Forecasting Volatility of Wind Power Production

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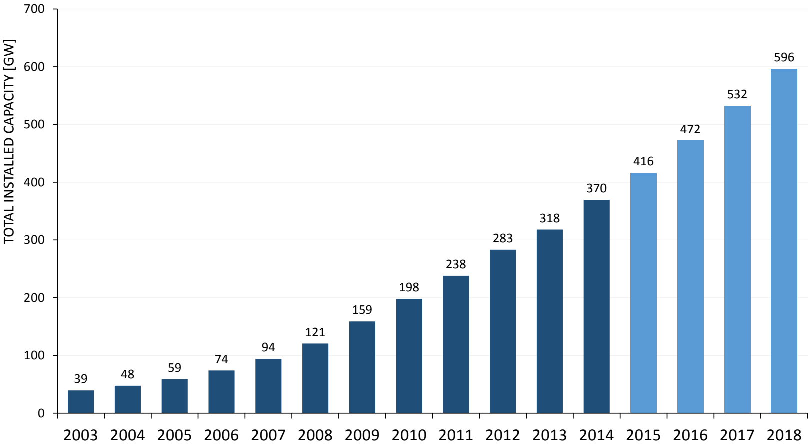
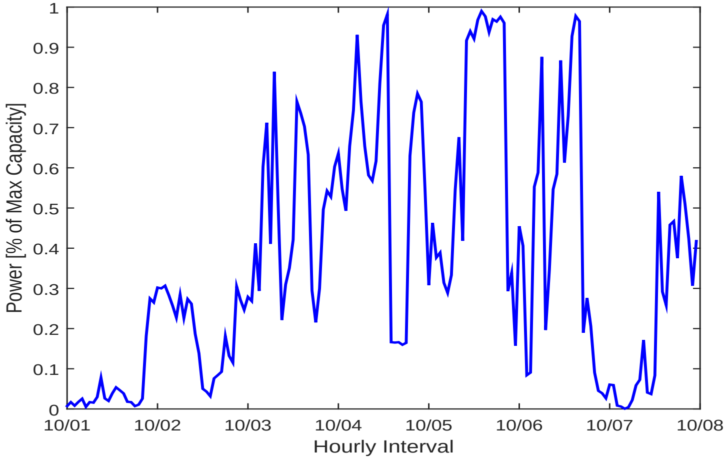
Wind Power Development



Data: GWEC (2014,2015)

Volatile Wind Power



Data: 4initia GmbH

Wind power forecasts

Motivation

- It fluctuates significantly due to changing weather condition.
- Volume risk: non-storable commodity.
- Importance in prediction of wind power production:
  - optimize power plant scheduling to balance supply and demand for a regional or national grid.
  - help energy traders make informed decision on how much they can offer or bid in the next trading cycle.

Wind power forecasts

Approaches

- Models
  - Meteorological models: Weather prediction + Power curve(Monterio et al., 2009);
  - Statistical models: time series, data mining such as neural networks or support vector machines (Giebel et al., 2011)

Wind power forecasts

Approaches

- Models
  - Meteorological models: Weather prediction + Power curve(Monterio et al., 2009);
  - Statistical models: time series, data mining such as neural networks or support vector machines (Giebel et al., 2011)
- Forecasts of wind power production
  - Point forecasts /Mean forecasts: a single value of conditional expectation of wind power production.
  - Probabilistic forecasts: such as quantile or interval forecast (Bremnes, 2004; Anastasiades and McSharry, 2013) ,or full predictive density forecasts (Lau and McSharry, 2010)

Wind power forecasts

Volatility forecast of wind power production

- Features of wind power production
  - not only wind speed but also wind volatility is time-varying.
  - time-varying heteroscedasticity similar to financial market (Lau and McSharry, 2010)
  - affected by ramp events[1]
- Volatility modelling
  - Seasonal: Fourier series (Alexandridis and Zapranis, 2013)
  - ARCH or GARCH models (Tastu et al., 2014; Liu et al., 2011; Lau and McSharry, 2010)

[1] ramp event: energy output changes by substantial fraction of the capacity within a short time

Objective

Research questions

- Which model performs best in terms of volatility forecasting?
- Traditional GARCH models may be too restrictive to capture random breaks and non-linear behaviour of wind power data.

Objectives

- Identify the best predictive model for the volatility of wind power production
- Develop Markov regime switching GARCH model to capture the dynamics of wind power production

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GARCH Models

Considering a time series of wind power y_t, we use the AR (k) model:

$$y_t = c + \sum_{i=1}^{k} \phi_i y_{t-i} + \epsilon_t \tag{1}$$

$$\epsilon_t = \eta_t h_t, \qquad \eta_t \sim (0, 1) \tag{2}$$

- GARCH

$$h_t^2 = \omega + \sum_{i=1}^{m} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j h_{t-j}^2 \tag{3}$$

- GARCH(1,1)

$$h_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \tag{4}$$

Asymmetric GARCH

- Exponential GARCH (EGARCH)

$$\ln(h_t) = \omega + \alpha(|\eta_{t-1}| - E[|\eta_{t-1}|]) + \gamma\eta_{t-1} + \beta \ln(h_{t-1}) \quad (5)$$

where α and γ capture the asymmetric effect of magnitude and sign of η on volatility. $\gamma < 0$, the negative value of ϵ results in higher volatility than the positive value. No restriction on the parameters.

Asymmetric GARCH

- Threshold GARCH (Zakoian,1994)[2]

$$h_t = \omega + \alpha(|\epsilon_{t-1}| - \gamma\epsilon_{t-1}) + \beta h_{t-1} \tag{6}$$

- GJR GARCH (Glosten et al., 1993)

$$h_t^2 = \omega + \alpha(|\epsilon_{t-1}| - \gamma\epsilon_{t-1})^2 + \beta h_{t-1}^2. \tag{7}$$

- Nonlinear GARCH (Engle and Ng, 1993)

$$h_t^2 = \omega + \alpha(\epsilon_{t-1} - \gamma h_{t-1})^2 + \beta h_{t-1}^2 \tag{8}$$

where γ reflects the asymmetric effect. $\gamma > 0$, all these three models indicated that negative error terms increase future volatility by larger amount than positive ones.

[2] The following formulas are rearranged and adopted from Ding et al., (1993).

Markov regime switching GARCH Model

Markov regime switching model (Hamilton, 1989)

- Multiple discrete regimes or states governed by a state variable s_t
- Different dynamics and sets of parameters on each regime
- Regime switching by a first-order Markov chain

The state variable s_t is assumed to evolve with transition probability:

$$\mathrm{P}(s_t = j | s_{t-1} = i) = p_{ij} \tag{9}$$

In the case of two regimes $s_t \in \{1, 2\}$, the transition matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix} \tag{10}$$

Markov regime switching GARCH model

Markov regime switching GARCH model

- Conditional mean: Markov regime switching AR(p)

$$y_t = c^{(j)} + \sum_{i=1}^{k} \phi_i^{(j)} y_{t-i} + \epsilon_t \qquad \epsilon_t = \eta_t h_t, \quad \eta_t \sim (0,1) \tag{11}$$

where subscript (j) denotes the regime of the process at t.

- Conditional variance: Markov regime switching GARCH(1,1)

$$h_t^{2|(j)} = \alpha_0^{(j)} + \alpha_1^{(j)} \epsilon_{t-1}^2 + \beta_1^{(j)} h_{t-1}^2 \tag{12}$$

h_{t-1} is a state-independent average of past conditional variances. Using expression of Klaassen (2002):

$$h_t^{2|(j)} = \alpha_0^{(j)} + \alpha_1^{(j)} \epsilon_{t-1}^2 + \beta_1^{(j)} \mathbb{E}_{t-1}\{h_{t-1}^{2|(j)} | s_t\} \tag{13}$$

Estimation: Markov regime switching GARCH model

Estimation

The conditional probability of s_t being in regime $j = 1, 2$, given the information set $\Omega_t = \{y_t, y_{t-1}...y_1\}$ and parameters Θ:

$$\xi^{(j)}_{s_t|\Omega_t} = \mathrm{P}(s_t = j|\Omega_t, \Theta) \tag{14}$$

To derive $\xi^{(j)}_{s_t|\Omega_t}$, reformulating (14) via conditional probability of y_t at $s_t = j$:

$$\xi^{(j)}_{s_t|\Omega_t} = \frac{f(y_t, s_t = j|\Omega_{t-1}, \Theta)}{f(y_t|\Omega_{t-1}, \Theta)} \tag{15}$$

where $f(y_t, s_t = j|\cdot)$: joint conditional density of y_t and regime j.

$$f(y_t, s_t = j|\Omega_{t-1}, \Theta) = \xi^{(j)}_{s_t|\Omega_{t-1}} f(y_t|s_t = j, \Omega_{t-1}, \Theta) \tag{16}$$

$$\xi^{(j)}_{s_t|\Omega_{t-1}} = \mathrm{P}(s_t = j|\Omega_{t-1}, \Theta) \tag{17}$$

$$\boldsymbol{\xi}_{s_t|\Omega_{t-1}} = \mathbf{P}^T \boldsymbol{\xi}_{s_{t-1}|\Omega_{t-1}} \tag{18}$$

where $\xi^{(j)}_{s_t|\Omega_{t-1}}$ forecast of probability of s_t being in j; $\mathbf{P}^T$ transition matrix .

Estimation: Markov regime switching GARCH model

Log-likelihood function

The conditional density of y_t:

$$f(y_t|\Omega_{t-1},\Theta) = \sum_{j=1}^{2} f(y_t, s_t = j|\Omega_{t-1},\Theta)$$
$$= \sum_{j=1}^{2}\sum_{i=1}^{2} p_{ij}\xi_{t-1}^{j} f(y_t|s_t = j, \Omega_{t-1},\Theta) \quad (19)$$

The log-likelihood function:

$$L(y_1, y_2, ...y_T|\Theta) = \sum_{t=1}^{T} \log f(y_t|\Omega_{t-1},\Theta) \quad (20)$$

Forecasting Evaluation

Measures (Patton, 2011; Byun and Cho, 2013)

$\hat{h}_t^2$: conditional volatility forecast at t and $\hat{\sigma}_t^2$ is the *ex post* proxy of conditional variance representing the actual volatility at t. Statistical loss functions:

- Root Mean Square Error (RMSE) $\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=T+1}^{T+N}(\hat{\sigma}_t^2 - \hat{h}_t^2)^2}$
- RMSE-LOG $\text{RMSE-LOG} = \sqrt{\frac{1}{N}\sum_{t=T+1}^{T+N}(\log\hat{\sigma}_t^2 - \log\hat{h}_t^2)^2}$
- Mean Absolute Error (MAE) $\text{MAE} = \frac{1}{N}\sum_{t=T+1}^{T+N}|\hat{\sigma}_{t+1}^2 - \hat{h}_{t+1|t-k}|$
- MAE-LOG $\text{MAE-LOG} = \frac{1}{N}\sum_{t=T+1}^{T+N}|log\hat{\sigma}_{t+1}^2 - \log\hat{h}_{t+1|t-k}|$
- QLIKE $\text{QLIKE} = \frac{1}{N}\sum_{t=T+1}^{T+N}(\log(\hat{h}_t^2) + \frac{\hat{\sigma}_t^2}{\hat{h}_t^2})$

In this paper, we use realized volatility as $\hat{\sigma}_t^2$.

Forecasting Evaluation

Diebold-Mariano (DM) test

The DM test is applied to determine if the predictive accuracies of competing models are significantly different.

- Define loss function differential between model a and model b:

$$d_t = g(e_{a,t}) - g(e_{b,t})$$

where $g(\cdot)$ means a loss function and $e_{\cdot,t}$ corresponds to forecast errors from the competing models.

- Two typical loss functions: $g(e_t) = e_t^2$ or $g(e_t) = |e_t|$.
- H_0: $E(d_t) = 0$, DM test statistic:

$$DM = \bar{d}/\sqrt{\hat{V}(\bar{d})} \sim N(0,1)$$

where $\bar{d} = \frac{1}{n}\sum_{t-1}^{n} d_t$ and $\hat{V}(\bar{d}) = \frac{2\pi \hat{f}_d(0)}{T}$

Contents

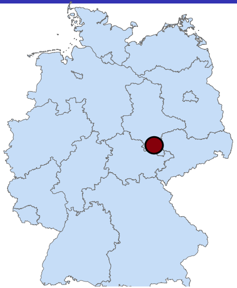
Data

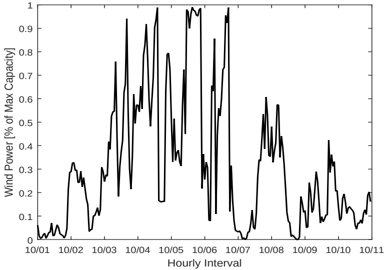
Wind power production

- Hourly interval data for 1.10.2012–31.12.2013 (Estimation) 1.1.2014–07.01.2014 (Evaluation)
- 6 turbines with capacity 2.3MW
- Normalization by max. capacity

Realized volatility

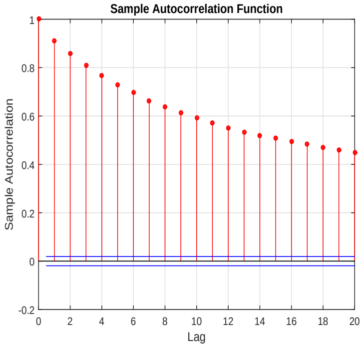
- Cumulative squared deviation over different time intervals.
- 10 mins interval data: $y_{t,i}$ $(i = 0, ..., 6)$
- RV: $\hat{\sigma}^2_{RV,t} = \sum_{i=1}^{6}(y_{t,i} - y_{t,i-1})^2$

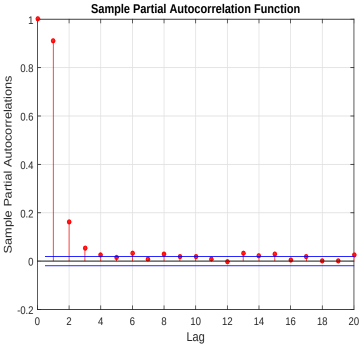




Data analysis

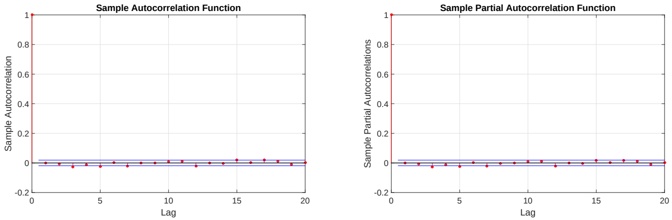
Wind power data:





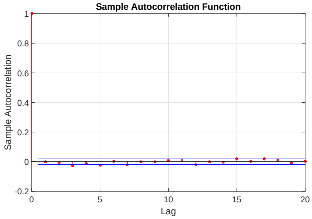
Data analysis

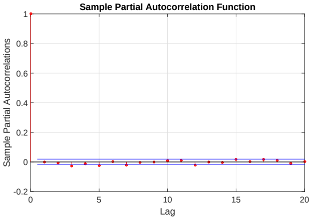
AR(3): ϵ



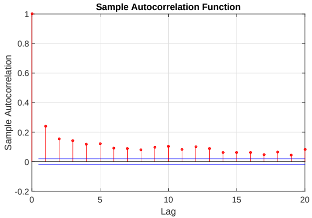
Data analysis

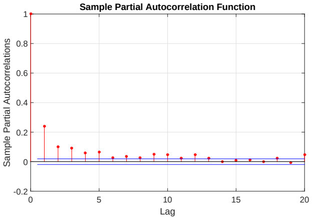
AR(3): ϵ



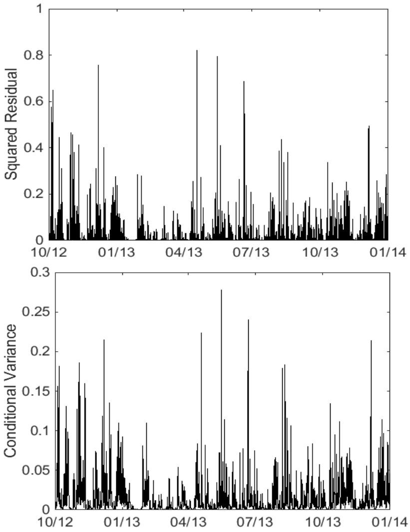


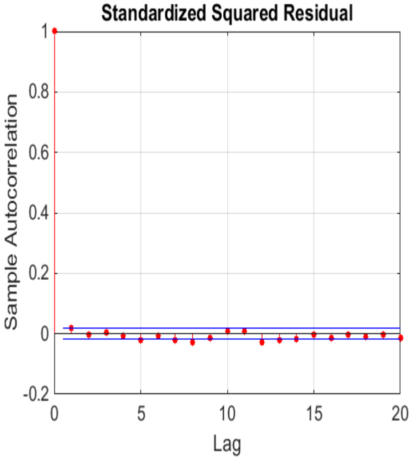
AR(3): ϵ^2





SGARCH models: AR(3)-GARCH(1,1)





Model comparison: In-sample

Table : In-sample goodness-of-fit statistics

Model	# Par.	Persistence	AIC	BIC	Log-likelihood	Rank
SGARCH	7	0.999	-2.167	-2.163	11690.55	6
EGARCH	8	0.959	-2.290	-2.284	12351.78	3
TGARCH	8	0.993	-2.297	-2.292	12394.37	2
GJR-GARCH	8	0.999	-2.233	-2.227	12043.92	5
NGARCH	8	0.945	-2.269	-2.264	12240.36	4
MRS-GARCH	16	0.999	-2.783	-2.773	15021.21	1

Note: for MRS-GARCH only highest persistence is reported. AIC is calculated as $(-2\log(L) + 2k)/T$, where k is the number of parameters and T is the number of observations. BIC is calculated as $(-2\log(L) + k\log(T))/T$.

Model comparison: Out-of-sample

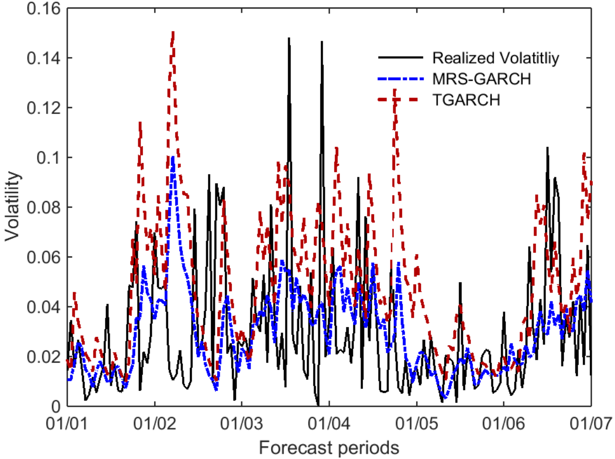


Fig. Realized volatility and 1-step ahead volatility from MRS-GARCH and TGARCH

Model comparison: Out-of-sample

Table : Out-of-sample evaluation for 1-step ahead volatility forecast

Model	RMSE (Rank)	RMSE-LOG (Rank)	MAE (Rank)	MAE-LOG (Rank)	QLIKE (Rank)
SGARCH	0.0335 (3)	1.9332 (2)	0.0244 (3)	0.9195 (4)	-2.443 (3)
EGARCH	0.0328 (2)	1.9845 (3)	0.0243 (2)	0.9117 (2)	-2.4592 (2)
TGARCH	0.0416 (6)	2.3421 (6)	0.0308 (6)	1.0139 (6)	-2.4227 (6)
GJR-GARCH	0.039 (5)	2.1738 (5)	0.0283 (5)	0.9687 (5)	-2.4321 (4)
NGARCH	0.0344 (4)	2.0626 (4)	0.0254 (4)	0.9134 (3)	-2.4813 (1)
MRS-GARCH	0.0295 (1)	1.7597 (1)	0.0207 (1)	0.8406 (1)	-2.4244 (5)

Model comparison: Out-of-sample

Table : Out-of-sample evaluation for 5-step ahead volatility forecast

Model	RMSE (Rank)	RMSE-LOG (Rank)	MAE (Rank)	MAE-LOG (Rank)	QLIKE (Rank)
SGARCH	0.1260 (3)	0.5487 (4)	0.0882 (4)	0.5694 (4)	-2.3679 (4)
EGARCH	0.1001 (2)	0.4279 (2)	0.0701 (2)	0.4770 (2)	-2.3737 (3)
TGARCH	0.1732 (6)	0.6709 (6)	0.1219 (6)	0.6300 (6)	-2.3677 (5)
GJR-GARCH	0.16 (5)	0.6468 (5)	0.1062 (5)	0.5951 (5)	-2.3507 (6)
NGARCH	0.1281 (4)	0.4805 (3)	0.0881 (3)	0.5133 (3)	-2.4104 (2)
MRS-GARCH	0.0982 (1)	0.4066 (1)	0.0676 (1)	0.4696 (1)	-2.4287 (1)

Model comparison: Out-of-sample

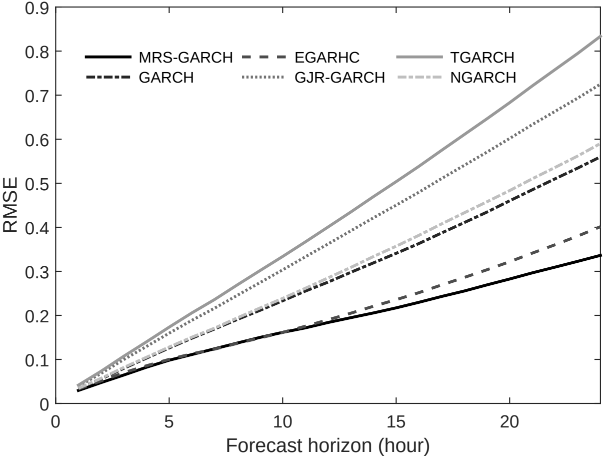


Fig. Forecast horizon and root mean square error

Model comparison: Out-of-sample

Table : Diebold-Mariano Test (1-step-ahead)

Model	Square error loss	Absolute error loss
MRS-GARCH	Benchmark	
SGARCH	-2.49^{***}	-3.64^{***}
EGARCH	-2.02^{***}	-3.44^{***}
TGARCH	-4.24^{***}	-5.54^{***}
GJR-GARCH	-3.86^{***}	-4.82^{***}
NGARCH	-2.86^{***}	-3.79^{***}

Note: the negative sign implies that benchmark's loss is lower than loss implied by other models. Asterisks *** denote significance at the 1% level .

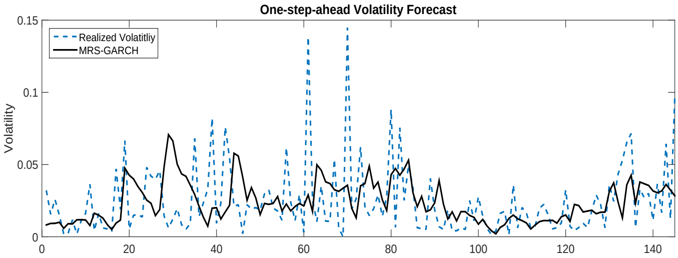
Model comparison: Out-of-sample

Table : Diebold-Mariano Test (1-step-ahead)

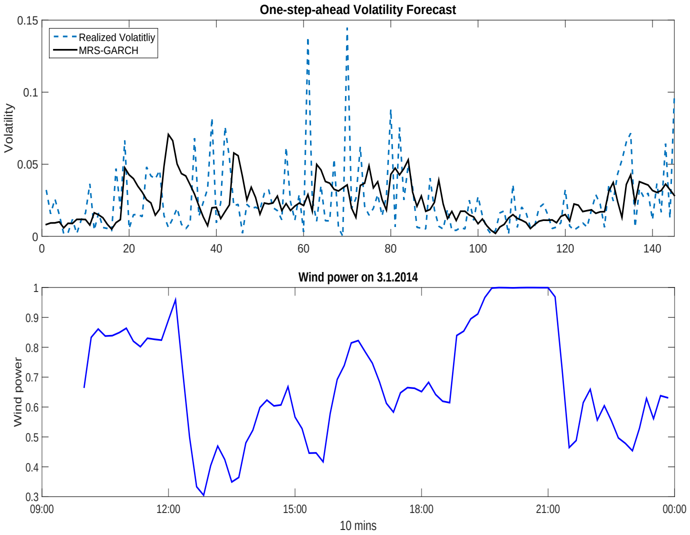
Model	Square error loss	Absolute error loss
EGARCH	Benchmark	
MRS-GARCH	2.01^{***}	3.44^{***}
SGARCH	-0.48^{*}	-0.06^{*}
TGARCH	-4.60^{***}	-5.81^{***}
GJR-GARCH	-4.13^{***}	-4.67^{***}
NGARCH	-1.12^{***}	-1.58^{***}

Note: the negative sign implies that benchmark's loss is lower than loss implied by other models. Asterisks *** and * denote significance at the 1% level and 10% level, respectively.

Model comparison: Jumps in realized volatility



Model comparison: Jumps in realized volatility



Conclusion

Findings

- MRS-GARCH model outperforms traditional GARCH models in in-sample and out-of-sample comparisons.
- EGARCH model might also be a fair choice to forecast the volatility of wind power production due to computational effort and information gain.
- The abrupt changes in realized volatility cannot be predicted due to the big instantaneous jumps in higher frequency wind power production.

Extensions

- Non-normal heavy tailed distributions
- Incorporating Markov regime switching with asymmetric GARCH models
- Validation using other wind farm data

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Appendx: Equations

$$\mathbb{E}_{t-1}\{h_{t-1}^{2|(j)} \mid s_t\} = \sum_{i=1}^{2} \bar{p}_{ij,t-1}[(\mu_{t-1}^{(i)})^2 + h_{t-1}^{2|(i)}] - [\sum_{i=1}^{2} \bar{p}_{ij,t-1}\mu_{t-1}^{(i)}]^2$$

$$\bar{p}_{ij,t-1} = \Pr(s_t = i \mid s_{t+1} = j, \Omega_{t-1}) = \frac{p_{ij}\Pr(s_t = i \mid \Omega_{t-1})}{\Pr(s_{t+1} = j \mid \Omega_{t-1})} = \frac{p_{ij}p_{i,t}}{p_{j,t+1}}$$

Appendix: Equations

In Diebold Mariano Test,

$$f_d(0) = \frac{1}{2\pi}(\sum_{k=-\infty}^{\infty} \gamma_d(k)) \tag{21}$$

is the spectral density of loss differential at frequency 0. $\gamma_d(k)$ is autocovariance of loss differential at lag k.

MRS-GARCH: estimated sequence of regimes probabilities

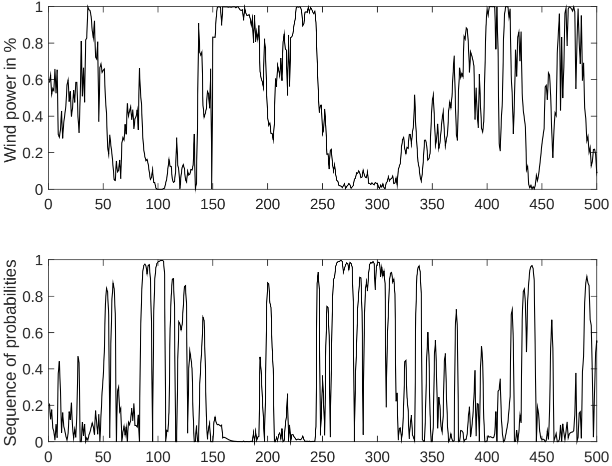

Fig. Wind power production and the estimated sequence of probabilities of being in regime 1 (i.e., low regime)