Discussion of the talk

Efficient and semi-positive definite pre-averaging realized covariance estimator

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CRC 649 Conference · Motzen · July 17, 2015

- Estimation of the integrated volatility matrix Σ in the setting of high sampling frequency:

$$d\mathbf{X}_t = \boldsymbol{\mu}_t dt + \boldsymbol{\sigma}_t^\top d\mathbf{B}_t, \quad t \in [0,1],$$

$$\Sigma \stackrel{\text{def}}{=} \int_0^1 \boldsymbol{\sigma}_t^\top \boldsymbol{\sigma}_t dt,$$

$\mathbf{X}_{t_j} \stackrel{\text{def}}{=} \left(X_{1,t_j}, \ldots, X_{p,t_j}\right)^*$ are efficient log-prices, they are regularly spaced ($t_j = j/n$, $j = 0, \ldots, n$), synchronous and noise free but unobservable.

- $\mathbf{X}_{t_j}$ are contaminated with microstructure noise:

$$\mathbf{Y}_{t_j} \stackrel{\text{def}}{=} \mathbf{X}_{t_j} + \boldsymbol{\epsilon}_{t_j}.$$

- The assets' log-prices are observed asynchronously:

$$\mathbf{Y}_{t_{j_1}} = \left(Y_{1,t_{j_1}}, Y_{2,t_{j_1}}, \cancel{Y_{3,t_{j_1}}}, \ldots, Y_{p-1,t_{j_1}}, Y_{p,t_{j_1}}\right)^*,$$

$$\mathbf{Y}_{t_{j_2}} = \left(\cancel{Y_{1,t_{j_2}}}, Y_{2,t_{j_2}}, Y_{3,t_{j_2}}, \ldots, \cancel{Y_{p-1,t_{j_2}}}, Y_{p,t_{j_2}}\right)^*.$$

Challenges

- The microstructure noise causes biasness of the realized volatility. The bias-correction approaches may destroy the nonnegative definiteness of the covariance estimator.
- The asynchronicity causes problems with efficiency of the estimates and their nonnegative definiteness.
- **The goal** is to estimate the integrated volatility matrix Σ with the optimal rate $O_p(n^{-1/4})$ ensuring the nonnegative definiteness of the estimator.

The approach and the theoretical results

- The novel synchronizing technique: **High Frequency Filtering**: the algorithm minimizes the distance of the available asynchronous data and an underlying synchronous filtered series.
- Efficient semi-positive definite integrated covariance matrix estimator based on the corrected data: the eigenvalues in the spectral decomposition of the initial estimator are replaced with their absolute values.

Questions

- Confidence estimation.
- High dimensional case: the number p of assets grows with the sample size n.
- Closeness of the obtained estimator and the target matrix in e.g. spectral norm.