Easy EZ in DSGE*

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Abstract

Epstein-Zin preferences (or "EZ" preferences) have become increasingly popular in recent asset pricing work. Dynamic stochastic general equilibrium (DSGE) models which feature Epstein-Zin preferences are typically considered technically challenging, often thought to require sophisticated numerical solution methods to solve them and considerable additional thought to understand them. The purpose of this paper is to make DSGE modeling with Epstein-Zin preferences easy, relying on log-linearization to the equations characterizing the equilibrium dynamics and exploiting log-normality for asset pricing.

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The paper therefore provides a benchmark, from which to explore and understand the added benefit of higher-order approximations.

Keywords:

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1 Introduction

Epstein-Zin (1989) preferences (or "EZ" preferences) have become increasingly popular in recent asset pricing work, see in particular Bansal-Yaron (2004), Pizzesi-Schneider (2007a, 2007b) and Hansen-Heaton-Li (2009). Dynamic stochastic general equilibrium (DSGE) models which feature Epstein-Zin preferences are typically considered technically challenging, often thought to require sophisticated numerical solution methods to solve them and considerable additional thought to understand them. Among the key references are Tallarini (2000) and Guvenen (2009), who combine a real business cycle model with Epstein-Zin preferences to successfully match both a number of macroeconomic and asset pricing facts: both papers rely heavily on computational methods to calculate their results. Likewise, say, Rudebusch-Swanson (2009) use higher-order approximations and van Binsbergen et al (2009) proceed with e.g. particle filter methods to compute likelihood functions.

Numerical methods as e.g. described in Judd (1998) and these papers surely ought to be considered part of the required tool box of modern quantitative macroeconomics. Moreover, an increasing number of the off-the-shelf numerical tools such as Dynare make the calculation of higher-order approximations and the resulting solutions to DSGE models easy. Nonetheless, it is often desirable to be able to proceed with simpler tools too: as a starting point of an investigation, as a framework to provide the key insights, or as a benchmark for a more sophisticated investigation. For example, log-linearization methods are often used to analyze the dynamics of DSGE models, building on the fairly complete theory of linear dynamic models. With the additional assumption of joint log-normality, this dynamics can in turn be exploited to compute asset pricing implications as in, say, Lettau-Uhlig (2002). While still relying on some numerical means, the results have a semi-analytic character, and lend themselves more easily to clean insights,

see e.g. Uhlig (2007a).

It turns out that this is feasible for Epstein-Zin preferences as well[1] A related exercise can be found in Backus et al (2007), who point to their appendix for the "gruesome" details. The purpose of this paper is to show to how use log-linearization to deal with Epstein-Zin preferences in a straightforward manner and how to compute approximate asset pricing implications, providing insights into the connections along the way. I.e., the purpose of this paper is to make DSGE modeling with Epstein-Zin preferences easy, relying on log-linearization to the equations characterizing the equilibrium dynamics and exploiting log-normality for asset pricing.

I shall build on a simple benchmark business cycle model with a representative agent, allowing for "wedges" as in Chari-Kehoe-McGrattan (2007). As the asset pricing literature has focussed on issues of long-run risk, see Bansal-Yaron (2004) and Hansen-Heaton-Li, the model will be formulated with long-run growth and potential shocks to the long-run growth rate: the preferences are therefore chosen to be consistent with long-run growth. The specific requirements on preferences as well as their log-linearizations are studied in more generality in Uhlig (2007b). That paper, as this one, furthermore points to the importance of the utility in leisure for pricing assets, when using Epstein-Zin preferences, see also Swanson (2009).

I provide the calculations to obtain the dynamic implications and provide insights into asset pricing. The paper uses the notation and exploits some of the tools in Hansen-Heaton-Li (2009). I shall formulate the model with fairly general functional forms, only relying on certain derivates along the balanced growth rates for the calculations. The exercise is related to Uhlig (2007a) for preferences with habit formation. Some of the algebra and results for the asset pricing implications are similar to Piazzesi-Schneider (2007a). As an

[1] I recall a related paper by Andrew Levin, perhaps with co-authors, but was not able to find it yet.

extension, I shall consider a version of the model by Guvenen (2009), and calculate its asset pricing implications with the methods here.

2 Preferences

The preferences are the crucial ingredient of the analysis here. I assume that agents enjoy lifetime utility $\pm V_t$, where V_t is given recursively by

$$V_t = \left(\left(1-\tilde{\beta}\right)\left(c_t\Phi(n_t)\right)^{1-\rho} + \beta\mathcal{R}_t^{1-\rho}\right)^{\frac{1}{1-\rho}} \tag{1}$$

where $\rho > 0$, $\rho \neq 1$, $0 < \beta < 1$, $0 < \tilde{\beta} < 1$ and where

$$\mathcal{R}_t = \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{2}$$

and where $\Phi(\cdot)$ is assumed to be positive, thrice differentiable, decreasing and concave. Note that V_t is increasing in c_t, if $\rho < 1$ and decreasing, if $\rho > 1$: I shall therefore assume that lifetime utility is $-V_t$, if $\rho < 1$ and is given by $-V_t$, if $\rho > 1$. The literature has generally tacitly assumed this sign convention, see e.g. Hansen-Heaton-Li (2009) and so shall I: what matters in the end are the first-order conditions, and they will be correct either way.

The parameter β is the discount factor, whereas $1-\tilde{\beta}$ is a scaling constant. We shall allow for growth in the model: it will there be convenient to choose a particular value for $\tilde{\beta}$, to be defined in equation (13).

For macroeconomic modeling purposes, it is important to allow for variations in n_t to affect utility. In order for labor-augmented Epstein-Zin preferences to be consistent with long-run growth, one can show that the preferences must be of the form (1), see Uhlig (2007b). As pointed out there, this generates asset price implications from labor fluctuations in two ways. First, labor enters preferences non-separably, and will therefore enter the pricing kernel, even if preferences are time-separable, i.e., if $\gamma = \rho$. Second, if preferences are non time-separable, i.e. if $\gamma \neq \rho$, then labor fluctuations

generate fluctuations in the value V_t, even if labor entered temporary utility in a separable way (as will be the case, when $\rho \to 1$).

3 A benchmark model

Time is discrete. The representative agent maximizes his or her lifetime utility, subject to the budget constraints

$$(1+\tau_t^c)c_t + (1+\tau_t^x)x_t = (1-\tau_t^w)w_t n_t + (1-\tau_t^k)r_t k_t + s_t \tag{3}$$

where c_t, x_t, w_t, n_t, r_t and k_t denote consumption, investment, wages, labor, capital rental rates and capital. Further, $\tau_t^c, \tau_t^x, \tau_t^w, \tau_t^k$ may simply represent tax rates on consumption, investment, labor income and capital income, and s_t represents lump sum transfers, as in Trabandt-Uhlig (2009). Alternatively, these may represent wedges for the resulting first-order conditions and the resource constraint, due to non-modeled distortions as in Chari-Kehoe-McGrattan (2007): in that case, it may be more appropriate to think of s_t as taking negative values. I return to these interpretations, their ramifications and possible issues of endogeneity in section 6.

Capital is accumulated according to

$$k_{t+1} = \left(1 - \delta + h\left(\frac{x_t}{k_t}\right)\right) k_t \tag{4}$$

where $h(\cdot)$ is a capital adjustment cost function as in Jermann (1998). Part of the literature has instead pursued adjustment costs to the level of investment, notably Christiano et al (2005). Recent industry-level evidence in Groth-Khan (2007) argues against an investment-change-based adjustment cost, however. It would not be difficult to modify the specifications here to allow for such an adjustment cost or even a combination of the two, if so desired.

The capital adjustment cost function is assumed to be concave, differentiable thrice and to satisfy

$$\tilde{\delta} = h(\tilde{\delta}),\ 1 = h'(\tilde{\delta}), \omega = -h''(\tilde{\delta})\tilde{\delta} > 0 \tag{5}$$

for some parameter $\tilde{\delta}$, to be specified in equation (12) for consistency with long-run growth, and for some parameter ω, measuring the degree of curvature. In the benchmark case of linearity, $\omega = 0$. Jermann (1998) denotes the negative of the inverse of $h''(\tilde{\delta})$ with ξ, so that

$$\omega = \frac{\tilde{\delta}}{\xi} \tag{6}$$

Production takes place in a competitive sector of firms with the production function

$$y_t = f\left(\frac{A_t n_t}{k_t}\right) k_t \tag{7}$$

where y_t denotes output and A_t is a (labor-augmenting) technology parameter. I assume that f is positive, concave and differentiable thrice. The usual first-order condition deliver wages and capital rental rates,

$$w_t = A_t f'\left(\frac{A_t n_t}{k_t}\right) \tag{8}$$

$$r_t k_t = y_t - w_t n_t \tag{9}$$

Due to the wedges and/or taxes, there is a difference between total output and spending on consumption and investment, given by

$$g_t = y_t - c_t - x_t \tag{10}$$

The most straightforward interpretation of the model so far is to assume $\tau_t^c, \tau_t^x, \tau_t^w, \tau_t^k$ and s_t to be exogenous, and to consider g_t as endogenously determined government spending, which enters preferences in a separable way or which is simply thrown away. The latter interpretion is also consistent with the interpretation of $\tau_t^c, \tau_t^x, \tau_t^w, \tau_t^k$ and s_t as wedges. The easiest case, of course, is to simply set $\tau_t^c \equiv \tau_t^x \equiv \tau_t^w \equiv \tau_t^k \equiv s_t \equiv 0$ throughout: this simplifies a number of expressions throughout.

I assume that there is long run growth in the model. Define

$$\zeta_t = \frac{A_t}{A_{t-1}} \tag{11}$$

In the non-stochastic case, I assume that

$$\zeta_t \equiv \bar{\zeta} > 1$$

With this, define

$$\tilde{\delta} = \bar{\zeta} - 1 + \delta \tag{12}$$

$$\tilde{\beta} = \beta \bar{\zeta}^{1-\rho} \tag{13}$$

I assume that $\beta, \bar{\zeta}$ and ρ are such that $0 < \tilde{\beta} < 1$.

Equilibrium is defined in the usual way as a sequence of allocations and prices

$$(g_t, c_t, x_t, y_t, k_t, n_t, r_t, w_t, V_t, \mathcal{R}_t)$$

satisfying the equations above as well as maximizing the agents' life-time utility.

A list of parameters of the model for the purpose of calculating the log-linearized dynamics as well as the asset pricing implications will be provided in 4.2.

4 Analysis

A reader interested in investigating the log-linearized dynamics can directly skip to subsection 4.2. The resulting asset pricing implications are discussed in section 5

4.1 First-order conditions

To obtain the log-linearized dynamics, one must investigate the first-order conditions and then de-trend everything appropriately.

To obtain first-order conditions from preferences, calculate

$$\begin{aligned}
\frac{\partial V_t}{\partial c_t} &= (1-\tilde{\beta})V_t^{\rho}c_t^{-\rho}\Phi(n_t)^{1-\rho} \\
\frac{\partial V_t}{\partial c_{t+1}} &= \frac{\partial V_t}{\partial c_t}\frac{\Phi'(n_t)c_t}{\Phi(n_t)} \\
\frac{\partial V_t}{\partial c_{t+1}} &= (1-\tilde{\beta})\beta V_t^{\rho}\left(\frac{V_{t+1}}{\mathcal{R}}\right)^{\rho-\gamma} c_{t+1}^{-\rho}\Phi(n_{t+1})^{1-\rho}
\end{aligned}$$

From that and taking into account the consumption wedge $1+\tau_t^c$ in equation (3), one obtains the stochastic discount factor

$$\begin{aligned}
M_{t+1} &= \frac{\partial V_t/\partial c_{t+1}}{\partial V_t/\partial c_t} \qquad (14) \\
&= \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{\rho-\gamma}\left(\frac{\Phi(n_{t+1})}{\Phi(n_t)}\right)^{1-\rho}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}
\end{aligned}$$

Observe that the bracket involving n_{t+1} and n_t disappears, as $\rho \to 1$: this is intentional, as one then obtains the case of a per-period utility function given by $\log(c_t)$ plus a function of n_t. To describe this limiting case as part of the preference specification (1) involves a bit more work, but can be done: see Uhlig (2007b).

For an asset with a return of R_{t+1} in terms of before-tax consumable resources in period $t+1$ for every unit of before-tax consumable resource invested in t, one has the inter-temporal first-order condition or Lucas asset pricing equation

$$1 = E_t\left[M_{t+1}R_{t+1}\right] \qquad (15)$$

This equation holds in particular for the return $R_{t+1}^{(k)}$ of investing in capital. As elsewhere in the literature and after some tedious calculation, one obtains

$$R_t^{(k)} = \frac{1}{q_{t-1}}\left((1-\tau_t^k)r_t + q_t\left(1-\delta+h\left(\frac{x_t}{k_t}\right)\right) - (1+\tau_t^x)\frac{x_t}{k_t}\right) \qquad (16)$$

where I define q_t per

$$q_t = \frac{1+\tau_t^x}{h'\left(\frac{x_t}{k_t}\right)} \qquad (17)$$

The economic interpretation of q_t is that it is the amount of additional resources required to create an additional unit of capital. It is therefore the market price of a marginal unit of capital, i.e. it is Tobin's q.

Considering the tradeoff between working and consuming, one obtains the intra-temporal first-order condition for labor supply

$$\frac{-\Phi'(n_t)c_t}{\Phi(n_t)} = \frac{1-\tau_t^w}{1+\tau_t^c} w_t \tag{18}$$

The dynamics in the 13 endogenous variables

$$(g_t, c_t, x_t, y_t, k_t, n_t, r_t, w_t, V_t, \mathcal{R}_t, M_{t+1}, R_t^{(k)}, q_t)$$

is characterized by the 13 equations (1,2,3,4,7,8,9,10,14,15,16,17,18), where the return $R_{t+1}^{(k)}$ is to be used in the asset pricing equation (15). For convenience, they have been restated in appendix A, somewhat re-ordered.

As productivity A_t is growing, it is convenient to restate all variables and the equations in de-trended form. Define

$$\tilde{V}_t = \frac{V_t}{A_t}, \tilde{\mathcal{R}}_t = \frac{\mathcal{R}_t}{A_t}, \tilde{c}_t = \frac{c_t}{A_t}, \tilde{x}_t = \frac{x_t}{A_t}, \tilde{k}_t = \frac{k_t}{A_t}, \tilde{y}_t = \frac{y_t}{A_t}, \tilde{w}_t = \frac{w_t}{A_t}, \tilde{g}_t = \frac{g_t}{A_t}, \tilde{s}_t = \frac{s_t}{A_t}$$

keeping $M_{t+1}, r_t, n_t, R_t^{(k)}$ and q_t unchanged. The equations in terms of these de-trended variables are stated in appendix A.

A balanced growth equilibrium in the non-stochastic case obtains for constant

$$\tau_t^c \equiv \overline{\tau^c},\ \tau_t^x \equiv \overline{\tau^x},\ \tau_t^w \equiv \overline{\tau^w},\ \tau_t^k \equiv \overline{\tau^k},\ \tilde{s}_t \equiv \bar{s},\ \zeta_t \equiv \bar{\zeta}$$

In that case all de-trended variables as well as $M_{t+1}, r_t, n_t, R_t^{(k)}$ and q_t are constant as well. Appendix A provides the list for the steady state equations as well as a strategy for solving them, given the parameters of subsection 4.2. One may wish to adjust these steady state equations for risk premia on returns. I shall return to this issue in section 5.

abbrev.	expression	equality	sugg. value
	$h(\tilde{\delta})$	per assumption:	$\tilde{\delta}$
	$h'(\tilde{\delta})$	per assumption:	1
$\omega =$	$-h''(\tilde{\delta})\tilde{\delta}$	free: cap. adj. cost parameter	$\in [0, 4\tilde{\delta}]$
$\theta =$	$\frac{f'\left(\bar{n}/\bar{k}\right)\bar{n}/\bar{k}}{f\left(\bar{n}/\bar{k}\right)}$	$= \bar{w}\bar{n}/\bar{y}$ (labor share, observed)	2/3
$1/\eta_D =$	$\frac{-f''\left(\bar{n}/\bar{k}\right)\bar{n}/\bar{k}}{f'\left(\bar{n}/\bar{k}\right)}$	free: inv. labor demand elast.	1/3
	$\Phi(\bar{n})$	Normalization:	1
$\nu =$	$\frac{-\Phi'(\bar{n})\bar{n}}{\Phi(\bar{n})}$	$= \frac{1-\tau^w}{1+\tau^c}\theta\frac{\bar{y}}{\bar{c}}$ (observed)	1
$1/\eta_S =$	$\left(\frac{\Phi''(\bar{n})\bar{n}}{\Phi'(\bar{n})} - \nu\right)$	free: inv. labor supply elast.	2

Table 1: values for the three functions $\Phi(\cdot), f(\cdot), h(\cdot)$

4.2 Parameterization and the log-linear dynamics

Log-linearizing the de-trended equations around the non-stochastic balanced growth path is straightforward. Before proceeding, it is convenient to introduce some additional abbreviations or parameters, however.

The parameters introduced so far have been $\bar{\zeta}, \beta, \delta, \rho, \gamma$ with the resulting $\tilde{\delta} = \bar{\zeta} - 1 + \delta$ as well as well as $\tilde{\beta} = \beta\bar{\zeta}^{1-\rho}$ as well as the steady state tax levels or wedge levels $\overline{\tau^c}$, $\overline{\tau^x}$, $\overline{\tau^w}$, $\overline{\tau^k}$, $\bar{s}$.

There are three as-of-yet unspecified functions, namely $\Phi(\cdot), f(\cdot), h(\cdot)$. Table 1 highlights a few key values for these functions, introducing new symbols for convenience. Note the assumptions about $h(\cdot)$ per (5): for completeness, this information is restated here. Some of these values can be calibrated or estimated from observed quantities such as the consumption share $\bar{c}/\bar{y}$ or the labor share $\bar{w}\bar{n}/\bar{y}$, say, as well as assumptions (or observations) about the steady state "tax" levels $\overline{\tau^c}$, $\overline{\tau^x}$, $\overline{\tau^w}$, $\overline{\tau^k}$, $\bar{s}$.

For the log-linearized dynamics around the non-stochastic balanced growth path, it will turn out that the values in table 1 are sufficient as a description

of the functions $\Phi(\cdot), f(\cdot), h(\cdot)$.

A few remarks are in order. The table shows that only η_D, η_S and ω are "free", which means here that they are not tied down by typical calibration exercises. Obviously one may seek information about these parameters from data as well. The interpretations in the table (e.g. "inv. labor demand elasticity") emerges either from the equations about balanced growth or from the log-linearization.

For ξ in (6), Jermann (1998) has suggested values as low as 0.25, resulting in the listed upper bound of $4\tilde{\delta}$ for the suggested values for ω.

In many DSGE models, aggregate production is assumed to be Cobb-Douglas, i.e. $f(x) = x^\theta$. In that case, θ equals the labor share, regardless of the state of the economy, and often set to a value near 2/3. Typically, θ is considered known from observations in standard DSGE exercises. Furthermore, $1/\eta_D = 1 - \theta$ is the inverse labor demand elasticity: with $\theta = 2/3$, one then obtains $1/\eta_D = 1/3$. More generally, one may wish to employ a CES specification, $f(x) = (a + bx^\alpha)^{1/\alpha}$, where a, b, α are parameters. Given balanced growth path observations about the labor share θ, the output-capital ratio and the output-labor ratio then imply that $a = (\bar{y}/\bar{k})^\alpha(1 - \theta)$ and $b = (\bar{y}/\bar{n})^\alpha\theta$. In turn, this implies $1/\eta_D = (1 - \alpha)(1 - \theta)$. Well-known properties are implied. For example, the Cobb-Douglas value is achieved for $\alpha = 0$. For the linear specification $\alpha = 1$, the demand elasticity is infinite, whereas it approaches zero, as the CES function approaches the Leontieff specification per $\alpha \to -\infty$.

$\Phi(\bar{n})$ only plays a role in scaling $\bar{V}$ relative to $\bar{c}$: it might as well be normalized to unity, unless one seeks to compare, say, welfare across different specifications of the model. The parameter ν may be more interesting, but it is in turn tied down by assumptions (or observations) about the balanced growth wedges or tax levels, the labor share as well as the consumption-output ratio. If the wedges are assumed to be near zero and with $\bar{c}/\bar{y}$ near

2/3, then $\nu = 1$ is a reasonable value. As shall emerge from the log-linearized equations, the labor supply elasticity η_S is the uncompensated (or Marshallian) short-run elasticity with respect to wage income. For the compensated (or Hicks) elasticity, i.e. when holding income constant, Raj Chetty (2009) has recently suggested a value of 1/2 in order to reconcile micro and macro observations. For a temporary wage change, this should be near the Marshallian elasticity, and I have therefore listed that as the suggested value, though this may require some further thought. This elasticity is also different from the often-employed Frisch elasticity of labor supply, which holds the marginal utility of consumption constant. One may wish to calculate the relationship between these three elasticities in order to bring the literature evidence on these quantities to bear on the calculations here.

For the log-linearization, I let "hat" on a variable denote its log-deviation from the balanced growth path. For example, $\hat{y}_t = \tilde{y}_t - \bar{y}$, where the variables on the right-hand side refer to de-trended output $\tilde{y}_t$ and its steady state value $\bar{y}$. One may wish to read $100\hat{y}_t$ as the percent deviation. Tax rates or wedges are already stated "in percent", and it is therefore more sensible to use percentage points rather than "percent of percent" to denote their deviation. I.e., I let $\hat{\tau}_t^w = \tau_t^w - \overline{\tau^w}$, for example.

With the usual tools, the log-linearized versions of the de-trended equations (58) to (70) can now be stated directly. The appendix A lists the log-linearized equations, including the movements in the wedges or tax rates. For the benchmark case, that all wedges or tax rates are zero, the thus simplified equations are stated below. These equations have straightforward interpretations. The first three equations provide the recursion for the Epstein-Zin value function and the resulting stochastic discount factor,

$$\hat{V}_t = \left(1 - \tilde{\beta}\right)(\hat{c}_t - \nu \hat{n}_t) + \tilde{\beta}\hat{\mathcal{R}}_t \tag{19}$$

$$\hat{\mathcal{R}}_t = E_t\left[\hat{\zeta}_{t+1} + \hat{V}_{t+1}\right] \tag{20}$$

$$\hat{M}_{t+1} = -\gamma\hat{\zeta}_{t+1} - \rho\left(\hat{c}_{t+1} - \hat{c}_t\right) - (1-\rho)\nu\left(\hat{n}_{t+1} - \hat{n}_t\right) + (\rho-\gamma)\left(\hat{V}_{t+1} - \hat{\mathcal{R}}_t\right) \tag{21}$$

Equation (21) provides some key features. First, there is a direct impact of labor fluctuations on the stochastic discount factor, provided $\rho \neq 1$: in that case, the temporal felicity function needs to be non-separable in consumption and leisure in order to be consistent with long-run growth. In the data, labor falls in recessions as does consumption: if so, the labor fluctuations amplify the impact of consumption movements on the stochastic discount factor, if $\rho < 1$, i.e., if the intertemporal elasticity of substitution $1/\rho$ is high, and mitigate them for $\rho > 1$. Second, the Epstein-Zin term enters only if $\rho \neq \gamma$, i.e., if the inverse of the intertemporal elasticity of substitution differs from risk aversion. We typically suppose that $\rho < \gamma$: in that case, $\hat{V}_{t+1}$ affects $\hat{M}_{t+1}$ negatively. From equation (19), this leads to an amplification of the impact of consumption fluctuations on the stochastic discount factor, as expected. That same equation also shows that labor fluctuations must enter the stochastic discount factor: indeed, with labor decreasing when consumption does, these labor fluctuations now mitigate the stochastic discount factor movements, if Epstein-Zin preferences (with $\gamma > \rho$) are present. Intuitively, the added leisure provides some insurance against the consumption decline in recessions.

The remaining equations are standard, though they may rarely be stated with the general function form assumptions as above.

$$0 = E_t\left[\hat{M}_{t+1} + \hat{R}^{(k)}_{t+1}\right] \tag{22}$$

$$\hat{n}_t = \eta_S\left(\hat{w}_t - \hat{c}_t\right) \tag{23}$$

$$\hat{y}_t = \theta\hat{n}_t + (1-\theta)\hat{k}_t \tag{24}$$

$$\hat{k}_{t+1} = (1-\tilde{\delta})\hat{k}_t + \tilde{\delta}\hat{x}_t - \hat{\zeta}_{t+1} \tag{25}$$

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{x}}{\bar{y}}\hat{x}_t \tag{26}$$

$$\hat{n}_t = \hat{k}_t - \eta_D \hat{w}_t \tag{27}$$

$$\hat{r}_t = -\frac{\theta}{1-\theta}\hat{w}_t \tag{28}$$

$$\hat{R}_t^{(k)} = -\hat{q}_{t-1} + \frac{\bar{r}}{\overline{R^{(k)}}}\hat{r}_t + \frac{\zeta}{\overline{R^{(k)}}}\hat{q}_t \tag{29}$$

$$\hat{q}_t = \omega(\hat{x}_t - \hat{k}_t) \tag{30}$$

It is interesting to note that the Epstein-Zin preference specification impacts on the DSGE dynamics only through the asset pricing equation (22). The Epstein-Zin preference specification can produce dynamics and predictable movements in the stochastic discount factor, which therefore must generate predictable movements in the return to capital and therefore capital and labor itself.

Note that there are three state variables emerging from the DSGE dynamics, i.e. $\hat{k}_{t-1}$ and $\hat{x}_{t-1}$ per (25), lagged once[2] and per $\hat{q}_{t-1}$ in equation (29), wit the latter only arising if $\omega \neq 0$. If $\hat{M}_{t+1}$ in equation (22) is replaced with (21), no further state variables are needed. However, if (21) is kept and lagged once, it may be convenient to introduce the summary variable

$$\hat{\lambda}_t = -\rho\hat{c}_t - (1-\rho)\nu + (\rho-\gamma)\hat{\mathcal{R}}_t \tag{31}$$

noting that then

$$\hat{M}_t = \hat{\lambda}_t - \hat{\lambda}_{t-1} + (\rho-\gamma)\left(\hat{V}_t - \hat{\mathcal{R}}_t\right) - \gamma\hat{\zeta}_t \tag{32}$$

This equation is similar to the typical equation in the time-separable case, expressing the log-stochastic discount factor as the first difference of the logs

[2]One can get by with $\hat{k}_{t-1}$ only by a different timing convention, i.e. by shifting the date on k_t and $\hat{k}_t$ back by one. This then also requires scaling k_t differently, dividing the new k_t by A_t and therefore the "old" k_t by A_{t-1}, generating additional ζ_t-terms in some of the equations. Saving a state variable can be desirable. Here, I shall proceed with the notation commonly used in the literature, however.

of marginal utilities of consumption, adjusted here by a stochastic growth rate due to de-trending.

With equations (19) through (32), with the latter replacing (21), or the corresponding equations in appendix A, plus a stochastic law of motion for the exogenous variables, the dynamics can now be solved for with standard tools, e.g. Uhlig (1999). Let z_t be a list of the exogenous variables. In the full version of the model, this includes $\hat{\zeta}_t$ as well as $\hat{\tau}_t^c$, $\hat{\tau}_t^w$, $\hat{\tau}_t^x$, $\hat{\tau}_t^k$, $\hat{s}_t$: one may wish to include additional lags of these in order to capture higher-order dynamics. Assume that z_t follows a first-order VAR.

Combine z_t with the endogenous state variables (or, alternatively, all variables) to form the vector X_t (not to be confused with investment x_t above): with the methods in, say, Uhlig (1999) and under saddle-path stability, the law of motion for the exogenous and endogenous state variable vector x_t can now be written as a joint VAR,

$$X_{t+1} = GX_t + H\epsilon_{t+1} \tag{33}$$

where ϵ_{t+1} are date-$(t+1)$ innovations, i.e. is a vector martingale difference sequence, adapted to the information filtration, and where X_t is "known" at date t. This has the same form as in Hansen-Heaton-Li (2008), equation (1): I have written X_t here instead of x_t there and I have used ϵ_{t+1} here rather than w_{t+1} there.

5 Asset Pricing and stochastic steady states

With (33), one can now exploit the tools and algebra in Hansen-Heaton-Li (2008). Like these authors, I shall assume that ϵ_{t+1} is iid and normally distributed, with identity variance-covariance matrix. We shall, however, proceed slightly differently for pricing assets than these authors, and directly rely on the log-linear recursions given by (19,20,21).

To that end, it is convenient to introduce the "surprise" operator $\mathcal{S}_{t+k|t}$ for any random variable x, given by

$$\mathcal{S}_{t+k|t} = E_{t+k}[x] - E_t[x] \tag{34}$$

I shall in particular write shortly $\mathcal{S}_{t+1}$ for $\mathcal{S}_{t+1|t}$. In particular, if $x = x_{t+1}$ is known at $t+1$ (or, more precisely, measurable with respect to the date-t sigma algebra of the information filtration), then

$$\mathcal{S}_{t+1}[x_{t+1}] = x_{t+1} - E_t[x_{t+1}]$$

extracts the "surprise" in x_{t+1} compared to its conditional expectation at date t. Note that

$$\mathcal{S}_{t+k|t} = \mathcal{S}_{t+k} + \mathcal{S}_{t+k-1} + \ldots + \mathcal{S}_{t+1} \tag{35}$$

The key to asset pricing is to understand the stochastic discount factor. Note first that (19) and (20) can be iterated to yield

$$\hat{V}_t + \hat{\zeta}_t = E_t \left[\sum_{j=0}^{\infty} \tilde{\beta}^j \left(\left(1 - \tilde{\beta}\right) \left(\hat{c}_{t+j} - \nu \hat{n}_{t+j}\right) + \hat{\zeta}_{t+j} \right) \right] \tag{36}$$

Replacing this in[3] (21) yields

$$\begin{aligned} \hat{M}_{t+1} &= -\rho \hat{\zeta}_{t+1} - \rho \left(\hat{c}_{t+1} - \hat{c}_t\right) - (1-\rho)\nu \left(\hat{n}_{t+1} - \hat{n}_t\right) \\ &\quad + (\rho - \gamma) \mathcal{S}_{t+1} \left[\sum_{j=0}^{\infty} \tilde{\beta}^j \left(\left(1 - \tilde{\beta}\right) \left(\hat{c}_{t+j} - \nu \hat{n}_{t+j}\right) + \hat{\zeta}_{t+j} \right) \right] \end{aligned} \tag{37}$$

where various cancellations lead to $-\rho\hat{\zeta}_{t+1}$ rather than $-\gamma\hat{\zeta}_{t+1}$ as in (21). Equation (37) shows how news about future consumption and future labor matters for the current stochastic discount factor.

[3] The similar calculation for the full version (86) is only slightly more complicated: one additionally needs to keep track of the tax term.

In the state space representation (33), assume that

$$\left(1-\tilde{\beta}\right)\left(\hat{c}_t-\nu\hat{n}_t\right)+\hat{\zeta}_t=c'X_t \tag{38}$$

for some coefficient vector c. This can always be achieved by carrying the lhs of (38) is one of the entries in X_t and thus allowing for X_t to encompass more than the minimal list of state variables. More generally, one can always find c_X and c_ϵ, so that the lhs of (38) equals $c_X'X_{t-1}+c_\epsilon'\epsilon_t$: that seems unnecessarily cumbersome notation, however. Likewise, assume that

$$\zeta_t = a'X_t \tag{39}$$

$$\hat{c}_t+\frac{1-\rho}{\rho}\nu\hat{n}_t = b'X_t \tag{40}$$

for some vectors a and b.

With the VAR representation (33), note that

$$\mathcal{S}_{t+1}[X_{t+j}]=G^{j-1}H\epsilon_{t+1}$$

Therefore, (37) can be rewritten as

$$\begin{aligned}\hat{M}_{t+1} &= -\rho(a'X_{t+1}+b'(X_{t+1}-X_t))+\\ &\quad +(\rho-\gamma)c'\mathcal{S}_{t+1}\left[\sum_{j=0}^{\infty}\tilde{\beta}^jX_{t+j}\right]\end{aligned}$$

and thus

$$\begin{aligned}\hat{M}_{t+1} &= -\rho\left((a+b)'(H\epsilon_{t+1}+GX_t)-b'X_t\right)\\ &\quad +(\rho-\gamma)c'(I-\tilde{\beta}G)^{-1}H\epsilon_{t+1}\end{aligned} \tag{41}$$

To obtain multiperiod asset pricing for an asset held from period t to $t+k$, note that appropriate stochastic discount factor is

$$M_{t+k,t}=M_{t+k}M_{t+k-1}M_{t+k-2}\ldots M_{t+1}$$

For the log-deviations, one obtains

$$\begin{aligned}\hat{M}_{t+k,t} &= \hat{M}_{t+k} + \hat{M}_{t+k-1} + \ldots + \hat{M}_t \\ &= -\rho\left(\sum_{j=1}^{k}\hat{\zeta}_{t+j}\right) - \rho\left(\hat{c}_{t+k} - \hat{c}_t\right) - (1-\rho)\nu\left(\hat{n}_{t+k} - \hat{n}_t\right) \\ &\qquad +(\rho-\gamma)\mathcal{S}_{t+k|t}\left[\sum_{j=0}^{\infty}\tilde{\beta}^j\left(\left(1-\tilde{\beta}\right)\left(\hat{c}_{t+j} - \nu\hat{n}_{t+j}\right) + \hat{\zeta}_{t+j}\right)\right]\end{aligned}$$

As above, this can be rewritten as

$$\begin{aligned}\hat{M}_{t+k,t} &= -\rho(a'\left(\sum_{j=1}^{k}X_{t+j}\right) + b'(X_{t+k} - X_t)) \\ &\qquad +(\rho-\gamma)b'\mathcal{S}_{t+k|t}\left[\sum_{j=0}^{\infty}\tilde{\beta}^j X_{t+j}\right]\end{aligned}$$

which in turn can be rewritten as a linear function of X_t and ϵ_{t+1} through ϵ_{t+k}, as in (41). The details are in appendix B.

With these expressions for the stochastic discount factor, and under the assumption of log-normality, it is now easy to do asset pricing, exploiting the standard asset pricing equation.

For the return R_t^f for a one-period safe real bond from period t to $t+1$, I have $1 = E_t[R_t^f M_{t+1}]$ or

$$\begin{aligned}\log R_t^f &= -\log E_t[M_{t+1}] \\ &= -\log E_t[\bar{M}\exp(M_{t+1})] \\ &= -\log(\bar{M}) - E_t[M_{t+1}] - \frac{1}{2}\mathrm{Var}_t[M_{t+1}] \\ &= -\log(\bar{M}) - b'X_t - \frac{1}{2}d'HH'd\end{aligned}$$

where

$$d = -\rho(a+b) + (\rho-\gamma)c'(I-\tilde{\beta}G)^{-1} \tag{42}$$

Consider some risky asset with return

$$R_{t+1} = \bar{R}\exp(\hat{R}_{t+1})$$

with

$$\hat{R}_{t+1} = f'H\epsilon_{t+1} + g'X_t$$

where the risk loading f is known, but where $\bar{R}$ and g need to be determined. The asset pricing equation

$$\begin{aligned} 1 &= E_t[M_{t+1}R_{t+1}] \\ &= \bar{M}\bar{R}E_t[\exp(\hat{M}_{t+1} + \hat{R}_{t+1}] \end{aligned}$$

delivers

$$\log \bar{R} + g'X_t = -\log(\bar{M}) - b'X_t - \frac{1}{2}h'HH'h \tag{43}$$

where

$$h = -\rho(a+b) + (\rho - \gamma)c'(I - \tilde{\beta}G)^{-1} + f \tag{44}$$

Taking unconditional expectations in (43) delivers

$$\begin{aligned} \log \bar{R} &= -\log(\bar{M}) - \frac{1}{2}h'HH'h \\ g'X_t &= -b'X_t \end{aligned}$$

Note that the conditional expected return of R_{t+1} is given by

$$\log E_t[R_{t+1}] = \log \bar{R} + g'X_t + \frac{1}{2}f'HH'f$$

Comparing these equations allows the calculation of Sharpe ratios.

5.1 Comparison to Hansen-Heaton-Li

The asset pricing equations provided in Hansen-Heaton-Li (2008) rely on a slightly different approach to approximation, investigating the Taylor-approximation around the $\rho = 1$ case, and relying on a VAR as in (33) for the evolution of the state. Le and Singleton (2010) have investigated higher order approximations, using the same approach. The purpose of this subsection is to compare the asset pricing results here to these two investigations.

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5.2 Adjustment of the steady state for a risk premium

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6 Wedges, taxes or preference shocks?

To pursue applications of the model above, it may be important to examine more precisely the interpretation and role of $\tau_t^c, \tau_t^x, \tau_t^w, \tau_t^k$ and s_t.

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7 Guvenen's model

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8 Numerical results

TO BE COMPLETED

9 Conclusions

TO BE COMPLETED

Appendix

A The equations characterizing the equilibrium

The equations characterizing the dynamics of the endogenous variables in equilibrium are

$$
\begin{aligned}
V_t &= \left(\left(1-\tilde{\beta}\right)\left(c_t\Phi(n_t)\right)^{1-\rho}+\beta\mathcal{R}_t^{1-\rho}\right)^{\frac{1}{1-\rho}} && (45)\\
\mathcal{R}_t &= \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} && (46)\\
M_{t+1} &= \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{\rho-\gamma}\left(\frac{\Phi(n_{t+1})}{\Phi(n_t)}\right)^{1-\rho}\frac{1+\tau_t^c}{1+\tau_{t+1}^c} && (47)\\
1 &= E_t\left[M_{t+1}R_{t+1}^{(k)}\right] && (48)\\
\frac{-\Phi'(n_t)c_t}{\Phi(n_t)} &= \frac{1-\tau_t^w}{1+\tau_t^c}w_t && (49)\\
y_t &= f\left(\frac{A_tn_t}{k_t}\right)k_t && (50)\\
k_{t+1} &= \left(1-\delta+h\left(\frac{x_t}{k_t}\right)\right)k_t && (51)\\
(1+\tau_t^c)c_t+(1+\tau_t^x)x_t &= (1-\tau_t^w)w_tn_t+(1-\tau_t^k)r_tk_t+s_t && (52)\\
w_t &= A_tf'\left(\frac{A_tn_t}{k_t}\right) && (53)\\
r_tk_t &= y_t-w_tn_t && (54)\\
R_t^{(k)} &= \frac{1}{q_{t-1}}\Big((1-\tau_t^k)r_t && (55)\\
&\qquad +q_t\left(1-\delta+h\left(\frac{x_t}{k_t}\right)\right)-(1+\tau_t^x)\frac{x_t}{k_t}\Big)\\
q_t &= \frac{1+\tau_t^x}{h'\left(\frac{x_t}{k_t}\right)} && (56)\\
g_t &= y_t-c_t-x_t && (57)
\end{aligned}
$$

In terms of the de-trended variables and recalling (11), they are

$$\tilde{V}_t = \left(\left(1-\tilde{\beta}\right)\left(\tilde{c}_t\Phi(n_t)\right)^{1-\rho} + \beta\tilde{\mathcal{R}}_t^{1-\rho}\right)^{\frac{1}{1-\rho}} \tag{58}$$

$$\tilde{\mathcal{R}}_t = \left(E_t\left[\zeta_{t+1}\tilde{V}_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{59}$$

$$M_{t+1} = \beta\zeta_{t+1}^{-\gamma}\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{-\rho}\left(\frac{\tilde{V}_{t+1}}{\tilde{\mathcal{R}}_t}\right)^{\rho-\gamma}\left(\frac{\Phi(n_{t+1})}{\Phi(n_t)}\right)^{1-\rho}\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \tag{60}$$

$$1 = E_t\left[M_{t+1}R_{t+1}^{(k)}\right] \tag{61}$$

$$\frac{-\Phi'(n_t)\tilde{c}_t}{\Phi(n_t)} = \frac{1-\tau_t^w}{1+\tau_t^c}\tilde{w}_t \tag{62}$$

$$\tilde{y}_t = f\left(\frac{n_t}{\tilde{k}_t}\right)\tilde{k}_t \tag{63}$$

$$\zeta_{t+1}\tilde{k}_{t+1} = \left(1-\delta+h\left(\frac{\tilde{x}_t}{\tilde{k}_t}\right)\right)\tilde{k}_t \tag{64}$$

$$(1+\tau_t^c)\tilde{c}_t + (1+\tau_t^x)\tilde{x}_t = (1-\tau_t^w)\tilde{w}_t n_t + (1-\tau_t^k)r_t\tilde{k}_t + \tilde{s}_t \tag{65}$$

$$\tilde{w}_t = f'\left(\frac{n_t}{\tilde{k}_t}\right) \tag{66}$$

$$r_t\tilde{k}_t = \tilde{y}_t - \tilde{w}_t n_t \tag{67}$$

$$R_t^{(k)} = \frac{1}{q_{t-1}}\Big((1-\tau_t^k)r_t + q_t\left(1-\delta+h\left(\frac{\tilde{x}_t}{\tilde{k}_t}\right)-h'\left(\frac{\tilde{x}_t}{\tilde{k}_t}\right)\frac{\tilde{x}_t}{\tilde{k}_t}\right)\Big) \tag{68}$$

$$q_t = \frac{1+\tau_t^x}{h'\left(\frac{\tilde{x}_t}{\tilde{k}_t}\right)} \tag{69}$$

$$\tilde{g}_t = \tilde{y}_t - \tilde{c}_t - \tilde{x}_t \tag{70}$$

Noting the particular cancellations in (73) and defining

$$\theta = \frac{\bar{w}\bar{n}}{\bar{y}} = 1 - \frac{\bar{r}\bar{k}}{\bar{y}}$$

the steady state equations are

$$\bar{V} = \bar{c}\Phi(\bar{\bar{n}}) \tag{71}$$

$$
\begin{aligned}
\bar{\mathcal{R}} &= \bar{\zeta}\bar{V} && (72)\\
\bar{M} &= \beta\bar{\zeta}^{-\rho} = \frac{\tilde{\beta}}{\bar{\zeta}} && (73)\\
1 &= \bar{M}\overline{R^{(k)}} && (74)\\
\frac{-\Phi'(\bar{n})\bar{n}}{\Phi(\bar{n})} &= \frac{1-\tau^w}{1+\tau^c}\theta\frac{\bar{y}}{\bar{c}} && (75)\\
\frac{\bar{y}}{\bar{k}} &= f\left(\frac{\bar{n}}{\bar{k}}\right) && (76)\\
\bar{\zeta} &= \left(1-\delta+\tilde{\delta}\right) \text{ and } \bar{x} = \tilde{\delta}\bar{k} && (77)\\
(1+\overline{\tau^c})\frac{\bar{c}}{\bar{y}} + (1+\overline{\tau^x})\frac{\bar{x}}{\bar{y}} &= (1-\overline{\tau^w})\theta + (1-\overline{\tau^k})(1-\theta) + \frac{\bar{s}}{\bar{y}} && (78)\\
\bar{w} &= f'\left(\frac{\bar{n}}{\bar{k}}\right) && (79)\\
\bar{r} &= \frac{\bar{y}}{\bar{k}}(1-\theta) && (80)\\
\overline{R^{(k)}} &= \frac{1-\overline{\tau^k}}{1+\overline{\tau^x}}\bar{r} + 1 - \delta && (81)\\
\bar{q} &= 1+\overline{\tau^x} && (82)\\
\frac{\bar{g}}{\bar{y}} &= 1 - \frac{\bar{c}}{\bar{y}} - \frac{\bar{x}}{\bar{y}} && (83)
\end{aligned}
$$

Given a value for θ and $\Phi(\bar{n})$ as well as values for $\overline{\tau^c}$, $\overline{\tau^x}$, $\overline{\tau^w}$, $\overline{\tau^k}$, $\bar{s}$, $\bar{\zeta}, \tilde{\delta} = \bar{\zeta} - 1 + \delta, \beta$ and a functional form for $f(\cdot)$ and $\Phi(\cdot)$, it is easy to calculate the solution: indeed, this is a standard procedure. Calculate $\bar{M}$ per (73). With that, calculate $\overline{R^{(k)}}$ per (74). With that and $\bar{a}$ from (82), calculate $\bar{r}$ per (81). That delivers the ratio $\bar{k}/\bar{y}$ per (80) and therefore $\bar{x}/\bar{y} = \tilde{\delta}\bar{k}/\bar{y}$. Use that to calculate the ratio $\bar{c}/\bar{y}$ per (78). Note that the left-hand side of (75) is a function of $\bar{n}$ only, whereas the right-hand side is now known. Solve it for a solution (typically: the only solution) $\bar{n}$. Furthermore, with knowledge of the ratio $\bar{k}/\bar{y}$, obtain the ratio $\bar{n}/\bar{k}$ per (76). Together, this now implies a value for $\bar{k}$, and thus $\bar{y}$, $\bar{c}$ and $\bar{x}$. Also, calculate $\bar{w}$ per (79) and $\bar{g}$ or $\bar{g}/\bar{y}$ per (83).

The log-linearized version of these equations are

$$\hat{V}_t = \left(1-\tilde{\beta}\right)(\hat{c}_t - \nu\hat{n}_t) + \tilde{\beta}\hat{\mathcal{R}}_t \tag{84}$$

$$\hat{\mathcal{R}}_t = E_t\left[\hat{\zeta}_{t+1} + \hat{V}_{t+1}\right] \tag{85}$$

$$\begin{aligned}\hat{M}_{t+1} &= -\gamma\hat{\zeta}_{t+1} - \rho\left(\hat{c}_{t+1} - \hat{c}_t\right) - (1-\rho)\nu\left(\hat{n}_{t+1} - \hat{n}_t\right) \\ &\quad +(\rho-\gamma)\left(\hat{V}_{t+1} - \hat{\mathcal{R}}_t\right) + \frac{1}{1+\overline{\tau^c}}\left(\hat{\tau}_t^c - \hat{\tau}_{t+1}^c\right)\end{aligned} \tag{86}$$

$$0 = E_t\left[\hat{M}_{t+1} + \hat{R}_{t+1}^{(k)}\right] \tag{87}$$

$$\hat{n}_t = \eta_S\left(\hat{w}_t - \hat{c}_t - \frac{1}{1+\overline{\tau^w}}\hat{\tau}_t^w - \frac{1}{1+\overline{\tau^c}}\hat{\tau}_t^c\right) \tag{88}$$

$$\hat{y}_t = \theta\hat{n}_t + (1-\theta)\hat{k}_t \tag{89}$$

$$\hat{k}_{t+1} = (1-\tilde{\delta})\hat{k}_t + \tilde{\delta}\hat{x}_t - \hat{\zeta}_{t+1} \tag{90}$$

$$\frac{\bar{c}}{\bar{y}}\left(\hat{\tau}_t^c + (1+\overline{\tau^c})\hat{c}_t\right) + \frac{\bar{x}}{\bar{y}}\left(\hat{\tau}_t^x + (1+\overline{\tau^x})\hat{x}_t\right) \tag{91}$$

$$\begin{aligned}&= \theta\left(-\hat{\tau}_t^w + (1-\overline{\tau^s})(\hat{w}_t + \hat{n}_t)\right) \\ &\quad +(1-\theta)\left(-\hat{\tau}_t^k + (1-\overline{\tau^k})(\hat{r}_t + \hat{k}_t)\right) + \frac{\bar{s}}{\bar{y}}\hat{s}_t\end{aligned} \tag{92}$$

$$\eta_D\hat{w}_t = \hat{k}_t - \hat{n}_t \tag{93}$$

$$\hat{r}_t = -\frac{\theta}{1-\theta}\hat{w}_t \tag{94}$$

$$\begin{aligned}\hat{R}_t^{(k)} &= -\hat{q}_{t-1} + \frac{\bar{r}}{\bar{q}\overline{R^{(k)}}}\left(\left(1-\overline{\tau^k}\right)\hat{r}_t - \hat{\tau}_t^k\right) \\ &\quad +\frac{1}{\overline{R^{(k)}}}\left(\zeta\omega(\hat{x}_t - \hat{k}_t) + \frac{1-\delta}{1-\overline{\tau^x}}\hat{\tau}_t^x\right)\end{aligned} \tag{95}$$

$$\hat{q}_t = \frac{1}{1+\overline{\tau^x}}\hat{\tau}_t^x + \omega(\hat{x}_t - \hat{k}_t) \tag{96}$$

$$\frac{\bar{g}}{\bar{y}}\hat{g}_t = \hat{y}_t - \frac{\bar{c}}{\bar{y}}\hat{c}_t - \frac{\bar{x}}{\bar{y}}\hat{x}_t \tag{97}$$

It may be convenient to define

$$\hat{\lambda}_t = -\rho\hat{c}_t - (1-\rho)\nu\hat{n}_t + (\rho-\gamma)\hat{\mathcal{R}}_t + \frac{1}{1+\overline{\tau^c}}\hat{\tau}_t^c \tag{98}$$

and then to replace (86) with

$$\hat{M}_t = \hat{\lambda}_t - \hat{\lambda}_{t-1} + (\rho-\gamma)\left(\hat{V}_t - \hat{\mathcal{R}}_t\right) - \gamma\hat{\zeta}_t \tag{99}$$

as in (32).

B Multiperiod asset pricing and the term structure of interest rates

TO BE COMPLETED

References

[1] Backus, David K., Bryan R. Routledge and Stanley E. Zin, 2007. "Asset pricing in business cycle analysis," draft, NYU.

[2] Bansal, Ravi and Amir Yaron (2004), "Risk for the Long Run: A Potential Resolution of Asset Pricing Puzzles," Journal of Finance, vol. 59, no. 4, 1481-1509.

[3] Binsbergen, Jules H., Jesús Fernándex-Villaverde, Ralph S.J. Koijen, Juan F. Rubio-Ramirez (2008). "Likelihood Estimation of DSGE models with Epstein-Zin preferences", draft, Duke University.

[4] Chari, V.V., Patrick J. Kehoe and Ellen R. McGrattan, 2007. "Business Cycle Accounting," Econometrica, Econometric Society, vol. 75(3), pages 781-836, 05.

[5] Chetty, Raj, 2009. "Bounds on Elasticities with Optimization Frictions: Reconciliation of Micro and Macro Labor Supply Elasticities," draft, Harvard University.

[6] Christiano, Lawrence, Martin Eichenbaum, and Charles Evans (2005), "Nominal rigidities and the dynamic effects of a shock to monetary policy", Journal of Political Economy, Vol. 113(1), pages 145.

[7] Epstein, L., and S. Zin (1989), "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," Econometrica 57: 937-968.

[8] Groth, Charlotta and Hashmat Khan, 2007. "Investment adjustment costs: evidence from UK and US industries." Bank of England Working Paper 332.

[9] Guvenen, Fatih, 2009. "A Parsiminonious Macroeconomic Model for Asset Pricing," Econometrica, vol. 77, no. 6, 1711-1750.

[10] Hansen, Lars Peter, John C. Heaton and Nan Li, 2008. "Consumption Strikes Back? Measuring Long-Run Risk," Journal of Political Economy, vol 116, no. 2, 260-302.

[11] Jermann, Urban J., 1998. "Asset pricing in production economies," Journal of Monetary Economics, Elsevier, vol. 41(2), pages 257-275, April.

[12] Judd, Kenneth L. (1998),*Numerical Methods in Economics*, MIT Press.

[13] Lettau, Martin and Harald Uhlig, 2002. "The Sharpe Ratio and Preferences: A Parametric Approach," Macroeconomic Dynamics. April; 6(2): 242-65.

[14] Le, Anh and Kenneth J. Singleton, 2010. "Long Run Risks and the Predictability of Bonds' Excess Returns," American Economic Review Papers and Proceedings.

[15] Piazzesi, Monika and Martin Schneider (2007a), "Equilibrium Yield Curves", NBER Macroeconomics Annual 2006, Kenneth Rogoff and Michael Woodford, eds., Cambridge MA: MIT Press, 389-442.

[16] Piazzesi, Monika, Martin Schneider and Selale Tuzel (2007b), "Housing, Consumption, and Asset Pricing", Journal of Financial Economics 83, March 2007, 531-569.

[17] Rudebusch, Glenn and Eric Swanson (2009). The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks. Federal Reserve Bank of San Francisco Working Paper 200831.

[18] Swanson, Eric T., (2009). "Risk Aversion, the Labor Margin, and Asset Pricing in DSGE models", draft, Federal Reserve Bank of San Francisco.

[19] Tallarini, Thomas D. Jr., "Risk-Sensitive Real Business Cycles," Journal of Monetary Economics. June 2000; 45(3): 507-32.

[20] Trabandt, Mathias and Harald Uhlig, September 2009. "How far are we from the slippery slope? The Laffer curve revisited." NBER Working Paper 15343, NBER, Cambridge, MA.

[21] Uhlig, Harald, 1999. "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in R. Marimon and A. Scott: Computational Methods for the Study of Dynamic Economies. Oxford and New York: Oxford University Press, pp. 30-61.

[22] Uhlig, Harald (2007a), "Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model," American Economic Association Papers and Proceedings, American Economic Review, vo. 97, no. 2, 239-243.

[23] Uhlig, Harald (2007b), "Leisure, Growth and Long Run Risk." Draft, University of Chicago.