Pricing Rainfall Derivatives at the CME

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Motivation

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Pricing Rainfall Derivatives at the CME
Motivation

Situation
- Rainfall risk affects many economic sectors.
- Rainfall risk can be insured with rainfall derivatives.
- The CME started trading rainfall derivatives in 2011.
  → Prices of exchange-traded rainfall derivatives are available for the first time.

CME monthly rainfall futures
- Index: Monthly sum of rainfall (inches)
- Tick size: $500 per index point
- Contract months: Mar, Apr, May, Jun, Jul, Aug, Sep, Oct
- Reference stations: Chicago, Dallas, Des Moines, Detroit, Jacksonville, Los Angeles, New York City, Portland, Raleigh
Motivation

Pricing rainfall derivatives

- Benth et al. (2007), Härdle/López Cabrera (2011): Pricing models for temperature futures including the Market Price of Risk (MPR)
  → But: Rainfall different from temperature
- Cao et al. (2004): Fair premium for rainfall futures, no MPR
- Leobacher/Ngare (2011): Indifference prices
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Goal

- Pricing model for CME rainfall futures including the MPR
Methods

Daily rainfall records → **Daily rainfall model** $R_t = r_t \cdot X_t$

Simulated daily rainfall

Aggregated rainfall index

Distribution of simulated index

Market price of risk $\neq 0$ → **Esscher transformation**

Price under $P$

Price under $Q$

**Figure**: Model for pricing rainfall derivatives
The daily rainfall amount $R_t$ at time $t$ is described as the product of a rainfall amount process $r_t$ and a rainfall occurrence process $X_t$.

$$R_t = r_t \cdot X_t, \quad X_t = \begin{cases} 0 & \text{if day } t \text{ is dry} \\ 1 & \text{if day } t \text{ is wet} \end{cases}$$
Daily Rainfall Model

$X_t$ is modelled as a first-order, two-state Markov process.

Transition probabilities:

$p_{t}^{01} = \Pr\{X_t = 1 | X_{t-1} = 0\}$
$p_{t}^{11} = \Pr\{X_t = 1 | X_{t-1} = 1\}$

Figure: Empirical and estimated transition probabilities for New York City

Figure: Empirical and estimated transition probabilities for Detroit
The process $r_t$ follows a mixed exponential distribution.


$$f[r_t] = \frac{\alpha_t}{\beta_t} \exp \left[ \frac{-r_t}{\beta_t} \right] + \frac{1-\alpha_t}{\gamma_t} \exp \left[ \frac{-r_t}{\gamma_t} \right]$$

with $\beta_t \geq \gamma_t > 0$ and $0 < \alpha_t < 1$. 

**Figure:** Parameters of the mixed exponential distribution for New York City

**Figure:** Parameters of the mixed exponential distribution for Detroit
Rainfall simulation

Occurrence process

Recursive simulation with starting value $X_0$ via uniform random variable $u_{1,t} \sim \mathcal{U}(0,1)$:

$$X_t^{\text{sim}} = \begin{cases} 1 & \text{if } u_{1,t} \leq p_t^{X_1}, \\ 0 & \text{otherwise}. \end{cases}$$

Amount process

Simulation via uniform random variables $u_{2,t}$ and $u_{3,t} \sim \mathcal{U}(0,1)$, independent from $u_{1,t}$:

$$r_t^{\text{sim}} = r_{\text{min}} - \delta_t \ln [u_{2,t}],$$

$$\delta_t = \begin{cases} \beta_t & \text{if } u_{3,t} \leq \alpha_t, \\ \gamma_t & \text{if } u_{3,t} > \alpha_t. \end{cases}$$
The Esscher transform allows to get an equivalent martingale measure for Lévy processes.

It corresponds to a Girsanov transform for the Brownian motion.

The density under the equivalent martingale measure is defined by

\[ f_t(x; \theta) = \frac{e^{\theta x} f_t(x)}{\int_{-\infty}^{\infty} e^{\theta y} f_t(y) dy} \]

Certain distributions retain their original form under the Esscher transform, e.g., the normal-inverse Gaussian distribution.
Normal-inverse Gaussian distribution NIG($\alpha, \beta, \mu, \delta$)

- Flexible distribution with 4 parameters
- Density
  $$f_X(x) = \frac{\alpha \delta \exp(\delta \sqrt{\alpha^2 - \beta^2} + \beta (x - \mu))}{\pi \sqrt{\delta^2 + (x - \mu)^2}} \cdot K_1 \left( \alpha \sqrt{\delta^2 + (x - \mu)^2} \right)$$
  - $\mu$ location
  - $\alpha$ tail heaviness
  - $\beta$ asymmetry parameter
  - $\delta$ scaling parameter
  - $K_1$ modified Bessel function of second kind
- After an Esscher transform with parameter $\theta$, an NIG($\alpha, \beta, \mu, \delta$) distributed random number is NIG($\alpha, \beta + \theta, \mu, \delta$) distributed.
Data

**Rainfall data**
- Daily rainfall amount 1980–2011
- Detroit, Jacksonville, New York City
- National Climatic Data Center (NCDC)

**Market data**
- Daily CME prices of futures on the monthly sum of rainfall (in inches)
- All contracts 2011 (March–October)
- Detroit, Jacksonville, New York City
- From Bloomberg via the RDC of SFB649
Fig.: Histogram of the simulated rainfall index, 03.01.2011
Fig.: Histogram of the simulated rainfall index, as well as the fitted NIG distribution, 03.01.2011
Fig.: Histogram of the simulated rainfall index, as well as the fitted and the transformed NIG distributions, 03.01.2011
Fig.: Histogram of the simulated rainfall index, as well as the fitted and the transformed NIG distributions, 03.01.2011
### Prices

#### New York City (03.01.2011)

<table>
<thead>
<tr>
<th>Methode</th>
<th>MPR $\theta$</th>
<th>Mar11</th>
<th>Apr11</th>
<th>May11</th>
<th>Jun11</th>
<th>Jul11</th>
<th>Aug11</th>
<th>Sep11</th>
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<td>–</td>
<td>4.20</td>
<td>4.40</td>
<td>3.20</td>
<td>5.00</td>
<td>4.50</td>
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<td>DRM</td>
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<td>1.74</td>
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<td></td>
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<td>2.86</td>
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<td>3.50</td>
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<td>0.30</td>
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<td>24.29</td>
<td>70.04</td>
<td>35.24</td>
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</table>

*Fig.:* Theoretical Prices for New York City, calculated on 03.01.2011, as well as the CME market prices the same day.
Fig.: Histogram of the simulated rainfall index, as well as the fitted and the transformed NIG distributions, 03.01.2011
**Prices**

<table>
<thead>
<tr>
<th>MPR $\hat{\theta}$</th>
<th>Mar11</th>
<th>Apr11</th>
<th>May11</th>
<th>Jun11</th>
<th>Jul11</th>
<th>Aug11</th>
<th>Sep11</th>
<th>Oct11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>-0.232</td>
<td>-0.216</td>
<td>0.198</td>
<td>0.014</td>
<td>-0.024</td>
<td>-0.235</td>
<td>-0.038</td>
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<tr>
<td>Jacksonville</td>
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<td>-0.165</td>
<td>-0.119</td>
<td>0.203</td>
<td>-0.054</td>
<td>-0.124</td>
<td>0.091</td>
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<td>New York City</td>
<td>0.138</td>
<td>0.117</td>
<td>-0.394</td>
<td>0.063</td>
<td>-0.063</td>
<td>-0.038</td>
<td>0.037</td>
<td>0.114</td>
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</tbody>
</table>

**Tab.:** Estimated MPR for different cities and contracts, 03.01.2011
Problem

Fig.: CME prices 2011 for rainfall futures, New York City
Conclusion

Summary
- Flexible and practicable model for pricing rainfall futures
- Approach not limited to NIG distribution
- Fitting to actual market prices via the market price of risk

Discussion
- CME prices not (yet) real market prices
- Weather forecasts
References

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*SFB 649 Discussion Paper 2011-055*.

A multi-period equilibrium pricing model of weather derivatives.

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Analysis of rainfall derivatives using daily precipitation models: Opportunities and pitfalls.