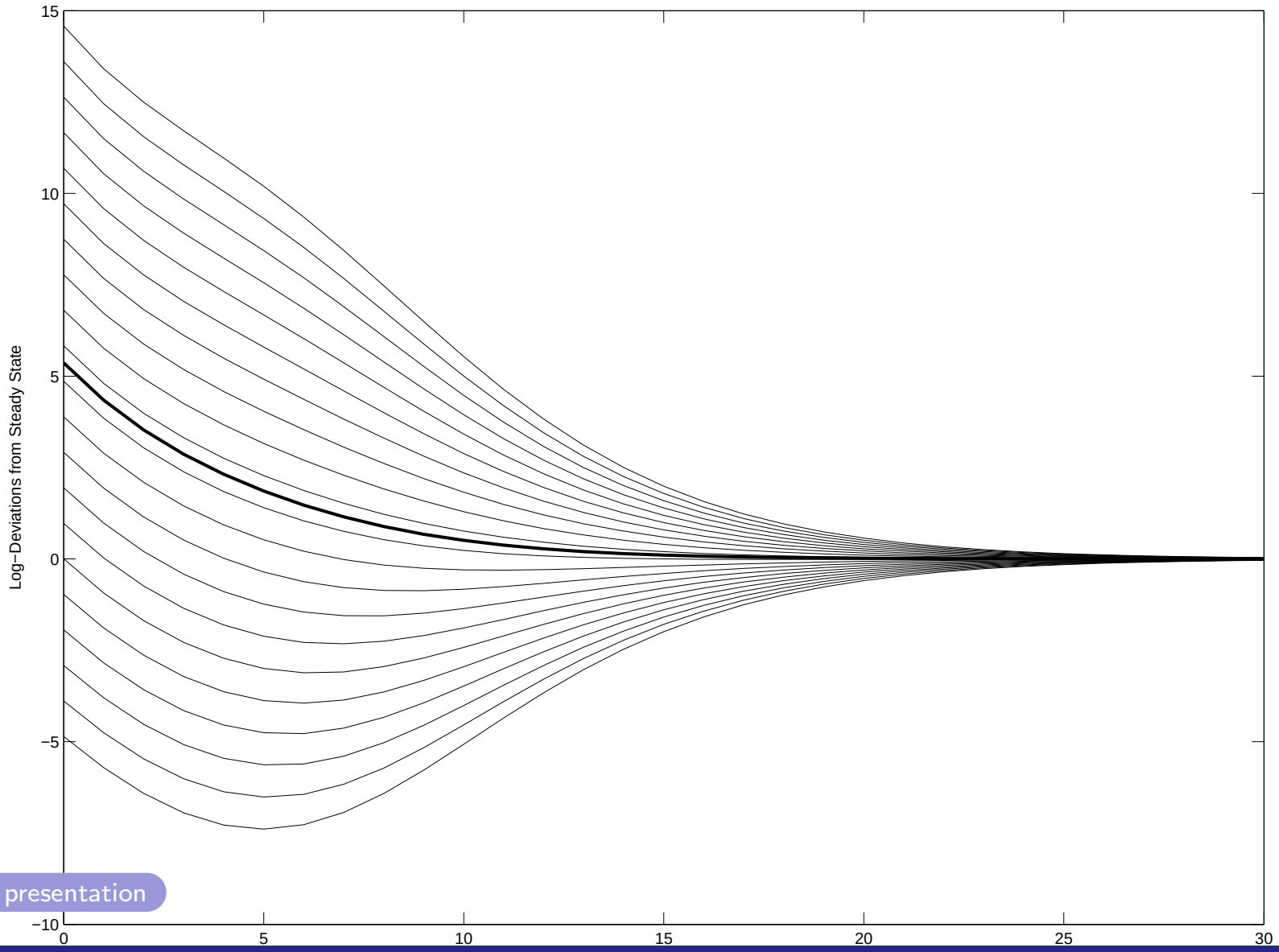

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Cochrane's (2007) Critique and Sticky Prices

First a quick sidetrack to the standard sticky-price model.

- Ironically, Cochrane's (2007) critique does not actually apply
- "[N]othing in economics rules out nominal explosions,[... e]conomics (transversality conditions) can rule out real explosions.
- But through violation of NRH, nominal explosion in NK model goes hand-in-hand with real explosion! Picture
- This is as dubious as the determinacy bounds in New Keynesian models
- As it relies on a stable long-run tradeoff between output and inflation to justify a particular equilibrium.

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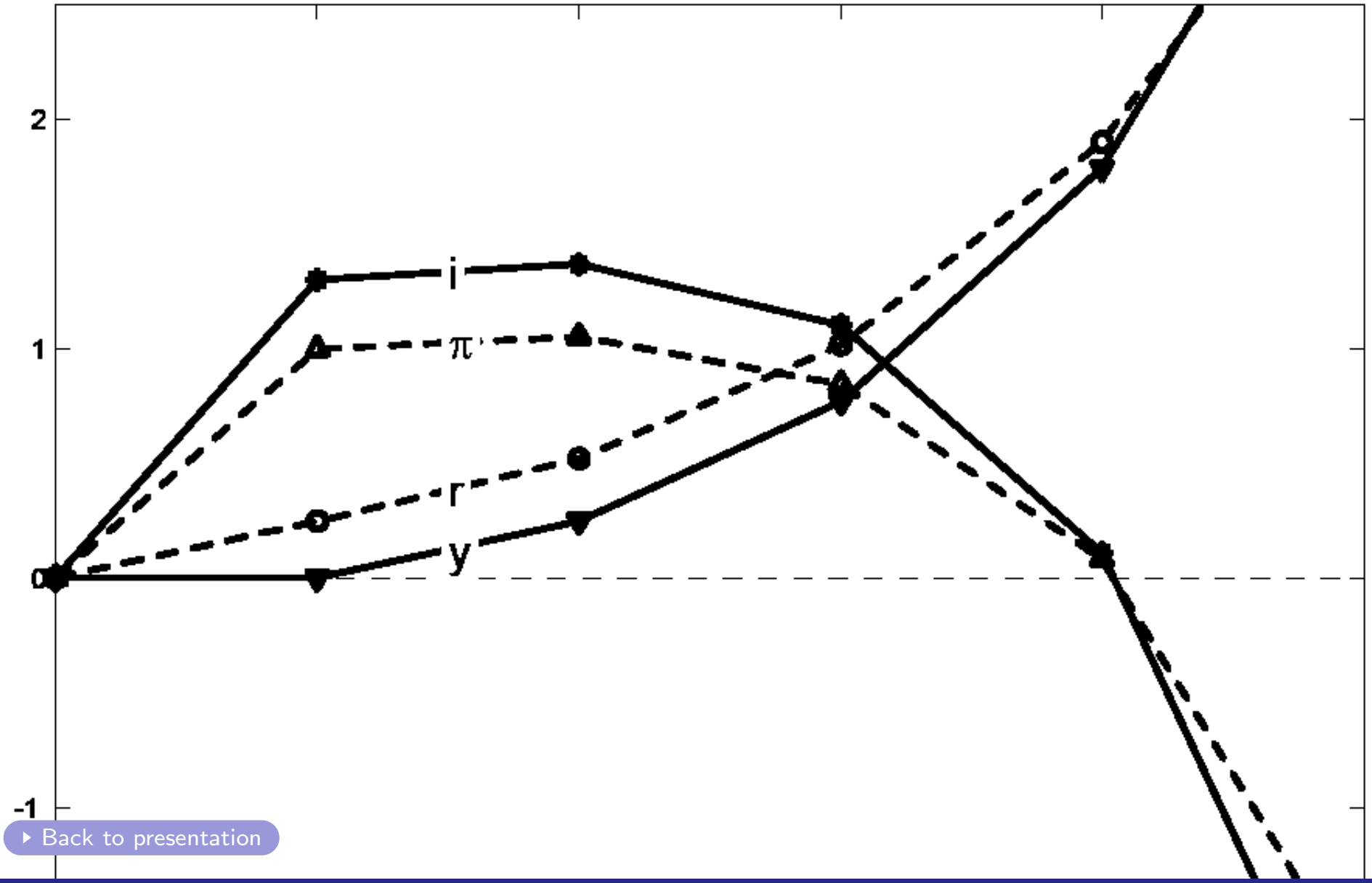

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Cochrane's (2007) Critique: an Example

Take the relevant NRH eq. $R_t = E_t[\pi_{t+1}]$ and simple Taylor rule $R_t = \phi_\pi \pi_t$, with $\phi_\pi > 1$ for determinacy.
Consider a sunspot $E_t[\pi_{t+1}] > 0$. This implies $R_t > 0$ and $0 < \pi_t < E_t[\pi_{t+1}]$.
If it realizes: $\pi_{t+1} > 0$, which implies $R_{t+1} > \pi_{t+1}$, thus $E_{t+1}[\pi_{t+2}] > E_t[\pi_{t+1}] > \pi_t > 0$.
Inflation is on an explosive path.
Why can we rule out such a path for a nominal variable?

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Ruling Out Explosive Real Balances

Take the relevant NRH eq. $R_t = E_t[\pi_{t+1}]$ and the money demand equation $\mu_t - \pi_t = \eta_R \Delta R_t$

Consider a sunspot $E_t[\pi_{t+1}] > 0$ that is not accommodated. Thus $\mu_{t+j} = 0$ and the sunspot implies $R_t > 0$ and $0 < \pi_t = \eta_R E_t[\pi_{t+1}] < E_t[\pi_{t+1}]$.

If it realizes: $\pi_{t+1} > 0$, which implies $R_{t+1} = \left(1 + \frac{1}{\eta_R}\right)\pi_{t+1}$, thus

$E_{t+1}[\pi_{t+2}] = \left(1 + \frac{1}{\eta_R}\right)\pi_{t+1} > \pi_{t+1} > \pi_t > 0.$

Inflation is on an explosive path.

Why can we rule out such a path for a nominal variable?

The real variable $\mu_{t+j} - \pi_{t+j}$ is on an explosive path.

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Stable Monetary Relations: A Straw Man for Money Supply Rules

Simplifying assumptions: From NRH, the output gap is necessarily stationary. Thus, we can neglect

- secular growth and money demand shocks

for the purposes of asymptotic behavior if it can be assumed that both are at least difference stationary, to arrive at (12). ▸ Stability of monetary relations.

"A steady rate of growth in the money supply will not mean perfect stability [...T]he available evidence [...] casts grave doubts on the possibility of producing any fine adjustments in economic activity by fine adjustments in monetary policy."(Friedman 1958 /1969 , pp. 185–186)

"I share very much the doubts [...] about the closeness of the monetary relations [...]
Indeed, that's the major reason why I'm in favor of a [constant money supply growth] rule. [...]

Nelson (2008, p. 1805)

- Though they no longer affect real interest rates,
- and no longer can affect nominal rates via *a liquidity effect*,

the central bank's open market operations continue in the long run to affect nominal money growth.

- So nominal money growth is unambiguously and undeniably susceptible to central bank influence even in the long run
- Reaching [an] inflation target means a specified quantity of open market operations in the steady state;
- There it is: the sense in which steady-state inflation can be regarded as pinned down by steady-state money growth.

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A Tempting Equivalent Measure

Why the detour to monetary aggregates? What is the difference to simply stating a steady-state inflation rate?

- Must differentiate from incomes policies.
  - The wage-price "guideposts" / "freezes" of the '60s and '70s a resounding failure.
  - Relying on "moral suasion" to anchor inflation expectations seems dangerous from experience.
  - Friedman (1966) , "[This] will not in fact cure inflation, but even if it did, it would be a cure that is worse than the disease."
- Belies the mechanism at work
  - MP does not have direct control over either the nom. interest rate or inflation in the long run
  - Relying on some sort of "permanent liquidity effect" for credibility cannot work. ▸ The price of money
- But MP can still control the money supply, even in the long run

The Nominal Interest Rate and the Price of Money

Isn't control over the nominal interest rate the same as control over the money supply?
The one determines the other, i.e. the interest rate is the "price" of money.

"The interest rate is not the price of money[...] it is the price of credit. The price of money is how much goods and services you have to give up to get a dollar [...—]the inverse of the price level." (Friedman 1969 in "Monetary vs. Fiscal Policy", pp. 74-75) Back to presentation