Bayesian Estimation of Autoregressive Moving-Average Processes as Exogenous Shock Processes in DSGE Models

Work in Progress

Alexander Meyer-Gohde   Daniel Neuhoff

Humboldt-Universität zu Berlin   CRC 649

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ÖKONOMISCHES
RISIKO

Introduction

Research question: What ARMA model describes exogenous processes in structural models best?

- Macro theory provides no guidance in this question
- Standard practice: AR(1) following Kydland and Prescott (1982) for technology
  - Often little empirical support for this practice
- Relax assumptions on shock processes to account for misspecification

Our contribution: Estimation of shock processes using Reversible Jump Markov Chain Monte Carlo

Preview of Results

- Estimated TFP-shock using US GDP per capita with Hansen (1985) model and calibration rejects AR(1)
- Accounting for noninvertible MA: Drop of hours in response to positive technology shock contained in the 80% credible set

Outline

ARMA(p,q) Processes and Stationarity

Reversible Jump Markov Chain Monte Carlo

ARMA Estimation of US GDP

Application to Hansen (1985) Neoclassical Growth Model

Conclusion

ARMA(p,q) Processes and Stationarity

Zero mean autoregressive moving average process with orders p,q:

$$y_t = P_1^p y_{t-1} + \ldots + P_p^p y_{t-p} + \epsilon_t + Q_1^q \epsilon_{t-1} + \ldots + Q_q^q \epsilon_{t-q} \quad (1)$$

- P^p and Q^q parameter vectors of the AR and MA polynomials for the orders p, q
- $\epsilon_t \sim N(0, \sigma^2)$
- Impose stationarity by reparametrizing the polynomials in terms of (inverse) partial autocorrelations

Overview

Standard practice in DSGE estimation: Metropolis-Hastings samplers

Now: Varying dimensionality of the parameter space

Reversible Jump Markov Chain Monte Carlo

- Pioneered by Green (1995) as generalization of M-H samplers
- Samples from a joint posterior distribution across different models and their corresponding parameter spaces
- Adaptation of acceptance probability to enable moves between parameter spaces of varying dimensionality
- Otherwise: Same as standard MCMC. M-H sampler a special case of RJMCMC

ARMA(p,q) Processes and Stationarity

Reversible Jump Markov Chain Monte Carlo

ARMA Estimation of US GDP

Application to Hansen (1985) Neoclassical Growth Model

Conclusion

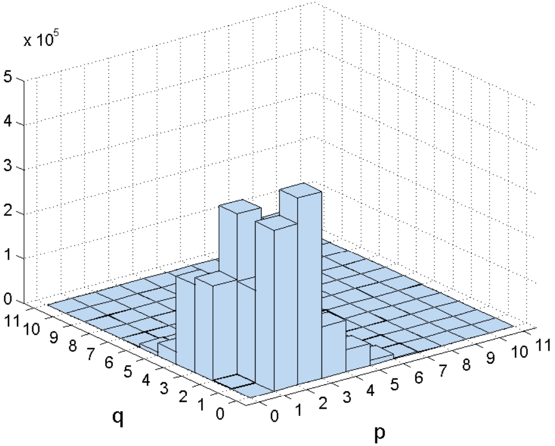
RJMCMC Estimate of US GDP

- First-differenced quarterly US log GDP per capita (1947:1 - 2013:3)
- Our prior over the orders (p, q): $U(0, 10)$
- 4 Mio. samples from posterior, 1 Mio. as burn-in

Parameter	Mean	Median
AR(1)	0.3186	0.3184
	(0.0616)	
AR(2)	0.1300	0.1297
	(0.0613)	
σ	0.9025	0.9010
	(0.0399)	
$\rho(1)$	0.3662	0.3659
$\rho(2)$	0.2467	0.2462

- Estimates from posterior conditional on $(p, q) = (2, 0)$
- Standard errors in parentheses
- $\rho(i)$ denotes the autocorrelation at lag i

Posterior over (p, q) for US GDP



Process at the mode: ARMA(2,0)

Monte Carlo Study

Question: How does RJMCMC compare to standard methods with regards to point estimates of the orders p and q?

Base synthetic data on posterior from application of RJMCMC US GDP

1. Based on the posterior generate 100 synthetic data sets with 250 observations each using
   1.1 The model at posterior mode

$$y_t = 0.3184 y_{t-1} + 0.1297 y_{t-2} + \epsilon_t; \epsilon_t \sim N(0, 0.9010)$$

   1.2 The model at every 30,000th draw
2. Apply RJMCMC to synthetic data: 1.5 Mio. draws, 1 Mio. burn-in
3. Identify orders at mode and compare with for AIC, AICC and SIC

Results Monte Carlo Study

Proportion of Correctly Identified Models

Method	Experiment 1 (Mode Model)	Experiment 2 (Posterior Draws)
RJMCMC	0.37	0.23
AIC	0.19	0.08
AICC	0.19	0.09
SIC	0.26	0.18

Hansen (1985) Neoclassical Growth Model

The Social Planner maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ln\left(c_t\right) + \psi ln\left(1 - l_t\right)\right],\ 0 < \beta < 1$$

subject to

$$\begin{aligned} y_t &= e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha} \\ c_t + i_t &= y_t \\ k_t &= \left(1 - \delta\right) k_{t-1} + i_t \\ z_t &= \text{Stochastic Productivity} \end{aligned}$$

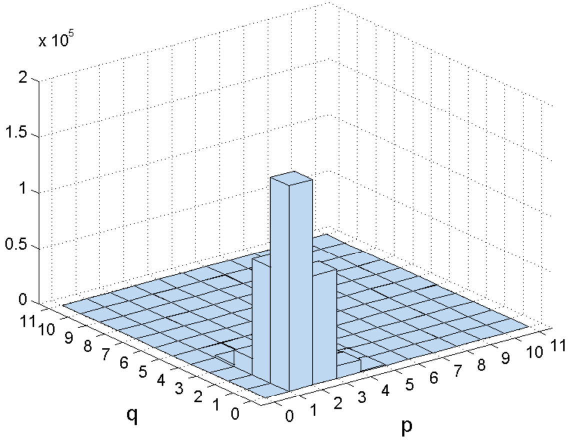
Setup

- Run estimation procedure using neoclassical growth model with calibration from Hansen (1985)
  - Synthetic data: $z_t = 0.95 z_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$
  - US GDP per capita HP filtered with $\lambda = 1600$ for (1947:1 - 2013:3)

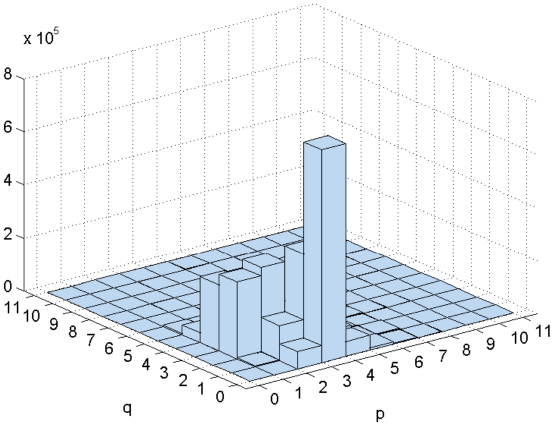
Variable	Prior	Proposal
p	U(0,10)	LaplaceD(p,2.2)
q	U(0,10)	LaplaceD(q,2.2)
AR PAC	TN(0,0.25)	TN(PAC,0.0016)
MA PAC	TN(0,0.25)	TN(PAC,0.0016)
σ	IG(1,1)	TN(σ,0.0025)

- **Draws:** 4,000,000
- **Burn-In:** 1,000,000

Posterior over p,q Synthetic Data AR(1) Shock



Posterior over p,q US GDP Data



Process at the mode: ARMA(3,0)

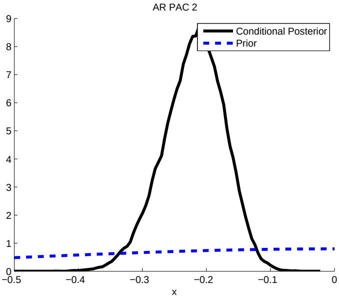
Parameter Estimates

Parameter	Mean	Median	Hansen
AR(1)	1.1689	1.1681	0.95
	(0.04)		
AR(2)	-0.0732	-0.0725	N/A
	(0.06)		
AR(3)	-0.1224	-0.1215	N/A
	(0.04)		
σ	0.5873	0.5733	0.712
	(0.08)		
$\rho(1)$	0.9804	0.9810	0.95
$\rho(2)$	0.9528	0.9542	0.9025

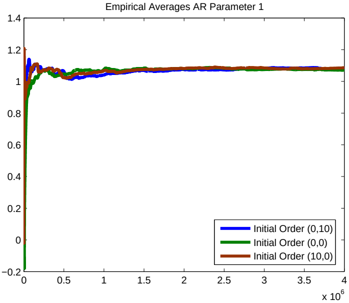
- Estimates from posterior conditional on $(p,q) = (3,0)$
- Standard errors in parentheses
- $\rho(i)$ denotes the autocorrelation at lag i

Priors, Posteriors and Convergence

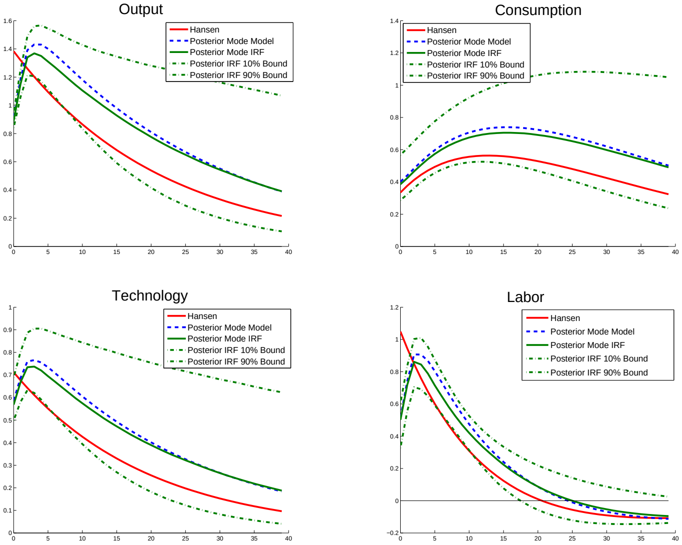
Prior and Posterior second PAC



Unconditional Recursive Average



Impulse Responses



Hump-shaped impulse responses, e.g. Cogley and Nason (1995)

Noninvertibility and Responses to Technology Shocks

What is the response of hours to a technology shock?

- Gali (1999), Francis and Ramey (2005) find a negative response
- Christiano et al. (2003), Chari et al. (2008) attribute the finding to misspecification
- Uhlig (2004) finds a mildly positive response

Possible mechanisms:

- Nominal and real rigidities: Galí and Rabanal (2004)
- Nontechnology shocks: Uhlig (2004)
- News shocks: Barsky and Sims (2011)

Noninvertible MA Representations

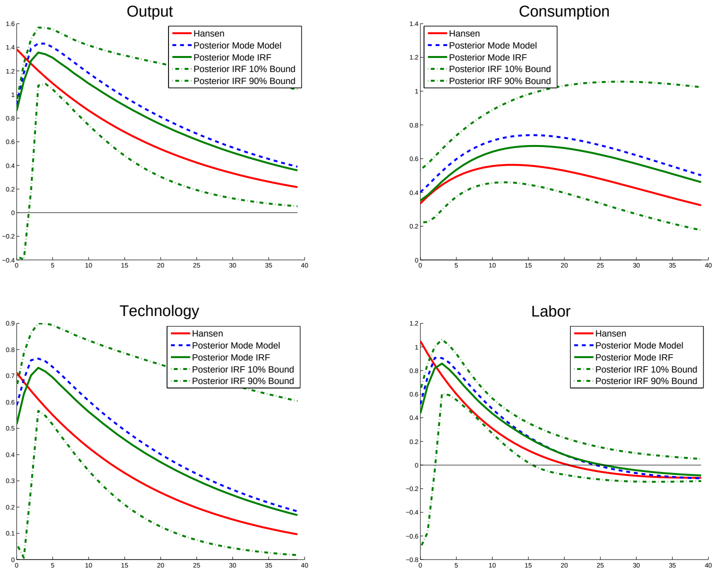
Till now, MA components invertible/fundamental, but

- Covariance equivalent representations with noninvertible MA representations,e.g. Lippi and Reichlin (1994)
- The data does not tell us anything about invertibility
- With noninvertible MA, a fall in hours in response to a positive technology shock possible even in neoclassical growth model

Implementation:

- With flat priors over orders and priors over PACS/inverse PACs
  - Posterior the same with invertible and noninvertible
- Sample uniformly from admissible invertible and noninvertible MAs

Noninvertible Impulse Responses



Conclusion

- RJMCMC enables identification of shock processes for DSGE models
- We reject AR(1) in Hansen's basic model
- Shock process generates hump-shaped responses
- Noninvertible MA: A positive technology shock leading to a fall in hours is contained in the credible set

Thank you for your attention!

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Appendix: Imposing Stationarity

In order to constrain sampling on the invertibility and stationarity region of the parameter spaces of each model we reparametrize the AR(p) polynomial in terms of partial autocorrelations following Barndorff-Nielsen and Schou (1973), Monahan (1984) and Jones (1987):

1. Introduce $p^k = (p_1^{(k)}, ..., p_k^{(k)}, k = 1, ..., p$
2. Draw $r = r_1, ..., r_p, r_i \in (0, 1)$ (inverse) partial autocorrelations
3. Set $p_1^{(1)} = r_1$
4. Run the recursion

$$p_i^{(k)} = p_i^{(k-1)} - r_k p_{k-i}^{(k-1)}, i = 1, ..., k-1$$

   with $p_k^{(k)} = r_k$ for $k = 2, ..., p$

5. Set $P^p = p^{(p)}$

P^p then contains the vector of AR(p) parameters associated with the partial autocorrelations r_i

Appendix: Standard Metropolis-Hastings Algorithm

Let ς denote a state of the Markov Chain, i.e. the current draw of model parameters

1. Set the initial state ς_0 of the Markov Chain
2. For $i = 1$ to N
   - **2.1** Set $\varsigma = \varsigma_{i-1}$
   - **2.2** Propose a new state from some proposal distribution $\gamma(\varsigma'|\varsigma)$
   - **2.3** Accept draw with probability

     $$\alpha(\varsigma, \varsigma') = \min(1, \chi)$$

     with

     $$\chi = \underbrace{\frac{\mathcal{L}(\varsigma')}{\mathcal{L}(\varsigma)}}_{\text{Likelihood Ratio}} \times \underbrace{\frac{\rho(\varsigma')}{\rho(\varsigma)}}_{\text{Prior Ratio}} \times \underbrace{\frac{\gamma(\varsigma|\varsigma')}{\gamma(\varsigma'|\varsigma)}}_{\text{Proposal Ratio}}$$

   - **2.4** If the draw is accepted set $\varsigma_i = \varsigma'$. If the draw is rejected set $\varsigma_i = \varsigma$

This algorithm defines a transition Kernel such that the Markov chain has the desired invariant distribution, i.e. the posterior.

Appendix: Detailed Balance

In order to have the correct stationary distribution:

$$\int_{\mathscr{A}_\varsigma} \pi(\varsigma) K\left(\varsigma, \mathscr{B}_{\varsigma'}, \alpha(\varsigma, \varsigma')\right) d\varsigma = \int_{\mathscr{B}_{\varsigma'}} \pi\left(\varsigma'\right) K\left(\varsigma', \mathscr{A}_\varsigma, \alpha(\varsigma', \varsigma)\right) d\varsigma' \qquad (2)$$

- States $\varsigma = (P^p, Q^q, \sigma, p, q)$
- $\mathscr{A}_\varsigma$, $\mathscr{B}_{\varsigma'}$ subsets of parameter spaces associated with ς and ς'

Problem: Dimensionality of ς and ς' differs if AR and/or MA orders change

Solution: Modify proposals such that both sides of the equation are of equal dimensionality Green (1995) using a bijection g

Acceptance probability chosen analogously to Metropolis-Hastings

Appendix: RJMCMC Algorithm

1. Set the initial state ς_0 of the Markov Chain
2. For $i = 1$ to N
   - **2.1** set $\varsigma = \varsigma_{i-1}$
   - **2.2** Propose a visit to model $(p,q)'$ with probability $\gamma_{pq}((p,q)'|(p,q))$
   - **2.3** Sample u from $\gamma_u(\varsigma,u)$
   - **2.4** Set $\varsigma' = g(\varsigma,u)$
   - **2.5** Accept draw with probability

$$\alpha = \min\big(1, \chi(\varsigma,\varsigma')\big)$$

with

$$\chi(\varsigma,\varsigma') = \underbrace{\frac{\mathscr{L}(\varsigma')}{\mathscr{L}(\varsigma)}}_{\text{Likelihood Ratio}} \times \underbrace{\frac{\rho(\varsigma')}{\rho(\varsigma)}}_{\text{Prior Ratio}} \times \underbrace{\frac{\gamma(\varsigma|\varsigma')}{\gamma(\varsigma'|\varsigma)}|g'(\varsigma,u)|}_{\text{Proposal Ratio}}$$

   - **2.6** If the draw is accepted set $\varsigma_i = \varsigma'$. If the draw is rejected set $\varsigma_i = \varsigma$

Appendix: Acceptance probability

Acceptance probability is chosen analogously to Metropolis-Hastings

$$\chi_{pp'}(\varsigma,\varsigma') = \underbrace{\frac{\mathscr{L}(\varsigma')}{\mathscr{L}(\varsigma)}}_{\text{Likelihood Ratio}} \underbrace{\frac{\rho(\varsigma')}{\rho(\varsigma)}}_{\text{Prior Ratio}} \underbrace{\frac{\gamma_p(p|p')\gamma_{p'p}(g_{pp'}(P^p,u))}{\gamma_p(p'|p)\gamma_{pp'}(P^p,u)}|g'_{pp'}(P^p,u)|}_{\text{Proposal Ratio}=\frac{\gamma(\varsigma|\varsigma')}{\gamma(\varsigma'|\varsigma)}|g'_{pp'}(P^p,u)|}$$

$|g'_{pp'}(P^p,u)|$ is the absolute value of determinant of the Jacobian of $g_{pp'}$ and equal to one with our mapping function

It shows up due to the application of the change-of-variable formula in the derivation of the acceptance probability

Appendix: RJMCMC Sampler Proposals for AR(p) Processes

Now proposals for the AR parameters and model order are constructed as follows:

1. Draw a new model order p' from $\gamma(p'|p)$
2. Draw a vector u with dimension p' from $\gamma_{pp'}(P^p, u)$
3. Map the proposal u to the new state using $g_{pp'}$

$$\begin{bmatrix} P^{p'} \\ u' \end{bmatrix} = g_{pp'}(P^p, u) = \begin{bmatrix} A(p,p')_{p' \times p} & I_{p' \times p'} \\ I_{p \times p} & 0_{p \times p'} \end{bmatrix} \begin{bmatrix} P^p \\ u \end{bmatrix} \tag{3}$$

where

$$A(p,p') = \begin{cases} \begin{bmatrix} I_{p \times p} \\ 0_{(p'-p) \times p} \end{bmatrix} & \text{if } p' > p \\ \begin{bmatrix} I_{p' \times p'} 0_{p' \times (p-p')} \end{bmatrix} & \text{if } p' < p \\ \begin{bmatrix} I_{p' \times p'} \end{bmatrix} & \text{if } p' = p \end{cases} \tag{4}$$

For $p = p'$ this mapping gives a standard Random-Walk sampler

Appendix: RJMCMC Sampler Kernel for AR(p) Processes

This algorithm defines a Markov Chain with Kernel

$$\mathscr{K}\left((P^p,p),\mathscr{B}_{\varsigma'}\right)=\underbrace{\gamma\left(p'|p\right)}_{(1)}\int\underbrace{\gamma_{pp'}(P^p,u|p')}_{(2)}\underbrace{\alpha_{pp'}\left((P^p,p),\left(p',g_{pp'}(P^p,u)\right)\right)}_{(3)}\times$$

$$\underbrace{g_{pp'}((P^p,p),u)}_{(4)}\underbrace{1\left(g_{pp'}(P^p,u)\in\mathscr{B}_{p'}\right)}_{(5)}du+P(\text{Rejecting the move and }\varsigma'\in\mathscr{B}_{\varsigma'})$$

- **(1):** Probability of proposing a visit to model p'
- **(2):** Probability of proposing u
- **(3):** Probability of accepting the proposal
- **(4):** Mapping from $((P^p,p),u)$ to $((P^{p'},p'),u')$
- **(5):** Indicator $=1$ if proposal in parameter space of model p'

Appendix: Detailed Balance: Solution

Green (1995) modifies the proposals by a change-of-variables such that both sides of the detailed balance condition are of equal dimensionality:

1. Introduce auxiliary variable u with proposal density $\gamma_{pp'}(P^p, u)$ together with an appropriately chosen differentiable bijection

$$(P^{p'}, u') = (g_{1pp'}(P^p, u), g_{2pp'}(P^p, u)) = g_{pp'}(P^p, u) \qquad (5)$$

   such that $\pi(P^p|p)\gamma_{pp'}(P^p, u)$ and $\pi(g_{1pp'}(P^p, u)|p')\gamma_{p'p}(g_{pp'}(P^p, u))$ are joint densities on spaces of equal dimensionality and $d\varsigma' du' = |g'_{pp'}(\varsigma, u)| d\varsigma du$
2. Plug into Kernel, do a change-of-variables and choose the appropriate acceptance probability

Appendix: Information Criteria

$$\text{AIC} = 2k - 2ln(\mathscr{L})$$

$$\text{AICC} = AIC + \frac{2k(k+1)}{n-k-1}$$

$$\text{BIC} = -2ln(\mathscr{L}) + kln(n)$$

with k being the number of model parameters and n the number of observations

Appendix: Priors and Proposals MC Study

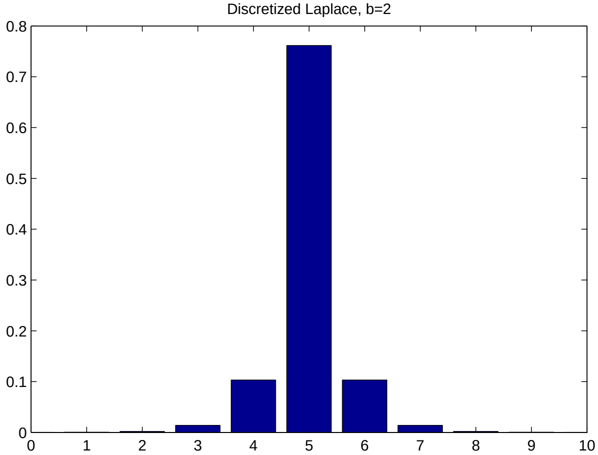
Variable	Prior	Proposal
p	U(0,10)	LaplaceD(p,2)
q	U(0,10)	LaplaceD(q,2)
AR PAC	TN(0,0.25)	TN(PAC,0.0025)
MA PAC	TN(0,0.25)	TN(PAC,0.0025)
σ	IG(1,1)	TN(σ,0.0025)

LaplaceD(μ, b) is a discretised Laplace distribution with location parameter μ and shape parameter b, such that $\gamma(p'|p) \propto exp(-b|p-p'|)$ with $p', p \in [1, 2, ..., p_{max}]$

TN is the truncated normal distribution and IG is the inverted gamma distribution

Appendix: Discretized Laplace

Discretized Laplace with b=2



Appendix: Study Setup

1. Generate 100 time series with 100 observations each from the ARMA(3,2) process given by

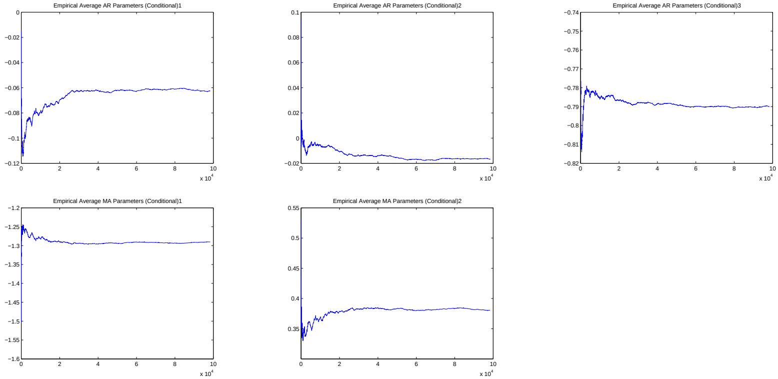
$$y_t = -0.75 y_{t-3} + \epsilon_t - 1.5 \epsilon_{t-1} + 0.5625 \epsilon_{t-2}$$

   with $\epsilon_t \sim N(0, 1.5^2)$ as in (Philippe, 2006)
2. For each data set generate 1,500,000 draws from the posterior, discarding the first 1,000,000 draws as burn-in
3. Identify the posterior mode in (p, q) giving the preferred model
4. Compare with model choice using MLE together with AIC, AICC and BIC (R routine auto.arima)

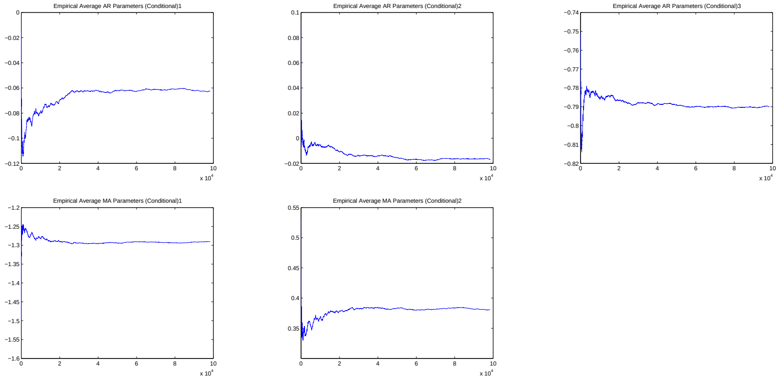
Appendix: Results

Method	Proportion of correctly identified models
RJMCMC	0.5
AIC	0.36
AICC	0.44
BIC	0.71

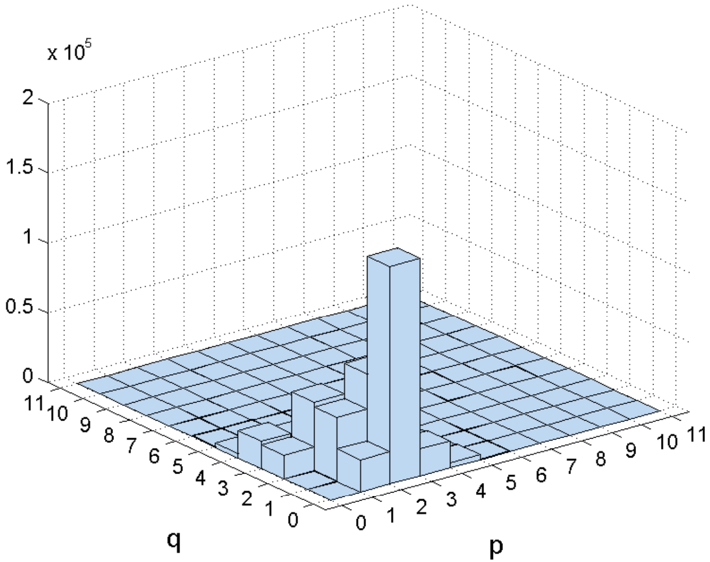
Appendix: Prior vs. Posterior ARMA(3,2) Synthetic



Appendix: Typical Conditional Empirical Averages MC Study

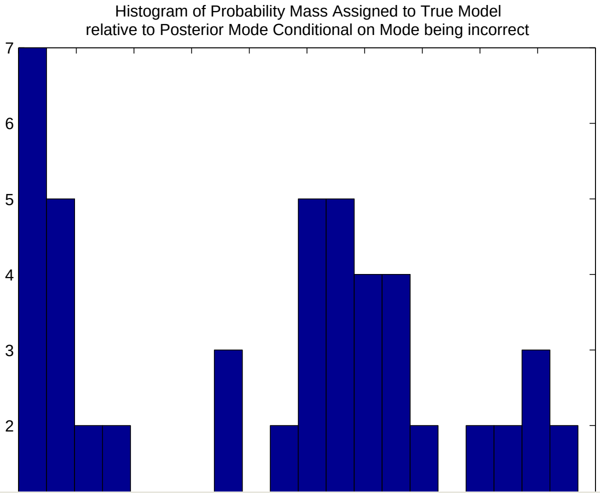


Appendix: A Typical Posterior



Appendix: Posterior Mass True Model vs. Model at Mode

Posterior Mass assigned to true model relative to posterior mass at mode conditional on the mode being wrong:



Appendix: Solution Method and Likelihood Function

Short

The solution method employed to solve the model under different specifications of the shock process is a method of undetermined coefficients approach giving a unique infinite moving average representation

$$X_t = \left(\underset{n_x \times n_x}{I} - \Lambda L \right)^{-1} \left[\Phi(L) P(L)^{-1} Q(L) + \Theta(L) \right] \epsilon_t \tag{6}$$

Given the model solution, the Likelihood is calculated as follows:

1. Calculate the spectrum of the process
2. Apply an inverse Fourier Transform to recover the sequence of autocovariances
3. Calculate Likelihood treating the vector of observations as one draw from a multivariate normal distribution

Appendix: Solution Method

The solution method employed to solve the model under different specifications of the shock process is a method of undetermined coefficients approach

We express the exogenous processes in vector form as

$$\underset{n_z \times 1}{Z_t} = P_1 Z_{t-1} + P_2 Z_{t-2} \ldots + P_p Z_{t-p} + Q_0 \underset{nz \times 1}{\epsilon_t} + Q_1 \epsilon_{t-1} \ldots + Q_q \epsilon_{t-q} \tag{7}$$

where p is the highest autoregressive order and q the highest moving average order present.

The solution for the endogenous variables is given by

$$X_t = \Lambda X_{t-1} + \Phi_0 Z_t + \Phi_1 Z_{t-1} \ldots + \Phi_{\tilde{p}-1} Z_{t-(\tilde{p}-1)} + \Theta_0 \epsilon_t + \Theta_1 \epsilon_{t-1} \ldots + \Theta_{q-1} \epsilon_{t-(q-1)} \tag{8}$$

The end result is a unique infinite moving average representation given by

$$X_t = \left(\underset{n_x \times n_x}{I} - \Lambda L \right)^{-1} \left[\Phi(L) P(L)^{-1} Q(L) + \Theta(L) \right] \epsilon_t \tag{9}$$

Appendix: Likelihood Function

Given the model solution, the Likelihood is calculated as follows:

1. Calculate the spectrum of the process
2. Apply an inverse Fourier Transform to recover the sequence of autocovariances
3. Calculate Likelihood treating the vector of observations as one draw from a multivariate normal distribution

Appendix: Model Application Setup

- Use neoclassical growth model with calibrated model parameters from Hansen (1985)

$\overline{L}$	$\frac{1}{3}$	Steady state employment 1/3 of total time endowment
$\overline{Z}$	1	Normalization of productivity
α	0.36	Capital share
δ	0.025	Depreciation rate for capital
$\overline{R}$	1.01	One percent real interest rate per quarter

- These values are taken from great ratios, i.e. the capital share for the calibration of ρ
- Run estimation procedure holding the model parameters fixed using HP-filtered quarterly US GDP per capita as in Hansen (1985) with $\lambda = 1600$ (263 observations)
- Analyze posterior over models and parameters

Appendix: Hansen (1985) Neoclassical Growth Model

FOCs

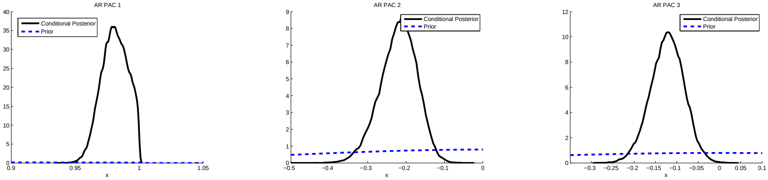
First order conditions:

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(1 - \delta + \alpha e^{z_{t+1}} \left(\frac{l_{t+1}}{k_t} \right)^{1-\alpha} \right) \right]$$

$$\frac{\psi}{1 - l_t} = \frac{1}{c_t} (1 - \alpha) e^{z_t} \left(\frac{k_{t-1}}{l_t} \right)^{\alpha}$$

Appendix: Priors, Posteriors and Convergence

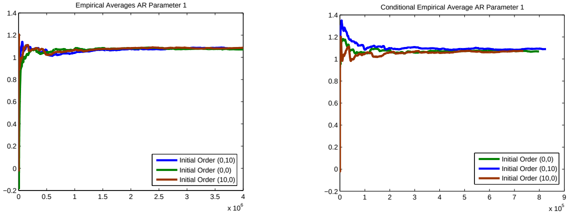
Priors and Posteriors for first Partial Autocorrelation



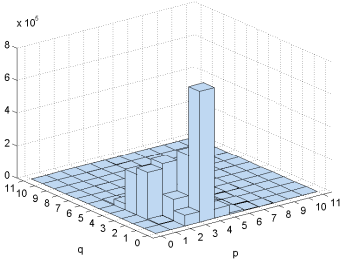
Recursive Averages

Unconditional

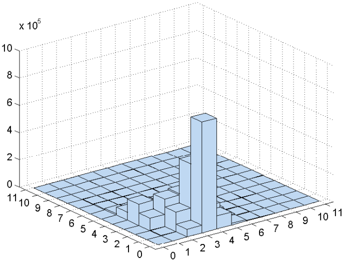
Conditional



Appendix: Posteriors over p,q both HP Filters



Two-Sided HP Filter



One-Sided HP Filter

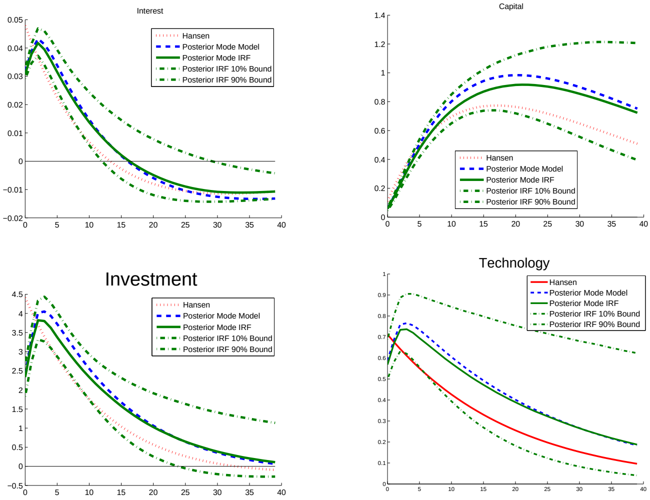
Process at the mode: ARMA(3,0)

Appendix: Parameter Estimates Both HP-Filters

Parameter HP-Filter	Mean 1	Median 1	Mean 2	Median 2	Hansen
AR(1)	1.1025 (0.05)	1.1034	1.1689 (0.04)	1.1681	0.95
AR(2)	-0.0913 (0.08)	-0.0921	-0.0732 (0.06)	-0.0725	N/A
AR(3)	-0.1679 (0.05)	-0.1679	-0.1224 (0.04)	-0.1215	N/A
σ	0.3303 (0.02)	0.3280	0.5873 (0.08)	0.5733	0.712
$\rho(1)$	0.8954	0.8957	0.9804	0.9810	0.95
$\rho(2)$	0.7453	0.7458	0.9528	0.9542	0.9025

All estimates are based on the posterior distribution conditional on the mode of the posterior in (p,q), that is $(p,q) = (3,0)$ from the chain started at $(p,q) = (0,0)$. Standard Errors in parentheses. $\rho(i)$ denotes the autocorrelation at lag i.

Appendix: Impulse Responses Two-Sided HP Filtering



Appendix: Correlation Structure Output

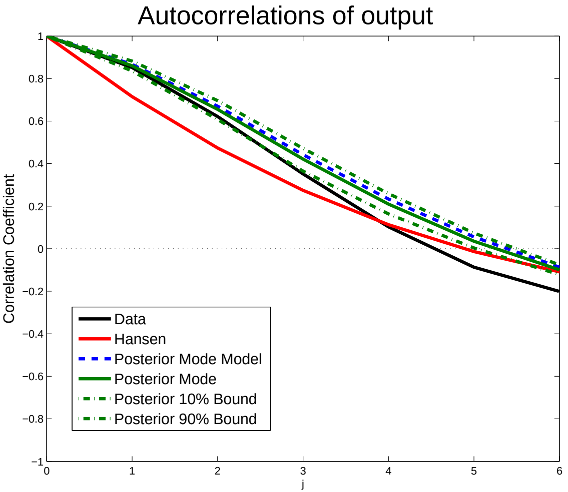


Figure A-1. Comparison of Autocorrelations of Output

Appendix: Sampling Noninvertible Representations

- With flat priors over orders and priors over PACS/inverse PACs
  - Posterior the same with invertible and noninvertible
- Take roots λ_i from

$$1 + Q_1^q L + \ldots + Q_q^q L^q = (1 - \lambda_1 L)(1 - \lambda_2 L)\ldots\left(1 - \lambda_q L\right)$$

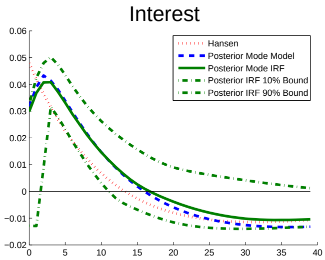
- Sample uniformly from admissible invertible and noninvertible MA representations

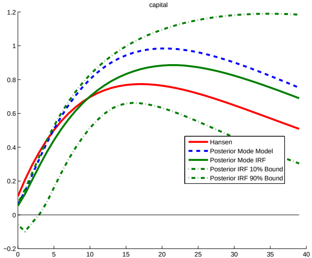
Appendix: Sampling Noninvertible Representations: Example

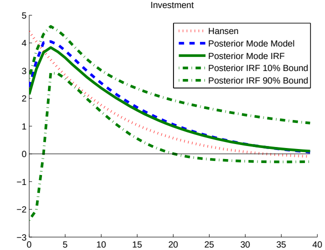
- Calculate the number different admissible - i.e. accounting for complex conjugate pairs - root flips $\tilde{n}$
- Draw candidates for flipping uniformly from $\{0, 1, \ldots, \tilde{n}\}$
- If e.g. the draw is associated with flipping roots λ_2, λ_3 the chosen MA representation is

$$\gamma_i(L) = (-\lambda_2)(-\lambda_3)\left(1 - \frac{1}{\lambda_2}L\right)\left(1 - \frac{1}{\lambda_3}L\right)(1 - \lambda_1 L)(1 - \lambda_4 L)\ldots(1 - \lambda_{q_i} L)$$

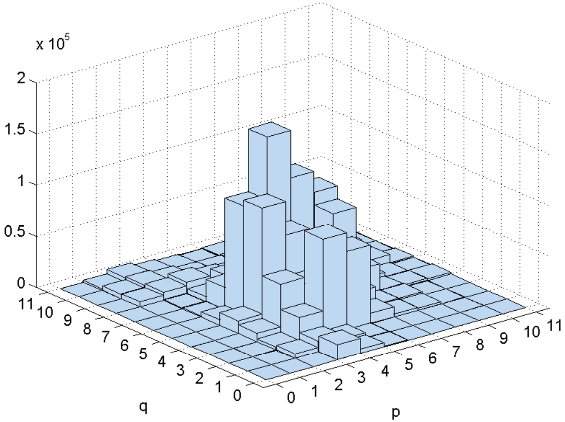
Appendix: Noninvertible Impulse Responses







Appendix: Posterior Over p,q Without Appropriate Filtering



Process at the mode: ARMA(4,5)

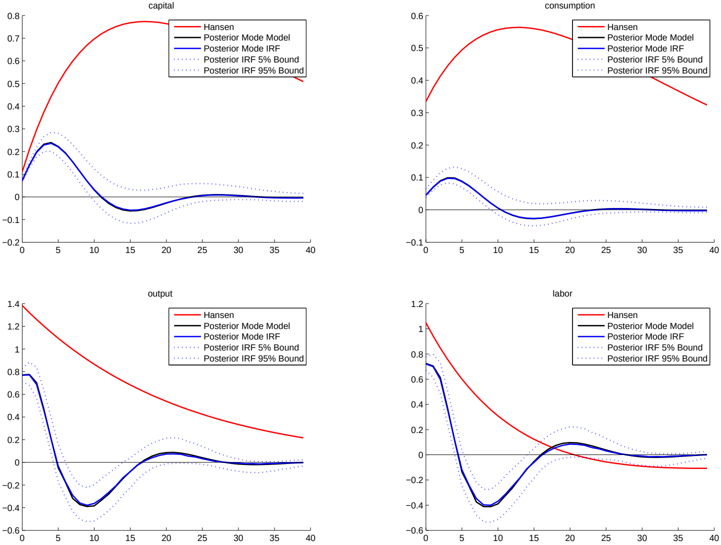
Appendix: Parameter Estimates without HP-Filtering

Model Output

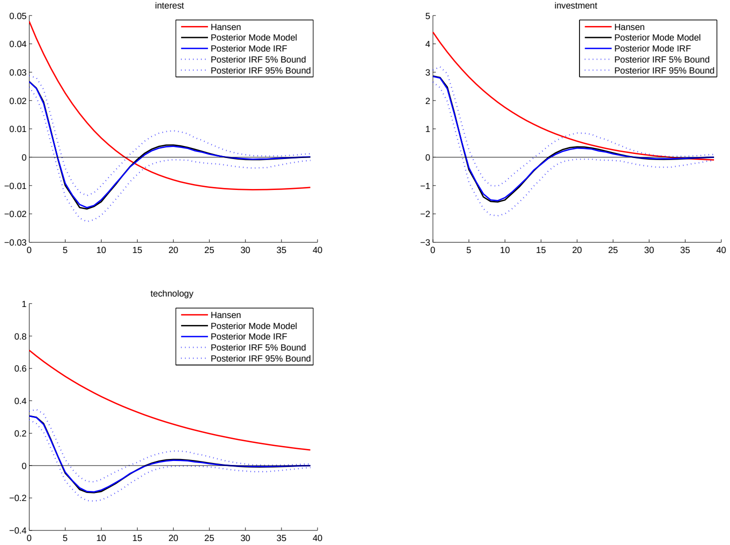
Parameter	Conditional Mean	Conditional Median	Hansen
AR(1)	0.54166	0.48353	0.95
AR(2)	0.64303	0.7219	N/A
AR(3)	0.019153	0.11174	N/A
AR(4)	-0.41535	-0.45589	N/A
MA(1)	0.43013	0.4824	N/A
MA(2)	-0.32231	-0.28857	N/A
MA(3)	-0.57495	-0.60161	N/A
MA(4)	-0.25796	-0.25725	N/A
MA(5)	-0.20054	-0.20275	N/A
σ	0.3053	0.3046	0.712
Autocorr(1)	0.84565	0.84658	0.95
Autocorr(2)	0.60802	0.64097	0.9025

All estimates are based on the posterior distribution conditional on the mode of the posterior in (p,q), that is $(p,q) = (4,5)$

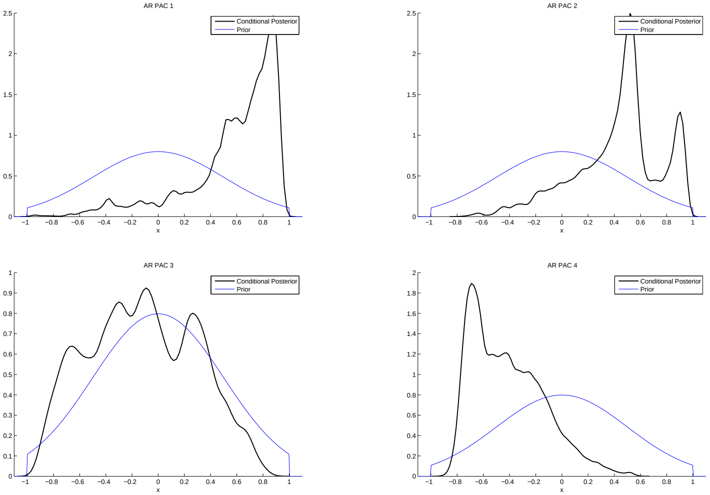
Appendix: Implied Impulse Responses (1 Std Dev) Without Appropriate Filtering



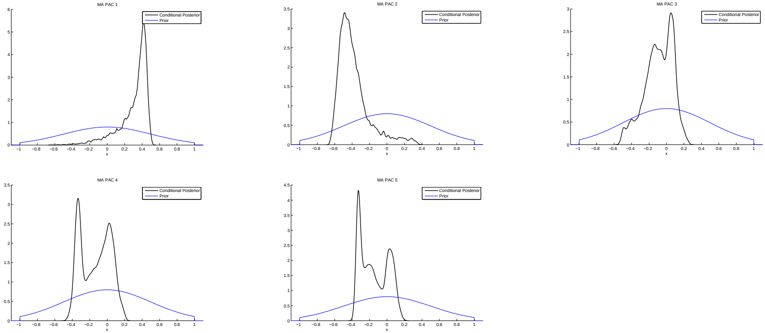
Appendix: Implied Impulse Responses (1 Std Dev) Without Appropriate Filtering cont.



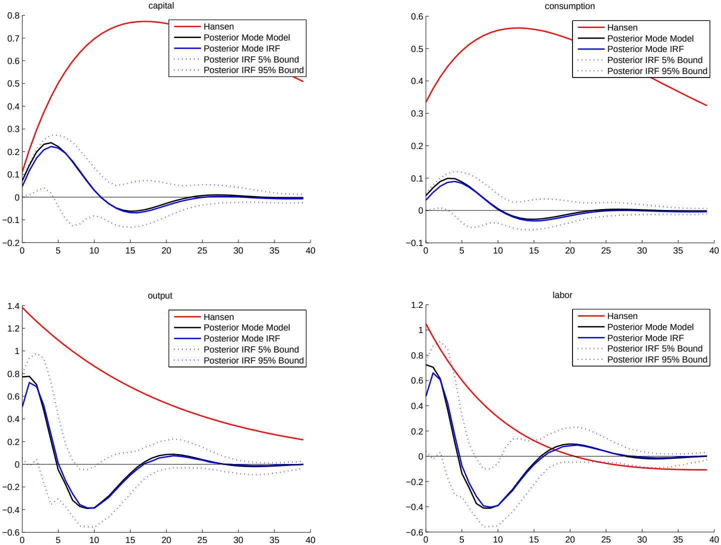
Appendix: Prior vs. Posterior PACs Without Appropriate Filtering



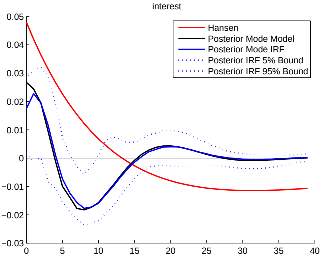
Appendix: Prior vs. Posterior PACs Without Appropriate Filtering

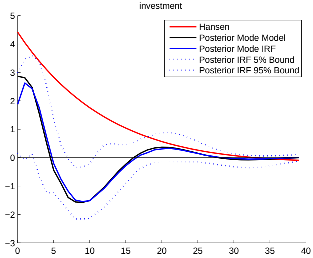


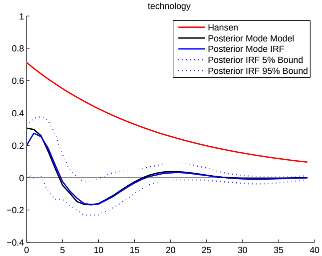
Appendix: Implied Impulse Responses with Nonfundamental Representations(1 Std Dev)



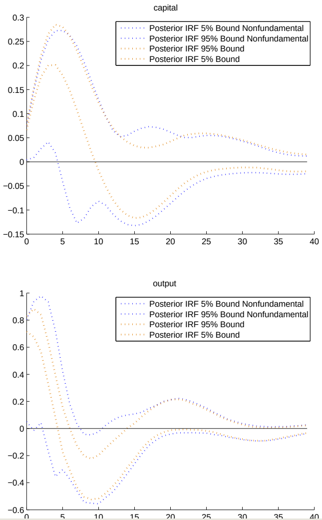
Appendix: Implied Impulse Responses with Nonfundamental Representations(1 Std Dev)



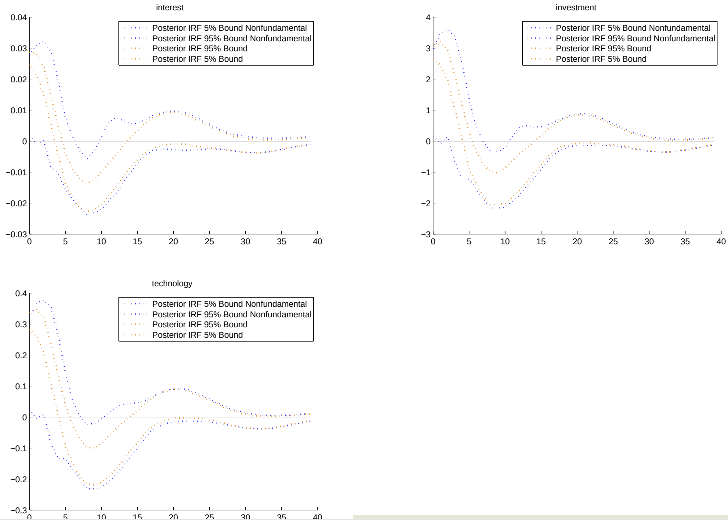




Appendix: Impulse Responses Credible Set Nonfundamental vs Fundamental Representations(1 Std Dev)



Appendix: Impulse Responses Credible Set Nonfundamental vs Fundamental Representations(1 Std Dev)



Appendix: Correlation Structures

Data	Hansen	Posterior Mode Model	Posterior Mode	90% Posterior Credible Set
2.8491	3.2574	2.8332	2.8182	2.1074 — 4.0965

Table A-1. Standard Deviation of Output, in %

Appendix: Correlation Structure Without Appropriate Filtering

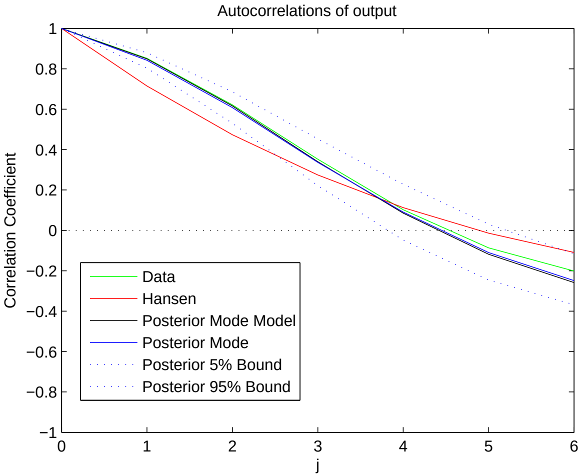


Figure A-2. Comparison of Autocorrelations of Output

Appendix: Correlation Structures cont.

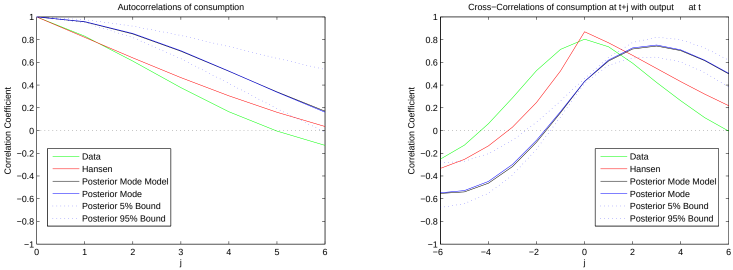


Figure A-3. Comparison of Autocorrelations of Consumption and of Cross-Correlations between Consumption and Output

Appendix: Future research

- Combine estimation of ARMA processes with Bayesian estimation of DSGE Models including parameters
- Improve proposals for DSGE estimation
- Assess forecasting performance of DSGE models with "optimal" shock processes and compare with VAR, BVAR as well as DSGE-VAR
- Estimation of non-stationary ARMA processes using an approximate likelihood function
- Impulse Responses for non-fundamental MA representations
- Model selection based on the comparison of spectra between models with "optimal" shock process and white noise disturbance following Watson (1993)
- Systematic exploration of "fixes" for lacking propagation