Is There the Technology Shock?

Confronting Sign Restrictions with the Properties of the Data

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Disentangling Demand and Supply Shocks in the Crude Oil Market: How to Check Sign Restrictions in Structural VARs[1]

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standard reduced-form VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t$$

n x 1

$$E\left[\mathbf{u}_t\right] = \mathbf{0} \qquad E\left[\mathbf{u}_t \mathbf{u}_t'\right] = \boldsymbol{\Sigma}_u$$

n x n

reduced form errors are by assumption uncorrelated from lagged ys and coefficient matrices can readily be estimated by OLS

standard reduced-form VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t$$

$$E[\mathbf{u}_t] = \mathbf{0} \qquad E[\mathbf{u}_t \mathbf{u}_t'] = \boldsymbol{\Sigma}_u$$

„structural“ VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{B}\boldsymbol{\varepsilon}_t$$

$$\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t \qquad E[\boldsymbol{\varepsilon}_t] = \mathbf{0} \qquad E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \mathbf{I}_{n \times n}$$

standard reduced-form VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \ldots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t$$

„structural“ VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \ldots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{B}\boldsymbol{\varepsilon}_t$$

The model is not really structural in the usual sense („everything depends on everything‘s past“)
The „structural form“ merely has uncorrelated errors.
Benefit: ceteris paribus compatible shocks

standard reduced-form VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \ldots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t$$

„structural“ VAR $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$ $E\left[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right] = \mathbf{I}_{n\times n}$

$$\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t \qquad E\left[\mathbf{u}_t\mathbf{u}_t'\right] = \boldsymbol{\Sigma}_u = \mathbf{B}\mathbf{B}'$$

If we could deduce B from this equation then we could compute uncorrelated („structural“) shocks

$$\boldsymbol{\varepsilon}_t = \mathbf{B}^{-1}\mathbf{u}_t$$

Identifying restrictions needed

$$\mathbf{\Sigma}_u = \mathbf{BB}'$$

n x n but only
[n x (n+1)]/2 unique elements

n x n elements

- exclusion restrictions (can be point-identifying)
- sign restrictions („do not identify the shocks uniquely but allow for a range of admissible shocks that are all in line with the sign restrictions.“
- neither can be tested

Example

$$\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$$

$$\begin{bmatrix} U_t^{\Delta prod} \\ U_t^{q} \\ U_t^{p} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_t^{oil-s} \\ \varepsilon_t^{aggr-d} \\ \varepsilon_t^{oil-d} \end{bmatrix}.$$

From L+N (2012): „ Kilian identies the shocks by assuming that

- oil-market specic demand shocks do not have an instantaneous effect on oil production and real activity
- and aggregate demand shocks have no immediate impact on oil production”

standard reduced-form VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t$$

$$E[\mathbf{u}_t] = \mathbf{0} \qquad E[\mathbf{u}_t \mathbf{u}_t'] = \boldsymbol{\Sigma}_u$$

Markov-switching reduced-form VAR

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t$$

$$\mathbf{u}_t \mid s_t \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_u(s_t)\right) \qquad s_t = \begin{cases} 1 \\ 2 \\ \vdots \\ M \end{cases} \qquad \text{Prob}(s_t \mid s_{t-1})$$

L+N (2013)

$$\mathbf{y}_t = \boldsymbol{\kappa}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{A}_3\mathbf{y}_{t-3} + \mathbf{u}_t$$

$$\mathbf{u}_t \mid s_t \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_u(s_t)\right)$$

Table 3: Estimated State Covariance Matrices of MS(m)-VAR(3) Models with State-Invariant B, $m = 2, 3, 4$, for $Y_t = (\Delta prod_t, q_t, p_t)'$

	$m = 2$	$m = 3$	$m = 4$
Σ_1	$\begin{bmatrix} 155.59 & & \\ 2.02 & 12.38 & \\ -9.43 & 2.36 & 58.29 \end{bmatrix}$	$\begin{bmatrix} 99.38 & & \\ 0.96 & 10.51 & \\ -0.86 & 1.09 & 42.84 \end{bmatrix}$	$\begin{bmatrix} 90.92 & & \\ 1.33 & 9.89 & \\ -0.98 & 0.49 & 40.27 \end{bmatrix}$
Σ_2	$\begin{bmatrix} 912.86 & & \\ 18.49 & 35.38 & \\ -0.56 & 0.56 & 2.94 \end{bmatrix}$	$\begin{bmatrix} 533.95 & & \\ 9.39 & 33.86 & \\ 5.07 & 0.64 & 2.45 \end{bmatrix}$	$\begin{bmatrix} 368.56 & & \\ 6.42 & 23.89 & \\ -0.60 & 1.06 & 1.52 \end{bmatrix}$
Σ_3		$\begin{bmatrix} 2499.03 & & \\ 65.38 & 37.42 & \\ 16.46 & 5.31 & 190.49 \end{bmatrix}$	$\begin{bmatrix} 947.64 & & \\ 17.89 & 39.83 & \\ -5.97 & 2.03 & 222.26 \end{bmatrix}$
Σ_4			$\begin{bmatrix} 2365.54 & & \\ 47.36 & 57.70 & \\ -3.55 & 2.49 & 9.59 \end{bmatrix}$

Identification via Heteroscedasticity

$$\mathbf{\Sigma}_u = \mathbf{B}\mathbf{B}'$$ becomes (if M=2)

$$\mathbf{\Sigma}_u(1) = \mathbf{B}\mathbf{B}' \qquad \mathbf{\Sigma}_u(2) = \mathbf{B}\,\mathbf{\Lambda}(2)\,\mathbf{B}'$$

where $\mathbf{\Lambda}(2)=\text{diag}(\lambda_{21},\ldots,\lambda_{2K})$

If the λ_{2s} are all distinct, this decomposition is unique apart from changes in the sign and permutations of the columns of B and corresponding changes in the ordering of the λ_{2s}.
Any additional restrictions from economics become over-identifying and can be checked against the data.

So what can we learned?

- For instance, exclusion restrictions can be checked with formal statistical tests.
- if a set of sign restrictions is postulated for the system to be suitable for the desired analysis, the impulse responses corresponding to the shocks identified via the MS structure must satisfy the prespecied sign restrictions that characterize the economic shocks of interest.
- Alternatively, if the shocks do not satisfy the sign restrictions, these restrictions are not compatible with the statistical properties of the data.

What be learned (continued)?

- The reasons could be: omitted variables, measurement errors, aggregation problems or distortions due to data transformations.

Shouldn’t we conclude that this is evidence against the theory that suggested the sign restrictions?

What be learned (continued)?

- If, however, the impulse responses satisfy the sign restrictions, labels may be attached to the shocks accordingly.

This seems to be the approach of the present paper. But it becomes a bit arbitrary. If we try enough, aren't we eventually going to find some sign restrictions that are compatible with the impulse responses identified via MS heteroscedasticity?

Stock & Watson

Until now, we have carefully distinguished between recursive and structural VARs: recursive VARs use an arbitrary mechanical method to model contemporaneous correlation in the variables, while structural VARs use economic theory to associate these correlations with causal relationships. Unfortunately, in the empirical literature the distinction is often murky. It is tempting to develop economic "theories" that, conveniently, lead to a particular recursive ordering of the variables, so that their "structural" VAR simplifies to a recursive VAR, a structure called a Wold causal chain. We think researchers yield to this temptation far too often. Such cobbled-together theories, even if superficially plausible, often fall apart on deeper inspection. Rarely does it add value to repackage a recursive VAR and sell it as structural.