Model risk and power plant valuation

Energy Finance Workshop 2013, Stolberg, 17-19 April 2013

Anna Nazarova
Joint work with Karl Bannör, Rüdiger Kiesel and Matthias Scherer | Chair for Energy Trading and Finance | University of Duisburg-Essen
Outlook

Motivation and Introduction

Model risk theory

Spread options and power plant valuation

Empirical investigation of the model risk

Questions and discussion

References
Motivation and Introduction

- **Model risk** - *is the risk of occurrence of a significant difference between the mark-to-model value of a complex and/or illiquid instrument, and the price at which the same instrument is revealed to have traded in the market.* (R. Rebonato, [6])

- Types of model risk (E. Derman, [4]):
  - model is inapplicable
  - model is incorrect
  - correct model, incorrect solution
  - correct model, inappropriate use
  - badly approximated solution
  - software bugs
  - unstable data
Motivation and Introduction

- How significant is the impact of the model’s choice on the value of a given instrument?
- How can we measure the model risk?
- How can we test the model’s boundaries?
- In the context of energy markets, how can we address the model uncertainty problem?
The general approach is to replace investments and to build more capacity in the power plant park. The financial streams are modelled as the spread option. Then the critical question is the need for reinvestment. To study this valuation problem together with the parameters uncertainty, we use the approach offered in K. Bannör and M. Scherer, [1].
General procedure

- Investor runs a gas-driven power plant, with 3-years investment period.
- She observes power, gas and carbon prices and wants to quantify the risk of such investment.
- With a pre-defined model she obtains prices. Further, she wants to measure the sensitivity of the power plant value w.r.t. the model parameters.
- With a help of risk-capturing functional she computes the risk-adjusted added value and obtains risk-captured prices bid and ask prices and relative delta of these prices.
- This analysis can be done for many parameters of the model to see which one is the biggest risk source.
General procedure

- Investor runs a gas-driven power plant, with 3-years investment period.

- She observes power, gas and carbon prices and wants to quantify the risk of such investment.

- With a pre-defined model she obtains prices. Further, she wants to measure the sensitivity of the power plant value w.r.t. the model parameters.

- With a help of risk-capturing functional she computes the risk-adjusted added value and obtains risk-captured prices bid and ask prices and relative delta of these prices.

- This analysis can be done for many parameters of the model to see which one is the biggest risk source.
General procedure

► Investor runs a gas-driven power plant, with 3-years investment period.

► She observes power, gas and carbon prices and wants to quantify the risk of such investment.

► With a pre-defined model she obtains prices. Further, she wants to measure the sensitivity of the power plant value w.r.t. the model parameters.

► With a help of risk-capturing functional she computes the risk-adjusted added value and obtains risk-captured prices bid and ask prices and relative delta of these prices.

► This analysis can be done for many parameters of the model to see which one is the biggest risk source.
General procedure

► Investor runs a gas-driven power plant, with 3-years investment period.

► She observes power, gas and carbon prices and wants to quantify the risk of such investment.

► With a pre-defined model she obtains prices. Further, she wants to measure the sensitivity of the power plant value w.r.t. the model parameters.

► With a help of risk-capturing functional she computes the risk-adjusted added value and obtains risk-captured prices bid and ask prices and relative delta of these prices.

► This analysis can be done for many parameters of the model to see which one is the biggest risk source.
General procedure

- Investor runs a gas-driven power plant, with 3-years investment period.
- She observes power, gas and carbon prices and wants to quantify the risk of such investment.
- With a pre-defined model she obtains prices. Further, she wants to measure the sensitivity of the power plant value w.r.t. the model parameters.
- With a help of risk-capturing functional she computes the risk-adjusted added value and obtains risk-captured prices bid and ask prices and relative delta of these prices.
- This analysis can be done for many parameters of the model to see which one is the biggest risk source.
Set-up

Let

- $(\Omega, \mathcal{F}, \mathbb{F})$ be a measurable filtered space;
- $S = S_t$ be a basic instrument;
- $X = F(S)$ be a contingent claim;
- $\mathcal{Q} = \{\mathbb{Q}_\theta, \theta \in \Theta\}$ be a parametrized family of (martingale) measures on $(\Omega, \mathcal{F})$;
- for $\theta \in \Theta$, a (risk-neutral) value of contingent claim is
  $$\mathbb{E}_\theta(X) := \mathbb{E}_{\mathbb{Q}_\theta}(X).$$
Risk capturing functional: definition

Let $\mathcal{D}^A$ denotes the space of all (exotic) $A$-admissible derivatives by

$$\mathcal{D}^A := \bigcap_{\theta \in \Theta} \{ L^1(Q_\theta) : \theta \to \mathbb{E}_\theta(X) \in A \}.$$ 

Define the mapping $\Gamma : \mathcal{D}^A \to \mathbb{R}$ as the model risk-capturing functional by

$$\Gamma(X) := \rho(\theta \to \mathbb{E}_\theta(X))$$

incorporating model’s uncertainty w.r.t. the model family $Q$ into prices and fulfils the following properties. $\rho$ is the generator of $\Gamma$ and $\Gamma(X)$ is the risk-captured price of $X$ given the functional $\Gamma.$
Risk capturing functional: properties

- **order preservation**: $\forall X, Y \in \mathcal{D}^A : X \geq Y \rightarrow \Gamma(X) \geq \Gamma(Y)$;

- **diversification**: 
  $\forall X, Y \in \mathcal{D}^A, \lambda \in [0, 1] : \Gamma(\lambda X + (1 - \lambda) Y) \geq \lambda \Gamma(X) + (1 - \lambda) \Gamma(Y)$;
Risk capturing functional: interpretation

- **ask price**: $\Gamma^u(X) = \sup_{Q \in \mathcal{Q}} E_Q$;

- **bid price**: $\Gamma^l(X) = \inf_{Q \in \mathcal{Q}} E_Q = -\Gamma^u(-X)$. 
Risk capturing functional: AVaR example

- $(\Omega, \mathcal{F}, \mathbb{F}), \beta \in (0, 1], \alpha \in (0, 1], X \in L^1(\mathbb{P})$;

- $\text{VaR}_\beta(X) = \inf\{x \in \mathbb{R} : P(X > x) \leq 1 - \beta\} = \inf\{x \in \mathbb{R} : F_X(x) \geq \beta\} = q^P_X(1 - \beta)$;

- $\text{AVaR}_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_u(X)du$;

- For $R$-distribution on $Q$ there are $L^1(R)$-admissible functionals and the space $\mathcal{D}^{L^1(R)}$ such that the risk-capturing functional generated by AVaR is the mapping $R \ast \text{AVaR}_\alpha(X) : \mathcal{D}^{L^1(R)} \rightarrow \mathbb{R}$

$$
\Gamma(X) = R \ast \text{AVaR}_\alpha(X) := \text{AVaR}_\alpha(\theta \rightarrow \mathbb{E}_\theta(X)).
$$
Risk capturing functional: AVaR example, Delta method

If for the estimator the distribution is known (e.g. MLE), then there is a Delta method

- let \((\theta_N)_{N \in \mathbb{N}}\) - asymptotically normal sequence of estimators for the true parameter \(\theta_0 \in \theta \subset \mathbb{R}^m\) with asymptotic positive definite covariance matrix \(\Sigma\), i.e.

  \[
  \sqrt{N}(\theta_N - \theta_0) \xrightarrow{N \to \infty} \mathcal{N}_m(0, \Sigma) \text{ weakly};
  \]

- let \(X \in \mathcal{D}^{L\infty}(\Theta)\);

- if \(\theta \to \mathbb{E}_\theta[X]\) is continuously differentiable and \(\nabla \mathbb{E}_0[X] \neq 0\), then

  \[
  \sqrt{N}(\mathbb{E}_{\theta_N}[X] - \mathbb{E}_{\theta_0}[X]) \xrightarrow{N \to \infty} \mathcal{N}(0, (\nabla \mathbb{E}_{\theta_0}[X])' \cdot \Sigma \cdot \nabla \mathbb{E}_{\theta_0}[X]) \text{ weakly}.
  \]
Risk capturing functional: AVaR example, Delta method

Then, approximating $\mathbb{E}_{\theta_N}[X]$ by r.v.

$$Y \sim \mathcal{N}\left(\mathbb{E}_{\theta_0}[X], \frac{(\nabla \mathbb{E}_{\theta_0}[X])' \cdot \Sigma \cdot \nabla \mathbb{E}_{\theta_0}[X]}{N}\right),$$

and using the calculation of the AVaR of a normal distribution in P. Embrechts, R. Frey and P. McNeil [5], we have that

$$\theta_N \ast AVaR_\alpha(X) \approx \mathbb{E}_{\theta_0}[X] + \sqrt{\frac{(\nabla \mathbb{E}_{\theta_0}[X])' \cdot \Sigma \cdot \nabla \mathbb{E}_{\theta_0}[X]}{N}} \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha},$$

where $\phi$ is the density of the standard normal distribution.
Spread option: general theory

- **The spark spread** is the difference between power and gas, modelling a gas fired power plant. See M. Burger, B. Graeber and G. Schindlmayer [2].

- The owner of a power plant can use a spread option model to hedge the market risk of the power plant using futures or he can directly sell a spread option offsetting risks of the power plant.

- The typical payoff of a spread option is

  $$\text{Payoff}_T = \max\{S_1(T) - S_2(T) - K, 0\}. $$
Spread option: energy markets

- We model the daily profit (or loss) of a power plant as
  \[ V_t = \max\{P_t - hG_t - \eta E_t, 0\}, \]
- \( P_t \) - is the power price;
- \( G_t \) - is the gas price;
- \( E_t \) - is the carbon certificate price;
- \( h \) - is the heat rate of the power plant;
- \( \eta \) - \( CO_2 \) emission rate of the power plant.
Spread option elements

Let
1. \((\Omega, \mathbb{P}, \mathcal{F}, \mathbb{F}_t, t \in [0, T])\) be a complete filtered probability space;
2. **carbon price**
   \[
   dE_t = \alpha^E E_t \, dt + \sigma^E E_t \, dW^E_t;
   \]
3. **gas price**
   \[
   G_t = e^{g(t) + Z_t},
   dZ_t = -\alpha^G Z_t \, dt + \sigma^G dW^G_t;
   \]
4. **power price**
   \[
   P_t = e^{f(t) + X_t + Y_t},
   dX_t = -\alpha^P X_t \, dt + \sigma^P dW^P_t,
   \]
   base signal:
   \[
   dY_t = -\beta Y_t \, dt + J_t \, dN_t.
   \]
5. **dependence structure**
   \[
   W^E, W^G \text{ and } N \text{ are mutually independent processes;}
   dW^P_t \, dW^G_t = \rho \, dt.
   \]
Spread option elements and their moments

After some trivial manipulations, one can obtain the following expressions for conditional mean and variance of the logarithmic prices:

\[
\begin{align*}
\mathbb{E}[\ln E_t] &= \ln E_0 + \left( \alpha^E - \frac{(\sigma^E)^2}{2} \right) t, \\
\text{Var}(\ln E_t) &= (\sigma^E)^2 t, \\
\mathbb{E}[\ln G_t] &= g(t) + (\ln G_0 - g(0)) e^{-\alpha^G t}, \\
\text{Var}(\ln G_t) &= \frac{(\sigma^G)^2}{2\alpha^G} \left( 1 - e^{-2\alpha^G t} \right), \\
\mathbb{E}[\ln P_t] &= f(0) + X_0 e^{\alpha^P t} + Y_0 e^{\beta t} + \frac{\lambda}{\beta} \left( 1 - e^{-\beta t} \right) \mathbb{E}[J], \\
\text{Var}(\ln P_t) &= \frac{(\sigma^P)^2}{2\alpha^P} \left( 1 - e^{-2\alpha^P t} \right) + \frac{\lambda}{\beta} \left( 1 - e^{-2\beta t} \right) \mathbb{E}[J^2].
\end{align*}
\]
Power price jump size assumption

- non-central Laplace distribution to capture heavy-tail nature of spikes;
- Gaussian distribution (see A. Cartea and M. G. Figueroa, [3]).
Parameters uncertainty

- The total set of parameters:
  \[ \{ \alpha^E, \sigma^E, g(t), \alpha^G, \sigma^G, f(t), \alpha^P, \beta, \sigma^P, \lambda, \mathbb{E}[J], \mathbb{E}[J^2], \rho \} . \]

- Hence, the hybrid model allows for several degrees of freedom.

- Consequently, the risk of determining parameters in a wrong way is considerable and it will turn out that even the determination of single parameters may lead to tremendous results for prices obtained in the model.
Data sources

- **Phelix day base**: it is the average price of the hours 1 to 24 for electricity traded on the spot market. It is calculated for all calendar days of the year as the simple average of the auction prices for the hours 1 to 24 in the market area Germany/Austria, EUR/MWh.

- **NCG daily price**: delivery is possible at the virtual trading hub in the market areas of NetConnect Germany GmbH & Co KG, EUR/MWh.

- **Emission certificate daily price**: one EU emission allowance confers the right to emit 1 tonne of carbon dioxide or 1 tonne of carbon dioxide equivalent, EUR/EUA.

- **Observation period**: covers 25.09.2009 - 08.06.2012.
Evolution of the power, gas, and carbon prices, 25.09.2009 - 08.06.2012.
Evolution of the clean spark spread, 25.09.2009 - 08.06.2012.
Estimating the model parameters

- deseasonalisation of gas and power;
- $\log E_t$ is normally distributed $\rightarrow$ use MLE;
- filtering out power base and spike signals;
- $\log(powerbase)$ and $\log(gas)$ are of bivariate normal distribution $\rightarrow$ MLE;
- spike size (Laplace, Gaussian) $\rightarrow$ MLE;
- spike intensity is taken as a frequency of the filtered jumps.
Measuring the model risk in

- Jump size distribution;
- Correlation (due to Fisher transformation);
- Gas and power base signal;
- Gas alone;
- Gas, power and emissions (all the parameters, except of jump size).
General procedure

- Consider the distributions of the parameters separately, by doing "sensitivity" analysis w.r.t. different parameters, disregarding the remaining parameter risk.

- Each parameter \( \theta_j = \hat{\theta}_j(X_1, \ldots, X_N) \) under the real-world measure, assuming other parameters \( \theta_1, \ldots, \theta_{j-1}, \theta_{j+1}, \ldots, \theta_N \) to be known.

- We calculate the risk-captured prices which are generated by the AVaR w.r.t different significance levels \( \alpha \in (0, 1] \).
Spark spread analysis

- For every $t \in [0,T]$ we simulate the spark spread:
  \[
  V_t = \max\{P_t - hG_t - \eta E_t, 0\}.
  \]

- Then, by fixing all the parameters except of chosen one and setting the shift value $\xi$ (e.g. 1%), we compute shifted up and down spark spread values:
  \[
  V_t^{up}(\theta + \xi) \text{ and } V_t^{down}(\theta - \xi).
  \]
Power plant analysis

▶ We compute the value of the power plant (VPP) by Monte Carlo simulations.

▶ For fixed large $N$ and $T = 3$ years we have

$$VPP(t, T) = \frac{1}{N} \sum_{i=1}^{N} VPP_i(t, T),$$

where

$$VPP_i(t, T) = \int_t^T e^{-r(s-t)} V_i(s) \, ds.$$ 

▶ For chosen $\xi$ we also compute

$$VPP_{up}(t, T; \theta) = VPP(t, T; \theta + \xi),$$

$$VPP_{down}(t, T; \theta) = VPP(t, T; \theta - \xi).$$
Power plant analysis

- We continue with sensitivity measuring for the VPP w.r.t. \( \theta \) by the central finite difference

\[
\nabla_{\theta} VPP(t, T) := \frac{\partial VPP}{\partial \theta} = \frac{VPP^{up}(t, T; \theta) - VPP^{down}(t, T; \theta)}{2\xi}
\]

- Then, we compute the risk-adjusted value

\[
RAV := \varphi\left(\Phi^{-1}(1-\alpha)\right) \sqrt{\frac{(\nabla_{\theta} VPP)' \cdot \Sigma_{\theta} \cdot \nabla_{\theta} VPP}{N}}
\]

- We finish by computing risk-adjusted bid and ask price by

\[
bidPrice = VPP(t, T) - RAV;
\]

\[
askPrice = VPP(t, T) + RAV;
\]

\[
\Delta = \frac{askPrice - bidPrice}{midPrice}.
\]
Risk-captured bid and ask prices w.r.t.

- Jump size distribution;
- Correlation (due to Fisher transformation);
- Gas and power base signal;
- Gas alone;
- Gas, power and emissions.
Parameter-risk implied bid-ask spread w.r.t. jump size distribution: Gaussian.
Parameter-risk implied bid-ask spread w.r.t. jump size distribution: Laplace.
Parameter-risk implied bid-ask spread w.r.t. correlation parameter, Gaussian jumps.
Parameter-risk implied bid-ask spread w.r.t. correlation parameter, Laplace jumps.
Parameter-risk implied bid-ask spread w.r.t. the gas and power base processes, Gaussian jumps.
Parameter-risk implied bid-ask spread w.r.t. the gas and power base processes, Laplace jumps.

- **Bid and ask prices accounting for the parameter risk in base power and gas signals with Laplace jumps**

- **Relative bid–ask spread width accounting for the parameter risk in base power and gas signals with Laplace jumps**

Anna Nazarova | University of Duisburg-Essen | April 19, 2013
Parameter-risk implied bid-ask spread w.r.t. the gas price process, Gaussian jumps.
Parameter-risk implied bid-ask spread w.r.t. the gas price process, Laplace jumps.
Parameter-risk implied bid-ask spread w.r.t. all the parameters, except of the Gaussian jump size.
Parameter-risk implied bid-ask spread w.r.t. all the parameters, except of the Laplace jump size.
Resulting values for the relative width of the bid-ask spread for various model risk sources. $\alpha_1 = 0.01$ (the highest risk-aversion), $\alpha_2 = 0.1$, $\alpha_3 = 0.5$.

<table>
<thead>
<tr>
<th>Model Risk</th>
<th>Jumps size distribution</th>
<th>Jumps</th>
<th>Correlation</th>
<th>Gas and power base</th>
<th>Gas</th>
<th>Gas, power and carbon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gaussian</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
</tr>
<tr>
<td>Jumps</td>
<td></td>
<td>111.9%</td>
<td>73.71%</td>
<td>33.51%</td>
<td>163.5%</td>
<td>107.7%</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td>6.95%</td>
<td>4.58%</td>
<td>2.08%</td>
<td>3.29%</td>
<td>2.17%</td>
</tr>
<tr>
<td>Gas and power base</td>
<td></td>
<td>6.48%</td>
<td>4.27%</td>
<td>1.94%</td>
<td>3.07%</td>
<td>2.02%</td>
</tr>
<tr>
<td>Gas</td>
<td></td>
<td>6.11%</td>
<td>4.03%</td>
<td>1.83%</td>
<td>2.89%</td>
<td>1.91%</td>
</tr>
<tr>
<td>Gas, power and carbon</td>
<td></td>
<td>8.21%</td>
<td>5.41%</td>
<td>2.46%</td>
<td>3.83%</td>
<td>2.52%</td>
</tr>
</tbody>
</table>
Conclusive remarks

- The major source of the model risk is the jump size distribution.
- We managed to define the lower boundary for the model risk. The upper boundary is impossible to catch.
- Real risks are even bigger.
- We can only quantify the risk in terms of the chosen model.
- Measurement of the total risk is impossible.
- In these terms it would be interesting to see which distribution (Gaussian, Laplace, exponential, Pareto, etc.) generates the biggest width of the spread.
References


Thank you for your attention!