Bayesian Estimation of a DSGE Model with Asset Prices*

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Abstract

The paper presents the estimation of a dynamic stochastic general equilibrium (DSGE) model which jointly explains business cycle and asset pricing facts. Therefore, we propose a novel estimation approach by using an 'augmented' prior distribution which accounts for observable stylized facts which are not included in the likelihood. Moreover, we explicitly take into account the agent's attitude to risk by approximating the model around its stochastic steady state. The estimated model replicates the observed Sharpe and other stylized asset pricing facts along with reasonable business cycle facts.

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1 Introduction

This paper presents an estimation of a dynamic stochastic general equilibrium (DSGE) model which explicitly takes into account the observed stylized asset pricing facts next to the observed time series. In particular, we focus on the Sharpe ratio which determines the difference of asset return expectations and their degree of risk. We form an 'augmented' prior distribution, which also takes into account observed stylized facts. This approach ensures that the DSGE model is estimated along a path of the parameter space which is also inline with additional information not included in the observations. Hence, the estimated DSGE model is in line with business cycle facts as well as the observed Sharpe ratio. Additionally, the present approach allows us to empirically investigate the ability of different model approaches as proposed by the recent literature in this field. For example, as shown by Boldrin, Christiano, and Fisher (2001) and Uhlig (2007) habit formation and labor rigidities are helpful to explain macroeconomic facts as well as asset pricing facts. For this reason, the model used in this paper is an extension of Uhlig (2007) which incorporates both approaches.

In common, prior choices for DSGE model estimation focuses on macroeconomic and microeconomic facts rather than on asset pricing facts. In particular, asset pricing is affected by the agents' attitude to risk, more precisely her precautionary savings and her possibility to smooth consumption. Hence, it is necessary to solve a DSGE model not only accurately with respect to first moments but also to to incorporate second moments, appropriately. However, including those risk terms into a standard estimation framework, due to observations, would require higher order approximations of the model. The contribution of the present paper is to propose a methodology of a 'augmented' prior which ensures that the model is estimated in line with observable asset pricing facts for a simple first-order log-linear model solution. This augmented prior allows us to estimate the model with respect to their ability to explain macroeconomic facts as well as asset prices, e.g. higher-order risk terms, simultaneously. Next to that, we solve the model around its stochastic steady state which takes the agent's uncertainty about the future into account. Moreover, we explicitly ignore recent attempts to overcome common problems of DSGE models with asset prices. In particular, we use power utility with habit formation and assume log-normality of shocks rather than recursive preferences, long-run risk, or rare disaster risk. Finally, the estimation proceeds on several steps. In the first step we estimate the model using our observations and standard prior distributions. Sampling from the corresponding posterior gives us an 'implied' distribution of the model's Sharpe ratio. Then, given this distribution as well as the distribution of the observed Sharpe ratio in the data we form our 'augmented' prior. In the final step, the resulting prior together with the likelihood is used to form a posterior distribution, which

ensures an 'implied' distribution of the model's Sharpe ratio equivalent to the one observed in the data. More precisely, the 'augmented' prior distribution is conditional on the model's conditional second moments and on observed stylized asset pricing facts not included in the likelihood.

The estimation results illustrate the success of our methodology. We can replicate the observed Sharpe ratio with the model's implied Sharpe ratio without ruin business cycle facts. In particular, the estimated model implies a risk premium which is ten times as high as in the benchmark model. Simultaneously, the volatility of the risk-free return is similar to the data and the 'risk-free rate puzzle' (Weil, 1989) is avoided. Of course, the model needs a high relative risk aversion to generate these results, but this goes in line without disturbing the business cycle characteristics of the model. Next to the 'augmented' prior approach the inclusion of agent's attitude to risk, by calculating the stochastic steady state, non-separability between consumption an leisure, and labor rigidities seems essential for the estimation results and the ability of a DSGE model to explain macroeconomic facts and asset prices jointly. For example, ignoring e.g. precautionary savings of agents leads a steady state which indicates a poorer economy as in the case of accounting for the agents' precautionary motives. This leads to different and distorted business cycle implications. Moreover, a separable utility in consumption and leisure would goes along with smoother consumption for higher relative risk averseness, while this is not the case for the nonseparable preferences used in the present paper. Finally, the estimates suggest that labor rigidities like the labor wedge and a small Frisch elasticity rather than external habits in consumption play an important role for the simultaneous explanation of macroeconomic facts and asset market facts. These results are in line with Uhlig (2007), who has shown that inelasticity of labor supply, a smaller elasticity of leisure substitution, and wage rigidities can help to explain the risk premium.

The present paper adds to the recent literature in various ways. Firstly, we present a novel procedure of a augmented prior to include stylized asset pricing facts into the estimation. This procedure adds to the literature about endogenous prior choice for Bayesian estimation of DSGE models as recently developed by Del Negro and Schorfheide (2008) and Christiano, Trabandt, and Walentin (2011). Secondly, we extend the discussion of accounting for agents' precautionary motive (see Coeurdacier, Rey, and Winant, 2011) and provide a first estimation of a DSGE which deals with this circumstance. Therefore, the results of the present paper indicate that high relative risk aversion generates similar dynamics of the macroeconomic variables as in the case of low relative risk aversion. This characteristic is similar to the findings of Tallarini (2000) or Rudebusch and Swanson (2012) for models with recursive preferences as postulated by Epstein and Zin (1989, 1991). Of course, because of

the characteristic of our preferences such high relative risk aversion is followed by a very small elasticity of intertemporal substitution compared to the suggestions by Hall (1988) or Vissing-Jørgensen (2002) but the macroeconomic dynamics are not disturbed. For this reason, our findings stay in contrast to the findings of Rudebusch and Swanson (2012, 2008) who emphasize that habit-based DSGE models fail in explaining financial facts and macroeconomic facts jointly. Moreover, our approach offers an alternative to explain financial and macroeconomic facts jointly compared to recent strands of literature dealing with long-run risk (e.g Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008) or rare disaster (e.g Barro, 2006; Gabaix, 2010).

The remaining paper is organized as follows. Section two introduces the model, while the third section investigates necessary asset pricing implications and introduces the approximation around the stochastic steady state. The fourth section presents the estimation methodology, characterizes the data, and describes the prior choice in more detail. Section five presents the estimation results and reports the asset pricing and business cycle characteristics of the model. The sixth section investigates the performance of the 'augmented' prior approach with a more simple version of the model. Finally, section seven concludes the paper.

2 Model

The present analysis bases upon the discounted stochastic growth economy proposed by Uhlig (2007). We extend the foregoing work by adding several stochastic processes, causing the fluctuations of the economy. In the following paragraphs we describe the model in detail and lay out the differences to Uhlig (2007) in more detail.

Form production is given by the following Cobb–Douglas production function,

$$y_t = k_{t-1}^{\theta} \left(e^{z_{P,t}} n_t\right)^{1-\theta} . \tag{1}$$

It depends on the capital k_{t-1} accumulated in the former period as well as on the hired amount of labor n_t in the current period. Furthermore, in period t the production depends on the technology level $z_{T,t}$ the firm can implement. We assume the technology to follow a random walk with drift

$$z_{P,t} = \gamma + z_{P,t-1} + \epsilon_{P,t}, \tag{2}$$

with γ reflecting the trend of the technology. In the linearized model we assume that $\epsilon_{P,t}$ is normally i.i.d. with standard error σ_P.

Under the assumption of competitive markets firm profit is equal to zero. Hence, the

first-order conditions of the firm's maximization problem yields the necessary conditions for the cost of hiring labor (market wages w_t) and borrowing capital (dividends d_t):

$$w_t = \frac{(1-\theta)\, y_t}{n_t} \quad \text{and} \tag{3}$$

$$d_t = \frac{\theta y_t}{k_{t-1}}. \tag{4}$$

Households are characterized by a representative agent with preferences characterized by the following utility function:

$$U = E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left((c_t - H_t)\left(A + (z_{L,t} l_t - F_t)^{\upsilon}\right)\right)^{1-\eta} - 1}{1-\eta}\right], \tag{5}$$

where c_t and l_t denote individual consumption and leisure respectively. The latter leisure is given by total time endowment minus labor supply n_t of the agent. For simplicity the total time endowment is scaled to unity,

$$n_t + l_t = 1.$$

In addition to the discount factor β there exist the parameters A, ν, and the power utility parameter η. Because of the monotonicity and concavity constraints, the parameters have to satisfy $\nu > 0$ and $\eta > \nu/(\nu+1)$. The variable $z_{L,t}$ represents a labor supply shock. In log-linearized terms the exogenous process can be expressed by a stochastic AR(1)process,

$$\hat{z}_{L,t} = \pi_L \hat{z}_{L,t-1} + \epsilon_{L,t} \tag{6}$$

where $\epsilon_{L,t}$ is a normally i.i.d. with standard deviation σ_L. Further, the utility of the representative agent depends on the economy-wide average level of consumption habit and leisure habit, H_t and L_t. These The exogenous habits evolve according to

$$H_t = e^{\gamma}\left((1-\rho_c)\,\chi C_{t-1} + \rho_c H_{t-1}\right), \tag{7}$$

$$F_t = (1-\rho_l)\,\psi L_{t-1} + \rho_l F_{t-1}, \tag{8}$$

where C_t and L_t are aggregate average levels of consumption and leisure. The parameters ρ_c and ρ_l denote the "depth" of the habits by implicitly reflecting the number of lags that are included in the representative agent's utility function. Finally, the parameters χ and ψ denote the habit persistence parameters corresponding to each of the mentioned exogenous habits.

The agent maximizes his utility by choosing leisure, consumption, and investments (x_t),

taking as given the exogenous habits, real wages w_t, and dividends d_t for providing capital to firms and pays lump-sum taxes T_t. Finally, he holds an initial endowment of capital k_{-1} and one unit of time. Hence, the budget constraint of the agent is

$$c_t + x_t + T_t = d_t k_{t-1} + w_t n_t. \tag{9}$$

Capital accumulation is affected by a depreciation rate δ and investment adjustment costs $g(\cdot)$,

$$k_t = \left(1 - \delta + g\left(z_{I,t}\frac{x_t}{k_{t-1}}\right)\right) k_{t-1}. \tag{10}$$

Following Jermann (1998) we assume the adjustment cost function $g(\cdot)$ to satisfy the following steady state conditions:

$$g\left(\tilde{\delta}\right) = \delta + e^{\gamma} - 1, \quad g'\left(\tilde{\delta}\right) = 1, \quad g''\left(\tilde{\delta}\right) = -\frac{1}{\zeta} \qquad \forall \quad \zeta > 0,. \tag{11}$$

with $\tilde{\delta}$ defined as $\tilde{\delta} = \exp(\gamma) + \delta - 1$. Besides, we assume that the adjustment costs are affected by the shock $z_{I,t}$ which is also defined as exogenous AR(1) process. The log-linearized counterpart can be written as:

$$\hat{z}_{I,t} = \pi_I \hat{z}_{I,t-1} + \epsilon_{I,t}, \tag{12}$$

where $\epsilon_{I,t}$ is a i.i.d. normal with standard deviation σ_I.

We assume real wage rigidities as postulated by e.g. Hall (2005), Shimer (2005), and Blanchard and Galí (2005). Under this assumption, the agent's first-order condition for labor supply yields the frictionless wage or the marginal rate of substitution,

$$w_t^f = \frac{U_L}{U_c} = \frac{z_{L,t}\upsilon\left(c_t - \chi c_{t-1}\right)}{A\left(z_{L,t}l_t - \psi l_{t-1}\right)^{1-\nu} + z_{L,t}l_t - \psi l_{t-1}}, \tag{13}$$

and the market wage,

$$w_t = \left(e^{\gamma} w_{t-1}\right)^{\mu} \left(e^{\varepsilon_{W,t}} w_t^f\right)^{1-\mu}. \tag{14}$$

The parameter $\varepsilon_{w,t} > 0$ represents the wage markup to ensure that $w > w^f$ locally around the steady state, and therefore the labor market is demand constraint . This specification also ensures that the workers receive more than their reservation wage if they decide to work. The parameter μ reflects the degree of frictions and real wage stickiness on the labor market. In the special case of $\mu = \varepsilon_w = 0$ there exist no frictions and the wages are fully flexible. Moreover, we assume that the wage markup follows a AR process, which log-linearized

counterpart can be written as

$$\hat{\varepsilon}_{W,t} = \pi_W \hat{\varepsilon}_{W,t-1} + \epsilon_{W,t}, \tag{15}$$

where $\epsilon_{W,t}$ is a normally i.i.d. with standard deviation σ_W.

Additionally, given the first order conditions of the household, we can evaluate the price of capital, which is defined as consumption cost of an additional unit of capital, as:

$$q_t = \frac{1}{g'\left(z_{I,t}\frac{x_t}{k_{t-1}}\right) z_{I,t}} \ , \tag{16}$$

while the same time q_t also illustrates the Tobin's q. Moreover, let λ_t be the marginal utility of consumption,

$$\lambda_t = U_c\left(c_t, l_t\right) \ , \tag{17}$$

and using the definition of the price of capital, the households' first order conditions desire the following relationship:

$$q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t}\left[\left(1-\delta+g\left(\cdot\right)\right) q_{t+1} - \frac{x_{t+1}}{k_t} + d_{t+1}\right]\right] \tag{18}$$

For simplicity we define the stochastic discount factor (SDF):

$$M_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \ , \tag{19}$$

and the return of investing in capital,

$$R_t^k = \frac{\theta \frac{y_t}{k_{t-1}} + \left(1-\delta+g\left(\cdot\right)\right) q_t - \frac{x_t}{k_{t-1}}}{q_{t-1}}. \tag{20}$$

Generally, for any asset in the economy the Lucas asset pricing equation has to hold:

$$1 = E_t\left[M_{t+1} R_{t+1}\right]. \tag{21}$$

As shown above, this equation holds in particular for R_{t+1}^k but has to hold as well for the risk-free return R_t^f,

$$1 = E_t\left[M_{t+1}\right] R_t^f. \tag{22}$$

Finally, for the government sector we assume that the fiscal policy is Ricardian, with a

budget balanced period by period through lump-sum taxes:

$$g_t = T_t \tag{23}$$

Furthermore, we model government expenditures exogenously, which can be expressed in linear terms as

$$\hat{g}_t = \pi_G \hat{g}_{t-1} + \epsilon_{G,t}, \tag{24}$$

where ϵ_G is a normally i.i.d. with standard deviation σ_G and π_G the autoregressive parameter.

Equilibrium: Given the initial values for $k_{-1} > 0$, $c_{-1} > 0$, $l_{-1} > 0$, $w_{-1} > 0$, $H_{-1} > 0$, $F_{-1} > 0$, and a Ricardian fiscal authority; a rational expectations equilibrium is a set of sequences $\{y_s,\ c_s,\ H_s,\ k_s,\ w_s,\ w_s^f,\ M_s,\ x_s,\ q_s,\ l_s,\ F_s,\ n_s,\ d_s,\ R_s^f,\ R_s^k\}_{s=t}^{\infty}$, which is satisfying the firms' first order condition, the households' first order condition, and the aggregate resource constraint, by clearing the labor market, clearing the market of capital, and clearing the final goods market, $y_s = c_s + x_s + g_s$, for $\{z_{P,s},\ z_{I,s},\ z_{L,s},\ g_s,\ \varepsilon_{W,s}\}_{s=t}^{\infty}$.

Finally, note that the variables k_t, y_t, c_t, H_t, w_t, w_t^f, x_t, λ_t, and g_t have to be productivity-detrended to solve the model. That is done by dividing each variable by $\exp(z_{P,t-1})$, except capital k_t, which is detrended with $\exp(z_{P,t})$ and λ_t which is detrended by $\exp(-\eta z_{P,t-1})$. Beside this, we assume l_t, F_t, n_t, q_t, R_t^f, R_t^k, M_t, and d_t to be stationary. In the following all detrended variables are marked with $\sim$.

3 Asset Pricing and Stochastic Steady State

In order to better understand the relationship between asset pricing and business cycles, we investigate these relation in this subsection in more detail. Given the model described before, we can derive for different asset returns.

Following Uhlig (1999) and Campbell (1994) we solve the model by using the method of undetermined coefficients.[1] We log-linearize the variables around their steady-state values to receive a linear approximation of the model. This solution technique yields the following recursive law of motion in the form:

$$\hat{y}_t = A\hat{h}_{t-1} + B\epsilon_t\ , \tag{25}$$

where $\hat{y}_t = \log(y_t) - \log(y^{ss})$ is a vector containing all log-linearized model variables and

[1] See Taylor and Uhlig (1990) for an overview of different methods to solve nonlinear stochastic models.

$\hat{h}_t = \log(h_t) - \log(h^{ss})$ is the vector containing all log-linearized state variables of the model, with y^{ss} and h^{ss} their corresponding steady state values. The matrices A and B contain the partial elasticities.

Following Lettau and Uhlig (2002) and using the representation above we can decompose e.g. the log pricing kernel into its conditional expectation and its innovations:

$$\hat{m}_{t+1} = E_t\left[\hat{m}_{t+1}\right] + b_m \epsilon_t \ , \tag{26}$$

where b_m indicates the row vector of matrix B with respect to the pricing kernel. We assume homoscedasticity and that the shock processes are independent and normally distributed with $\Sigma = \epsilon_t' \epsilon_t$ is the corresponding diagonal variance covariance matrix. Under these assumptions the conditional variance, σ_m^2, of the Pricing kernel is given by:

$$\sigma_m^2 = b_m \Sigma b_m' \tag{27}$$

Similarly, we can solve for the conditional variance of R_t^k and R_t^f respectively. Additionally, the conditional covariance of the pricing kernel and the return on capital, σ_{mR^k}, can be evaluated as

$$\sigma_{mR^k} = b_m \Sigma b_{R^k}' \ , \tag{28}$$

with b_{R^k} the row vector of matrix B with respect to the return on capital R^k.

With these finding in mind it is easy to do standard asset pricing and to illustrate relationship of asset returns and conditional second moments. Especially the differences between different asset returns as well as the risk-measure Sharpe ratio depend only on second moments.

For example, let's investigate the risk premium of the return on capital over the risk-free return. If we measure this premium as $E_t\left[R_{t+1}^k\right]/R_t^f$ we obtain that the premium depends only on the covariance between the log pricing kernel and the log return on capital:

$$E_t\left[\hat{r}_{t+1}^k\right] - \hat{r}_t^f = -\sigma_{mR^k} \ . \tag{29}$$

A similar result can be obtained for the Sharpe ratio:

$$\omega = \frac{E_t\left[\hat{r}_{t+1}^k\right] - \hat{r}_t^f}{\sigma_{R^k}} = -\frac{\sigma_{mR^k}}{\sigma_{R^k}} \ . \tag{30}$$

Following Hansen and Jagannathan (1997) and Campbell and Cochrane (2000) the highest possible Sharpe ratio is equal to σ_m, by assuming a correlation between the pricing kernel

and the return of capital equal to -1. Of course, it is unlikely that this restriction is satisfied by the data, but it illustrates the weakest assumption regarding the Sharpe ratio. These findings are common knowledge, but they illustrate the necessity of second moments for the evaluation of asset returns. To include these findings in our model we rewrite the Euler equations as follows for the risk-free return:

$$\frac{1}{R_t^n} = \exp\left(\log \bar{M} + E_t\left[\hat{m}_{t+1}\right] + \frac{\sigma_m^2}{2}\right), \tag{31}$$

and comparably for the return and capital:

$$\frac{1}{E_t\left[R_{t+1}^k\right]} = \exp\left(\log \bar{M} + E_t\left[\hat{m}_{t+1}\right] + \frac{\sigma_m^2}{2} + \sigma_{mR^k}\right). \tag{32}$$

In contrast to the common approach of linearization around the deterministic steady state, where the agents do not take into account the existence of future shocks, the approach in the present paper incorporates the agents' attitude towards risk (Juillard, 2010). In particular, the steady state is affected by future uncertainty. Similarly, Coeurdacier et al. (2011) call this point the risky steady state, which is the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date. Hence, we assume that the agents take into account the existence of future shocks, which makes the present approach significantly different from standard second-order approximation around the deterministic steady state (e.g. Schmitt-Grohe and Uribe, 2004).

Because the conditional second moments depend on the policy function, we get a fixed point problem by solving our model accurately with respect to the stochastic steady state. For this reason, we use an iterative procedure to achieve the steady state. We start with a set of second moments to solve for the steady state and to obtain the model's policy function. Afterwards, we use the implicit second moments of this solution as new starting values and solve the model again. As mentioned by Canton (2002) usually few iterations suffice to achieve convergence and to resolve the fixed point problem. Of course, the iteration algorithm has to be done for any set of structural parameters, which implies a significant increase of computational time.

4 Estimation

4.1 Data

The following paragraphs describe the time series, some calculations, and finally the implementation of the observed data into the model. The estimation of the model is based on five time series within the period from 1963:qI to 2008:qII. All data are quarterly and in real terms. Moreover, the Business Cycle data are measured in per capita terms.

In particular, we use the quarterly real Gross Domestic Product as measure for output.[2] Additionally we use the civilian noninstitutional population over 16 years from the Bureau of Labor Statistics (BLS) as a proxy for population to calculate per capita time series. Finally, we calculate the first differences of the real logarithmic output per capita and afterwards reduce the mean of this time series, $\Delta\hat{y}_t^{obs}$. We use the calculated mean to calibrate the growth rate of the economy γ. For consumption we use the expenditures on non-durables and services. The observable variable for investment is calculated as the sum of nominal gross private investment and personal durable consumption both provided by the Bureau of Economic Analysis (BEA). Both time series, consumption and investment, are transformed into real and per capita terms, by using the GDP deflator and the population series mentioned above. Altogether, we use the demeaned log-differences of these time series as observable variables during the estimation. In particular, the observed variables e.g. $\Delta\hat{y}_t^{obs}$, are implemented into the model as follows:

$$\Delta\hat{y}_t^{obs} = \hat{\tilde{y}}_t - \hat{\tilde{y}}_{t-1} + \epsilon_{T,t-1} \tag{33}$$

Additionally, we use observation for hours worked in the estimation. In particular, we use quarterly hours worked of of employees working in private non-farm business excluding non-profit business. This series is an updated version of the one used by Francis and Ramey (2009). The final logarithmic time series is demeaned. Moreover, we use observation for the real risk-free return. As proxy we use the quarterly returns calculated based on the monthly returns of the three month T-Bill returns provided by the Board of Governors of the Federal Reserve System. The returns are calculated in real terms, too, by using the implicit inflation given by the GDP price deflator. Furthermore, the final logarithmic return series is demeaned. Finally, we use the excess returns as observable variable. The excess returns are calculated as the log differences between the total returns of the S&P 500 and the three month T-Bill returns. Because we have no equivalent variable in our model to capture equity returns we define excess returns' log-linear deviations from steady state as

[2]See appendix A for details about source and description of any data used in this paper.

follows

$$\hat{R}_{q,t}^{obs} = -\sigma_{mR^q} + \frac{1}{1-\Omega}\left(\hat{R}_t^k - \hat{R}_t^f\right) + \epsilon_{Q,t} \ , \tag{34}$$

where Ω is parameter which can be interpreted as leverage and $\epsilon_{Q,t}$ is an i.i.d. error term. In comparison to the other observable variables we do not de-trend excess returns to put more asset pricing information into the likelihood. Similar to the asset pricing presented in section 3 we can derive the mean excess returns as the negative covariance between the pricing kernel and the excess returns, $-\sigma_{mR^q}$. Because, we can observe the Sharpe ratio of the total returns of the S&P 500 in the data, but we cannot observe a Sharpe ratio for the return on economy-wide capital in the data; we assume that in our economy all assets are priced along the implied market line and therefore excess returns and capital returns share the same Sharpe ratio. Hence, we use $\omega \cdot \sigma_{R^q}$ as a proxy for the mean excess returns. The following table summarizes some stylized asset pricing facts:[3]

	Mean	s.d.
Sharpe ratio	0.2049	0.0753
mean excess returns	0.0149	0.0054
s.d. excess returns	0.0727	0.0038

Table 1: Stylized asset pricing facts (quarterly).

4.2 Prior Choice

We find it useful to describe our prior in terms of an economically meaningful transformation of the model's parameter. Specifically, consider the Frisch elasticity of labor supply, defined as the elasticity of labor supply to frictionless wages by holding the marginal rate of consumption constant,

$$\tau = \frac{dn}{dw^f}\frac{w^f}{n}\bigg|_{\bar{U_c}} . \tag{35}$$

Given our preference assumptions, this yields

$$\tau = \frac{U_n}{\bar{n}\left[U_{nn} - \frac{U_{nc}^2}{U_{cc}}\right]} = \frac{1-\bar{n}}{\bar{n}} \cdot \frac{\eta\left(1+\alpha\right)\left(1-\psi\right)}{\eta\left(\alpha\left(1-\nu\right)+1+\nu\right)-\nu} \ , \tag{36}$$

[3]The estimates are based on the maximum likelihood estimation of the following data generating process, $R_t^q = \omega\sigma_{R^q} + \sigma_Q\epsilon_t \ \epsilon_t \sim N\left(0,1\right)$. The presented standard deviation are based on inverse Hessian which is a consistent estimator of the covariance matrix of the parameters.

where $\alpha = A(1-\psi)^{-\nu}\bar{l}^{-\nu}$. Therefore, rather than specifying a prior for A or ν we shall specify a prior for τ, and calculate the implied A and ν from τ as well as other parameter. In particular, we also assume that the steady state level of hours worked is $n = 1/3$ to solve get

$$\nu = 1 - (1-\psi)\frac{\bar{l}}{1-\bar{l}}\frac{1}{\tau} - \left(2 - \frac{1}{\eta}\right)\frac{1}{(1-\chi)\kappa}\ , \tag{37}$$

with

$$\kappa = \frac{\exp\bar{\varepsilon}_W}{1-\theta}\frac{1-\bar{l}}{\bar{l}}\frac{\bar{c}}{\bar{y}}\ , \tag{38}$$

where the $\bar{c}/\bar{y}$ is the steady state consumption share of output.[4] Additionally, we can solve for the remaining preference parameter:

$$\alpha = \frac{\kappa\nu(1-\chi)}{1-\psi} - 1 \tag{39}$$

$$A = \alpha(1-\psi)^{\nu}\bar{l}^{\nu} \tag{40}$$

While the Real Business cycle literature often models a relatively high Frisch elasticity of two or more (Prescott, 1986; King, Plosser, and Rebelo, 1988), recent papers of Bayesian DSGE model estimation found far smaller values for the Frisch elasticity in a New Keynesian model framework. For example Justiniano and Primiceri (2008) argue for values between 0.25 and 0.5. These findings are in line with some micro-data based studies, which also argue for small values in a range between 0 and 0.7 (see Pistaferri, 2003, and references therein). Finally, we use a prior about the inverse of the Frisch elasticity, which is Gamma distributed with mean 1.0 and a standard deviation of 0.75. This assumption covers the values used in the different strands of the literature as well as the empirical findings.

In explaining business cycle facts and asset pricing facts simultaneously, we expect that the discount factor β as well as the power utility parameter η play an important role. The Business cycle literature often uses values for the discount factor slightly smaller than one to ensure a positive time preference of the representative agent and steady-state risk-free returns comparable to observed returns. However, from an asset pricing perspective discount factors with much smaller values or values greater than one are postulated. These opposing assumptions are known as the risk-free rate puzzle (see Weil, 1989). However, Kocherlakota (1990) has shown that values for the discount factor above unity can be in line with positive time preference if the economy is growing. For this reason we decided to use no prior for β, instead we use a prior for the risk-less return return to ensure positive time preference and

[4] More details regarding steady state calculation can be found in appendix C.2.

solve recursively for the discount factor:

$$\beta = exp(\eta\gamma - \sigma_m^2/2 - \log(\bar{R}^f)) \tag{41}$$

Therefore we assume that the steady state risk-free real quarterly return is Inverted-gamma distributed with mean 0.005 and standard deviation 0.01, that ensures that the mass of the prior on positive real annual returns smaller than 4%. To estimate the model in line with a high Sharpe ratio, we expect high values for η. For this reason, we assume that the power utility parameter is uniformly distributed between 1 and 200 which implies most of the prior mass on high values. This is in contrast to the common business cycle literature which generally assumes small values and therefore uses quite informative prior.

The prior for the remaining deep model parameters are chosen according to the recent literature. We intend not to introduce too much curvature or too much down-weighting of some parameter spaces into the likelihood function by using wide priors for non-observable parameters like habit formation, and more restrictive priors for well-documented or observable parameters like the depreciation rate. The standard deviations of all shocks are assumed to be Inverted-gamma distributed at a level of 0.01, with a degree of freedom equal to 4. We assume for every autoregressive parameter to be Beta distributed, with mean 0.85 and standard deviation 0.1. An overview of the priors is given in table 2.

Next to the steady state labor supply we also calibrate the growth rate of the economy γ and the capital share θ. The growth rate is set equally to the observed value of 0.0044 per quarter and the capital share is calibrated to 0.33 as common.

4.3 The 'Augmented' Prior

If we want to estimate the model accurate with respect to financial markets; we want that the model is able to replicate this number. For this reason we have to estimate the model under the nonlinear constraint that the models' implied Sharpe ratio, ω is equivalent to the observed one. To illustrate the challenge which goes along with this ideas we want to expose the strategy. First, to incorporate a linear or nonlinear constraints is to estimate the model just between bounds for possible Sharpe ratios. This approach could be illustrate as follows:

$$p(\theta|X) \propto \begin{cases} p(\theta)\, p(X|\theta) & \text{if} \quad \omega(\theta) \in Q \\ 0 & \text{if} \quad \omega(\theta) \notin Q \end{cases} \tag{42}$$

$$\text{where} \qquad Q \equiv \{\theta : \; a \leq \omega(\theta) \leq b\} \tag{43}$$

However, this approach just truncates the posterior around the the bounds a and b.

		Domain	Density	Para(1)	Para(2)
MODEL PARAMETER					
μ	wage rigidity	$[0,1)$	Beta	0.75	0.1
η	power utility parameter	$\mathbb{R}^+$	Uniform	1	200
χ	consumption habit	$[0,1)$	Beta	0.5	0.23
ψ	leisure habit	$[0,1)$	Beta	0.5	0.23
ρ_c	consumption habit	$[0,1)$	Beta	0.5	0.23
ρ_l	leisure habit	$[0,1)$	Beta	0.5	0.23
δ	depreciation rate	$[0,1)$	Beta	0.02	0.005
ζ	investment adjustment costs	$\mathbb{R}$	Normal	4.0	1.0
$1/\tau$	inverse Frisch elasticity (s.s.)	$\mathbb{R}^+$	Gamma	1.00	0.750
$\log \bar{R}^f$	risk-free return (s.s.)	$\mathbb{R}^+$	InvGam	0.005	4.0
Ω	leverage	$[0,1)$	Beta	0.5	0.23
AUTOREGRESSIVE PARAMETER AND S.D. OF SHOCKS					
π_G	AR government shock	$[0,1)$	Beta	0.85	0.1
π_W	AR wage mark-up shock	$[0,1)$	Beta	0.85	0.1
π_I	AR investment shock	$[0,1)$	Beta	0.85	0.1
π_L	AR labor supply shock	$[0,1)$	Beta	0.85	0.1
ϵ_P	s.d. technology shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_W	s.d. wage mark-up shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_I	s.d. investment shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_L	s.d. labor supply shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_G	s.d. government shock	$\mathbb{R}^+$	InvGam	0.01	4.0
ϵ_Q	s.d. excess return shock	$\mathbb{R}^+$	InvGam	0.01	4.0

Table 2: Prior distribution for model parameter and additional parameter. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distribution, while for the Uniform distribution these values correspond to the lower and upper bounds. The shortcut s.s. indicates steady state values.

Next to the subjective decision about the bounds this approach also does not take care about the observed distribution of the Sharpe ratio and would give the same weights to every Sharpe ratio within the bounds. Moreover, because we suggest small Sharpe ratios implied by a DSGE model we would also run into maximization problems at the lower bound. Consequently, we use the following alternative solution and looking for a posterior distribution which implies the observed Sharpe ratio just by directly sampling from it. Let p^* be this distribution:

$$\theta_j \sim p^*(\theta|X) = \int p(\theta|X)\, p_\omega(\omega) \quad d\theta \tag{44}$$

where $p_\omega(\omega)$ is a kind of prior distribution which ensures that the implied cumulative distribution (G_p) for the Sharpe ratio by sampling from $p^*(\theta|X)$ is the same as the observed cumulative distribution F. The 'augmented prior' distribution $p_\omega(\omega)$ depends on the conditional second moments and therefore on the model parameter. Moreover, the 'augmented prior' affects the marginal prior distribution of the deep model parameter θ. In this respect the present approach is similar to the 'endogenous prior' approaches as postulated by Del Negro and Schorfheide (2008) or Christiano et al. (2011). Both approaches elicit prior distributions for DSGE model parameters from beliefs about second moments of observable variables. In particular, we do not follow this literature because we cannot observe the pricing kernel whose second moments are essential for asset pricing.

To find the prior distribution $p_\omega(\omega)$ we assume that the implied cumulative distribution for the Sharpe ratio of the posterior distribution $p(\theta|X)$ is given by:

$$G(\omega) = \int_{\{\theta|\Omega\leq\omega\}} p(\theta|X)\, d\theta \tag{45}$$

Additionally, under the assumption that $p^*(\theta|X)$ delivers the correct distribution for the Sharpe ratio the following holds:

$$G(\omega) \stackrel{!}{=} F(\omega) = \int_{\{\theta|\Omega\leq\omega\}} p_\omega(\omega)\, p(\theta|X)\, d\theta \tag{46}$$

Given these cumulative distributions we can approximate the corresponding probability densities for small changes in the Sharpe ratio Δ as:

$$g(\omega) = \frac{\partial G(\omega)}{\partial \omega} \approx \frac{G(\omega+\Delta) - G(\omega)}{\Delta} \tag{47}$$

Using this approximation we receive the following term for the implied density g:

$$\frac{G(\omega+\Delta) - G(\omega)}{\Delta} = \frac{\int_{\{\theta|\omega\leq\Omega\leq\omega+\Delta\}} p(\theta|X)\, d\theta}{\Delta} \tag{48}$$

and for the observed density f:

$$\begin{aligned} \frac{F(\omega+\Delta) - F(\omega)}{\delta} &= \frac{\int_{\{\theta|\omega\leq\Omega\leq\omega+\Delta\}} p_\omega(\omega)\, p(\theta|X)\, d\theta}{\Delta} \\ &\approx \frac{p_\omega(\omega) \int_{\{\theta|\omega\leq\Omega\leq\omega+\Delta\}} p(\theta|X)\, d\theta}{\Delta} \end{aligned} \tag{49}$$

which both can be reduced to:

$$h(\omega) \approx \frac{f(\omega)}{g(\omega)} \tag{50}$$

Finally, the overall prior can be written as:

$$p(\theta|\omega) \propto p_\omega(\omega)\, p(\theta)\,, \text{ with} \tag{51}$$

$$p_\omega(\omega) = \frac{h(\omega)}{c}\,, \tag{52}$$

where c is a normalization constant. The normalization constant c is required for posterior odds calculations and can be approximated with a Laplace approximation or with the modified harmonic mean estimator from Geweke (1999). A few practical considerations are worth mentioning. The normalization constant is a function of the parameter vector θ, which leads often to the so-called problem of an intractable normalization constant (Del Negro and Schorfheide, 2008). For this reason, we calculate the constant c by fixing the parameter with the exception of η at their posterior mode. Because, we mainly interested in positive risk premia, where the risky asset gives an higher return than the risk-less asset, the theoretical Sharpe ratio has to be positive to. Of course, this is a limitation because we also observe negative Sharpe ratios in the data. Moreover, we assume that $f(\omega)$ and $g(\omega)$ follow a Gamma distribution with a standard deviation of $g(\omega)$ bigger than the standard deviation of $f(\omega)$ to ensure a proper prior.

The implementation of the described methodology can be summarized by the following algorithm:

1. Estimation of the model by sampling from $p(\theta|X)$
2. Approximate the implied probability density function $g(\omega)$
3. Calculating the pdf of $p_\omega(\omega|\theta)$
4. Estimation of the model with 'augmented' prior by sampling from $p^*(\theta|X)$

5 Estimation Results

The following results are based on the presented model estimated with and without additional Sharpe ratio constraint. We estimate the posterior mode of the distribution and employ a random walk Metropolis-Hastings algorithm to approximate the distribution around the posterior mode. We run two chains, each with 300,000 parameter vector draws. The first 75% have been discarded. Table 3 shows detailed posterior statistics, e.g. posterior mean

and the HPD interval of 10% and 90%. The results indicate that the posterior distributions of all structural parameters are well approximated around the posterior mode.

	Benchmark Prior			Augmented Prior		
Parameter	Posterior Mean	HPD 10%	HPD 90%	Posterior Mean	HPD 10%	HPD 90%
Model parameter						
μ	0.2685	0.1392	0.3948	0.2882	0.1644	0.4209
η	4.9508	2.1058	8.0901	108.18	84.19	134.30
χ	0.8689	0.8096	0.9258	0.8402	0.7777	0.9086
ψ	0.8555	0.7951	0.9167	0.8438	0.7871	0.9022
ρ_c	0.6090	0.4967	0.7237	0.6551	0.5523	0.7646
ρ_l	0.0696	0.0044	0.1344	0.0723	0.0043	0.1367
Ω	0.1188	0.0099	0.2276	0.1073	0.0059	0.2059
ζ	7.8726	6.7565	9.0130	7.8361	6.7952	8.9570
δ	0.0172	0.0136	0.0206	0.0175	0.0139	0.0210
$1/\tau$	5.5308	3.6103	7.4448	7.1055	5.1878	9.0453
$\log(\bar{R}^f)$	0.0047	0.0027	0.0065	0.0032	0.0017	0.0046
autoregressive parameter and s.d. of shocks						
π_G	0.9116	0.8798	0.9450	0.9253	0.8909	0.9611
π_I	0.7065	0.6432	0.7711	0.7003	0.6417	0.7580
π_W	0.6008	0.4787	0.7119	0.9303	0.8909	0.9719
π_L	0.9240	0.8812	0.9693	0.6516	0.5400	0.7670
σ_P	0.0091	0.0083	0.0098	0.0090	0.0083	0.0097
σ_I	0.0135	0.0107	0.0162	0.0132	0.0108	0.0156
σ_L	0.0031	0.0027	0.0034	0.0029	0.0026	0.0032
σ_W	0.0205	0.0149	0.0256	0.0211	0.0155	0.0265
σ_G	0.0197	0.0171	0.0224	0.0190	0.0169	0.0211
σ_Q	0.0765	0.0696	0.0831	0.0738	0.0677	0.0802
Log marginal density	3439.74			3437.25		

Table 3: MCMC Results

Comparing the results of the estimation with benchmark prior and augmented prior we find the biggest difference for the power utility parameter η. This result is expected, because introducing the augmented prior changes the marginal prior distribution with respect to η significantly to high values. Additionally, for this class of preferences, this parameter is directly linked to the agents' relative risk aversion regarding consumption which can be

calculated as:

$$RRA = \frac{\eta}{1-\chi} \ , \tag{53}$$

	Benchmark Prior			Augmented Prior		
Parameter	Posterior Mean	HPD 10%	HPD 90%	Posterior Mean	HPD 10%	HPD 90%
β	1.0162	1.0043	1.0292	1.0436	0.9538	1.1388
ν	5.8312	3.4399	8.1339	5.1088	2.9349	7.4408
α	0.5953	0.3922	0.7917	0.6071	0.3247	0.8797
$\bar{x}/\bar{y}$	0.3232	0.2944	0.3513	0.3129	0.2939	0.3328
$\bar{c}/\bar{y}$	0.3968	0.3687	0.4256	0.4071	0.3872	0.4261
RRA	39.20	15.26	63.95	719.25	419.23	1032.3

Table 4: Distributions of implicit model parameter and steady state values.

We have calculated the relative risk aversion for every draw from the posterior. The implied posterior statistics can also be found in Table 4. This classical calculation of the relative risk aversion regarding consumption is recently criticized by Swanson (2009). In particular, Swanson (2009) argues that this measure ignores the labor margin which can lead to a inaccurate measure of the household's true attitudes toward risk, especially in the case of habit formation. However, the classical measure nicely illustrates the common fact that high relative risk aversion is needed to explain stylized asset pricing facts. We also find that for both estimations we identify similar volatilities of the exogenous shocks. This means both economies face the same 'economy-wide' risk. This finding has two consequences. First, the underlying model is able to explain asset pricing facts without high global risk. Secondly, therefore the necessity of a high relative risk aversion is unavoidable to explain financial facts and holds also for different preferences as shown by Rudebusch and Swanson (2008) or Lettau and Uhlig (2002). The advantage of e.g. recursive preferences as introduced by Epstein and Zin (1989) that relative risk aversion can be modeled independently from the intertemporal elasticity of substitution. In the habit-based DSGE model presented in this paper the elasticity of intertemporal substitution is the inverse of the relative risk aversion. Therefore, the elasticity of intertemporal substitution can be calculated as 0.026 and 0.0013 for the estimation with benchmark prior and augmented prior respectively. Following the findings by Hall (1988) or Vissing-Jørgensen (2002) we have to conclude that these values are too small. However, most recent estimated DSGE models postulate high parameter values for external or internal habit and therefore imply also elasticities much smaller than the suggested values. As shown by Uhlig (2007) wage rigidities are a helpful ingredient to explain a high risk premium in habit-based DSGE models. Our estimation shows that the degree of

wage rigidity is small and similar for both estimation. From an modelling point of view we loose a helpful feature to explain financial facts and therefore, we need a higher value for η to explain those stylized facts. While we find no high degree of wage stickiness in the data we find a smaller Frisch elasticity τ for the estimation with augmented prior distribution. The Frisch elasticity of labor supply τ decreases from 0.18 for the benchmark estimation to 0.14 for the estimation with augmented prior distribution. Both values are in line with findings of the microeconomic literature (see Pistaferri, 2003) but indicate higher labor market frictions for the estimation with augmented prior. These empirical results support the theoretical findings by e.g. Uhlig (2007) that labor market frictions are helpful to explain financial facts and macroeconomic facts jointly. Also the wage markup shock is more persistent for the estimation with augmented prior. This result supports the recent results about the labor wedge, which find that it has high importance for explaining macroeconomic fluctuations (Shimer, 2009).

5.1 Implied asset pricing facts

The next paragraphs focuses on the implied asset prices of the estimated models with and without augmented prior. The Table 5 shows the implied distribution of the first and conditional second moments for both estimations. The estimation with augmented prior delivers asset price facts similar to those observed in the data, while the estimation without the augmented prior can just explain the observed first and conditional second moments of the risk-free rate. Especially, the Sharpe ratio and the risk premium illustrate the well known problematic of standard DSGE models to explain those.

		Benchmark Prior			Augmented Prior		
			Posterior			Posterior	
		Mean	10%	90%	Mean	10%	90%
s.d. risk-free return	σ_{R^f}	0.39	0.35	0.42	0.38	0.35	0.42
s.d. return on capital	σ_{R^k}	1.19	0.97	1.41	1.18	0.98	1.37
s.d. excess returns	σ_{R^q}	7.86	7.20	8.54	7.59	6.98	8.22
s.d. pricing kernel	σ_M	4.16	1.90	6.82	92.15	72.13	110.92
Risk premium (R^k/R^f)	$-\sigma_{mR^k}$	0.024	0.015	0.034	0.240	0.200	0.280
eff. mean excess return (R^q)	$-\sigma_{mR^q}$	0.036	0.021	0.050	0.370	0.290	0.440
Sharpe ratio (R^k)	ω_k	0.0203	0.0140	0.0270	0.205	0.203	0.206
Sharpe ratio (R^q)	ω_q	0.0045	0.0027	0.0064	0.049	0.038	0.058

Table 5: Implied quarterly asset pricing facts by the estimated models. All values in percent with the exception of the Sharpe ratio.

The conditional second moments of the risk free rate similar for both estimations and comparable to the data, while the mean of risk free return for the estimation with augmented prior is a slightly smaller but still inline with the observations. Additionally, also the first and conditional second moments of the return of capital for both estimations are comparable with each other. As mentioned above we estimate a high relative risk aversion to match the observed Sharpe ratio. This observation is also illustrated by the higher precautionary motive of the households and the higher risk premium. Both measures are known to be directly linked to agents' attitude to risk as postulated by Kimball (1990) and Swanson (2009) respectively. This explains the huge conditional volatility of the stochastic pricing kernel σ_m. Interestingly, because this parameter is also the maximum Sharpe ratio implied by the model, it is approximately five times as high as the Sharpe ratio for the return on capital. Consequently, this high maximum Sharpe ratio implied by our model is able to explain higher Sharpe ratios for different asset classes as observed in the literature. In particular, Scholl and Uhlig (2008) and Piazzesi and Schneider (2009) find high Sharpe ratios for exchange rates and house prices, respectively. While the value seems high with respect to the relative risk aversion of the agent, the value seems reasonable with respect to asset pricing facts. The proposed estimation procedure generates a ten times higher risk premium in economy than the model estimated without the augmented prior. As mentioned in the section 4.1 we have assumed that both risky assets in the economy are priced along the marketline wiich implies the same Sharpe ratio. The mean of the excess returns as proxy for the observed excess returns was calculated as $\omega_k \cdot \sigma_{R^q}$. Given the results in Table 5 it is easy to figure out, that the estimation with benchmark priors fail to match the observed risk premium of 1.49% while the estimation with augmented prior delivers a similar value of 1.59% at the posterior mean. The effective mean of the excess returns is smaller for both estimation. Hence, just the introduction of the mean of the excess returns as additional observation into the likelihood is not enough to match asset prices.

5.2 Implied business cycle facts

In the following subsection we want to investigate the empirical performance of our estimated model with respect to Business Cycle statistic in more detail. To do so, we compare the predicted unconditional second moments of the DSGE models with those of a BVAR with two lags. We estimate the BVAR with the same set of observable variables as used for the DSGE estimation with the exception of the excess returns. Moreover, we assume a diffuse independent Normal-Whishart prior for the covariance matrix and the coefficient matrices of the BVAR. Afterwards, we draw 1200 parameter vectors from the posterior of

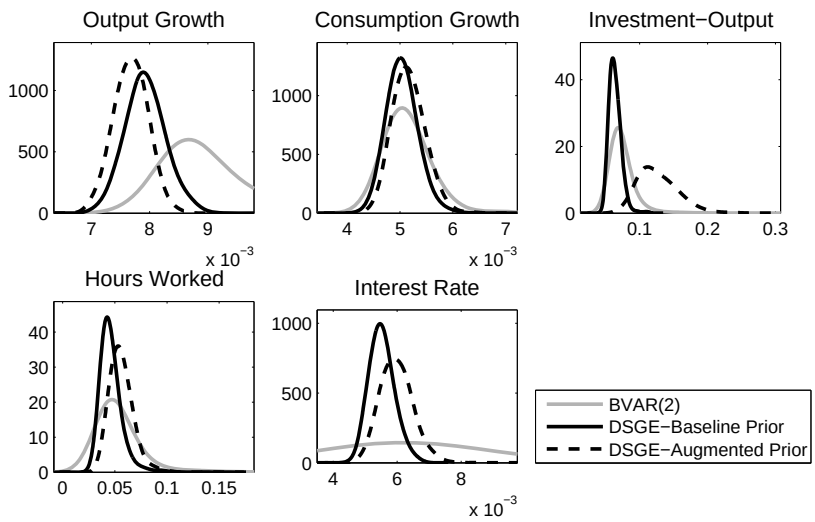

Figure 1: Implied standard deviations of the DSGEs and the BVAR(2) based on 1200 draws from the corresponding posterior.

the BVAR(2) as well as 1200 parameter vectors from the posterior distributions of both estimated DSGE models and calculate the unconditional second moments. Figure 1 shows the implied distributions for the standard deviations of the observable variables.

Both estimated DSGE models predict the similar standard deviations for output growth which are slightly smaller than those predicted by the BVAR(2). Also, the predicted standard deviation for consumption growth, hours worked, and the real risk-free interest rate is similar to each other for both estimation approaches. As common, both DSGEs overpredict the standard deviation of the real risk-free interest rate compared to the BVAR, while they match the standard deviation of the real quantities well (Christiano et al., 2011). The biggest difference can be found for the implied standard deviation of the investment-output ratio of the DSGE models. While the model estimated with the benchmark prior predicts values close to those of the BVAR, the model estimated with augmented prior predicts a bigger standard deviation. In this respect, the model estimated with augmented prior lose explanatory power with respect to business cycle facts. Additionally, this model also predicts a higher autocorrelation of the investment-output ratio in comparison to the BVAR and the benchmark DSGE. However, both models are quite successful in predicting the second moments of the observable variables compared to the BVAR, but the model estimated with benchmark priors slightly outperforms the model estimated with the augmented prior.

We think, next to the preference characteristics of the underlying model, the inclusion of agents attitude to risk is helpful in this respect. In particular, taking agents' precautionary motive into account allows to introduce a high relative risk aversion but also to linearize the

model around a steady-state which indicates a similar rich economy. This circumstance is illustrated by the implied distributions for the great ratios, namely the consumption-output ratio and investment-output ratio in Table 4. These ratios are similar for the estimation with and without augmented prior or with and without high relative risk aversion. More precisely, an higher interest rate due to a higher relative risk aversion would make the economy ceteris paribus poorer, but accounting for the agents' precautionary motive offsets this effect.

Therefore, in our model high relative risk aversion generates similar dynamics of the macroeconomic variables as in the case of low relative risk aversion. This characteristic is similar to the findings of Tallarini (2000) or Rudebusch and Swanson (2012) for models with recursive preferences.

5.3 Model comparison

In the following, it is our intention to compare the models, estimated with augmented prior and estimated with the benchmark prior. We use the Modified Harmonic Mean estimator by Geweke (1999) to calculate the marginal data density of each model i . The marginal data density for model i is usually:

$$p(X|M_i) = \int_{\Theta} p\left(X|\theta^M, M_i\right) p\left(\theta^M|M_i\right) d\theta^M \ , \tag{54}$$

where $p\left(X|\theta^M, M_i\right)$ is the posterior, $p\left(\theta^M|M_i\right)$ the prior, and Θ the parameter space of model i. We denote the benchmark model as M_1 and the model estimated with augmented prior as M_2. For model M_2 the augmented prior is a joint distribution

$$p\left(\theta^M|M_2\right) = p\left(\theta\right) p_{\omega}\left(\omega\right) \tag{55}$$

where $p_{\omega}\left(\omega\right)$ is the prior with respect to the Sharpe ratio as presented in section 4.3 and $p\left(\theta\right)$ is the prior with respect to the deep model parameters. The latter prior is equivalent to the prior of model $M1$, $p\left(\theta^M|M_1\right)$.

The difference in the marginal data density leads to posterior probabilities of 0.92 vs. 0.08 and a poster odds ratio of 12.1 in favor of the model estimated with benchmark prior. This result is not surprising because the model with augmented prior is the more restricted one. Moreover, the result is in line with the analysis in the former subsection. However, the investigation of the marginal data densities focuses on the explanatory power with respect to the observable variables which are dominated by business cycle variables. In this respect, the result just confirms our finding from the former subsection and our econometric intuition.

6 A more simple RBC model

In the following section we apply our method of an 'augmented prior' to a more simple version of the model in this paper. First, we assume that households are now characterized by a representative agent with preferences characterized by the following utility function which is separable in consumption and leisure:

$$U = E\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\eta}}{1-\eta} - \Psi_l \frac{z_{L,t} n_t^{1+1/\tau}}{1+\frac{1}{\tau}}\right], \tag{56}$$

where η is the relative risk averseness and τ is the Frisch elasticity with respect to labor supply. The variable $z_{L,t}$ is the labor supply shock and Ψ_l the scaling parameter for disutility of labor. In comparison to the former model we neglect any habit formation in consumption and leisure. Next to these nominal rigidities we also abstract from real wage rigidities by assuming that the real wage is equal to the marginal rate of substitution. Moreover, we neglect growth and solve the model around its deterministic steady state. Finally, we assume that the exogenous technology process can be expressed in log-linearized terms by a stochastic AR(1)process,

$$\hat{z}_{P,t} = \pi_P \hat{z}_{P,t-1} + \epsilon_{P,t} \tag{57}$$

where $\epsilon_{P,t}$ is a normally i.i.d. with standard deviation σ_P.

These assumptions have some consequences for the estimation. First, we cannot identify as many shocks as before. For this reason we reduce the numbers of observables to $X_t = [\Delta\hat{y}_t,\ \Delta\hat{c}_t,\ \hat{n}_t,\ \hat{x}_t/\hat{y}_t]$. We also reduce the number of estimated parameters. We calibrate the depreciation rate to $\delta = 0.025$ and the steady state risk-free rate to 1.0042 which is the mean of the corresponding times series used in the former part of this paper and this implies a discount factor $\beta = 0.9958$. We also changed the prior distribution for the inverse of the Frisch elasticity, which follows now a Gamma distribution with mean 5 and standard deviation of 2. This is necessary because with the tighter prior from the estimation before we get maximization problems. For the same reason, we loosen our additional Sharpe ratio constraint for the estimation with augmented prior. In particular, we assume that the observed Sharpe ratio is centered around 0.102 instead of 0.204 as in the data. The prior distributions of the remaining parameters are the same.

The following results are based on the estimation with and without additional Sharpe ratio constraint. We estimate the posterior mode of the distribution and employ a random walk Metropolis-Hastings algorithm to approximate the distribution around the posterior mode. We run two chains, each with 300,000 parameter vector draws. The first 75% have

	Benchmark Prior			Augmented Prior		
Parameter	Posterior Mean	HPD 10%	HPD 90%	Posterior Mean	HPD 10%	HPD 90%
Model parameter						
η	7.6114	4.3446	10.8825	43.5015	29.3441	57.4003
ζ	6.3621	5.0788	7.5694	5.8340	4.5777	7.0856
$1/\tau$	2.9802	1.7344	4.1938	12.1938	10.3318	14.1113
autoregressive parameter and s.d. of shocks						
π_G	0.9072	0.8846	0.9288	0.9270	0.9054	0.9489
π_I	0.9444	0.9219	0.9671	0.7978	0.7549	0.8418
π_P	0.9464	0.9287	0.9648	0.9931	0.9910	0.9953
π_L	0.9933	0.9882	0.9985	0.9964	0.9944	0.9984
σ_P	0.0098	0.0089	0.0106	0.0110	0.0100	0.0120
σ_I	0.0407	0.0270	0.0537	0.1786	0.1609	0.1957
σ_L	0.0532	0.0338	0.0707	0.2726	0.2065	0.3371
σ_G	0.0139	0.0127	0.0151	0.0137	0.0125	0.0149
Log marginal density	2443.91			2393.29		

Table 6: MCMC Results for the simple RBC model

been discarded. The table 6 show detailed posterior statistics, e.g. posterior mean and the HPD interval of 10% and 90%.

If we compare the estimation results for the deep model parameters we find similarities with the estimation of the richer model before. The power utility parameter, η, increases which is directly linked to a higher relative risk aversion. While this is an expected result we find also, that the Frisch elasticity of labor supply is strongly decreasing, from 0.31 to 0.08. In particular, we can observe the same mechanisms, higher relative risk aversion and higher labor market frictions, as for the estimation before. A more detailed investigation of the standard deviations of the shocks shows a main difference to the former model. The more simple RBC model needs higher volatility, higher 'economy-wide' risk, to explain the looser Sharpe ratio constraint. Because of the separability of the utility, the maximum Sharpe ratio is just a function of η and the conditional standard deviation of consumption, which also depends on the model parameters, $\omega^{max} = \eta\sigma_c$. Hence, an increase of the relative risk averseness makes consumption smoother, so we have an opposite effect on the maximum Sharpe ratio. Finally, a higher 'economy-wide' risk is needed to explain the asset pricing facts.

		Benchmark Prior			Augmented Prior		
			Posterior			Posterior	
		Mean	10%	90%	Mean	10%	90%
s.d. risk-free return	σ_{R^f}	0.27	0.17	0.37	3.26	2.90	3.63
s.d. return on capital	σ_{R^k}	3.05	2.00	4.09	13.83	12.58	14.95
s.d. pricing kernel	σ_M	3.87	2.33	5.39	22.25	16.43	27.93
Risk premium	$-\sigma_{mR^k}$	0.063	0.029	0.097	1.41	1.28	1.53
Sharpe ratio (R^k)	ω	0.020	0.0149	0.0253	0.102	0.100	0.104

Table 7: Implied quarterly asset pricing facts by the estimated models. All values in percent with the exception of the Sharpe ratio.

Table 7 shows the implied asset pricing facts of the model for both estimation approaches. We are successful in matching our Sharpe ratio constraint. Surprisingly, half the size of the observed Sharpe ratio is enough to approximately match the observed risk premium. However, the model estimated with augmented prior fails in matching the low volatility of the risk-free rate. In comparison to the former model, the present model estimation delivers a high volatility of the return on installed capital which is higher than the volatility of equity observed in the data. As highlighted by Gomme, Ravikumar, and Rupert (2011), the opposite would be in line with the data, the volatility of the return to installed capital is an order of magnitude smaller than that of the stock market return.

Finally, the posterior odds ratio is $9.64e^{21}$ to one in favor of the model estimated with the benchmark prior, compared to 12.1 for the former model. Of course, these number do also not account for the asset pricing facts but they illustrate impressively the poor performance of the model in line with asset pricing facts with respect to Business cycle facts.

7 Conclusion

In the present paper we estimate a DSGE model with asset prices. We discus the related challenges and propose solutions. In particular, we develop an 'augmented' prior distribution, which takes into account stylized asset pricing facts. In particular, our approach extends the recent literature of endogenously formed prior where a priori information are not only taken from marco- or microstudies. The applications in the paper show that our approach ensures that the DSGE model is estimated along a path of the parameter space which is also inline with the additional information which is not included in the observations per se. To that end, our estimated model is able to replicate the observed Sharpe ratio with the model's implied Sharpe ratio without ruin business cycle facts.

We show that a simultaneous explanation of asset pricing facts and business cycle facts is possible, but needs more than a econometric approach. However, we explicitly ignore recent attempts to overcome common problems of DSGE models with asset prices. In particular, we use power utility with habit formation and assume log-normality of shocks rather than recursive preferences, long-run risk, or rare disaster risk, as recently proposed helpful ways to overcome well studied problems. Our analysis shows that latter features are not necessarily needed. More precisely, we identify labor market frictions, non-separability between consumption and leisure, as well as the agents' attitude to risk as important features. Of course, the the estimates predicts high relative risk aversion which is in contrast to many existing literature. But, in our model high relative risk aversion generates similar dynamics of the macroeconomic variables as in the case of low relative risk aversion. This characteristic is similar to the findings of Tallarini (2000) or Rudebusch and Swanson (2012) for models with recursive preferences. Therefore our findings are in contrast to the findings of Rudebusch and Swanson (2012, 2008) who emphasize that habit-based DSGE models fail in explaining financial facts and macroeconomic facts jointly.

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A Data

Within this paper we use several macro and financial time series. This appendix describes some modifications and especially the source of the raw data.

Real GDP: This series is *BEA NIPA table 1.1.6 line 1 (A191RX1)*.

Nominal GDP: This series is *BEA NIPA table 1.1.5 line 1 (A191RC1)*.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of **Nominal GDP** to **Real GDP**.

Private Consumption: Real consumption expenditures for non-durables and services is the sum of the respective nominal values of the *BEA NIPA table 1.1.5 line 5 (DNDGRC1)* and *BEA NIPA table 1.1.5 line 6 (DNDGRC1)* and finally deflated by the deflator mentioned above.

Private Investment: Total real private investment is the sum of the respective nominal values of the series Gross Private Investment *BEA NIPA table 1.1.5 line 7 (A006RC1)* and Personal Consumption Expenditures: Durable Goods *BEA NIPA table 1.1.5 line 4 (DDURRC1)* and finally deflated by the deflator mentioned above.

Hours worked: The series measures the hours worked of employees working in private non-farm business excluding non-profit business. This series is an updated version of the one used by Francis and Ramey (2009) and is available on the authors' website. Source: `http://weber.ucsd.edu/~vramey/`

Civilian Population: This series is calculated from monthly data of civilian noninstitutional population over 16 years (CNP16OV) from the U.S. Department of Labor: Bureau of Labor Statistics.

S&P 500: The total returns of the S&P 500 are calculated by the monthly values of for the S&P price index and dividends calculated by Robert J. Shiller and provided on his website. Source: `http://www.econ.yale.edu/~shiller/data.htm`

Risk-free Rate: The quarterly risk-free return is calculated from monthly returns of the 3-Month Treasury Bill: Secondary Market Rate provided by Board of Governors of the Federal Reserve System. The finally used real returns are calculated with the implicit inflation rate of the price deflator series above.

B Tables and Figures (Not for publication)

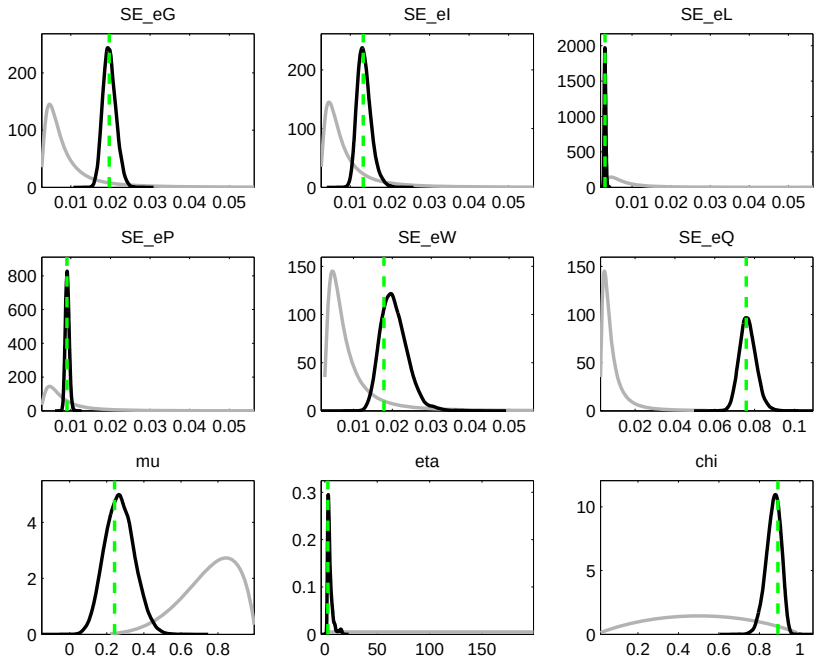

Figure 2: Prior and posterior distribution of the model with baseline prior. Vertical dashed line indicates the posterior mode.

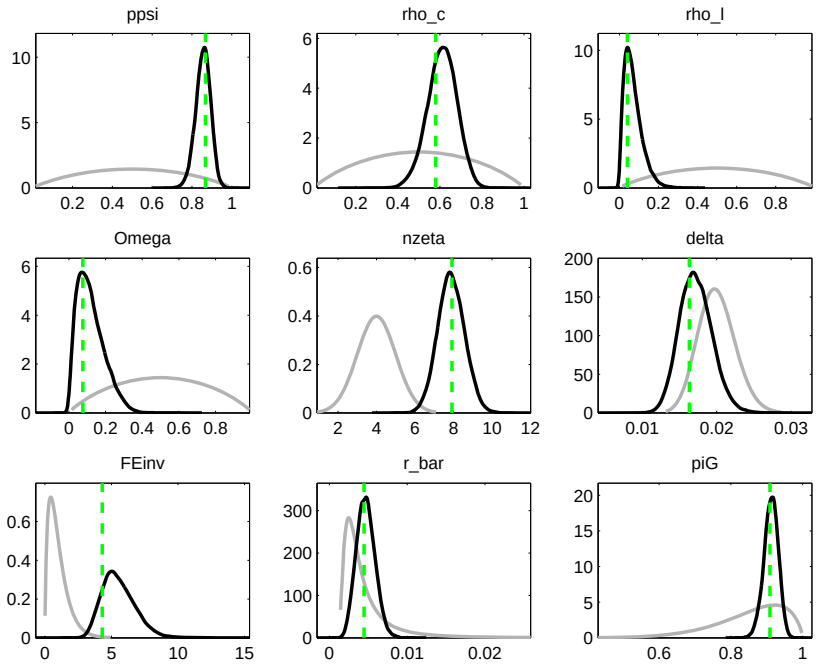

Figure 3: Prior and posterior distribution of the model with baseline prior. Vertical dashed line indicates the posterior mode.

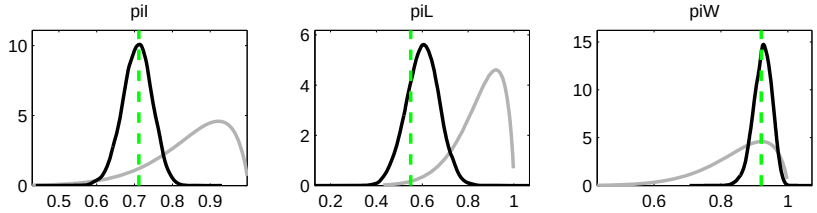

Figure 4: Prior and posterior distribution of the model with baseline prior. Vertical dashed line indicates the posterior mode.

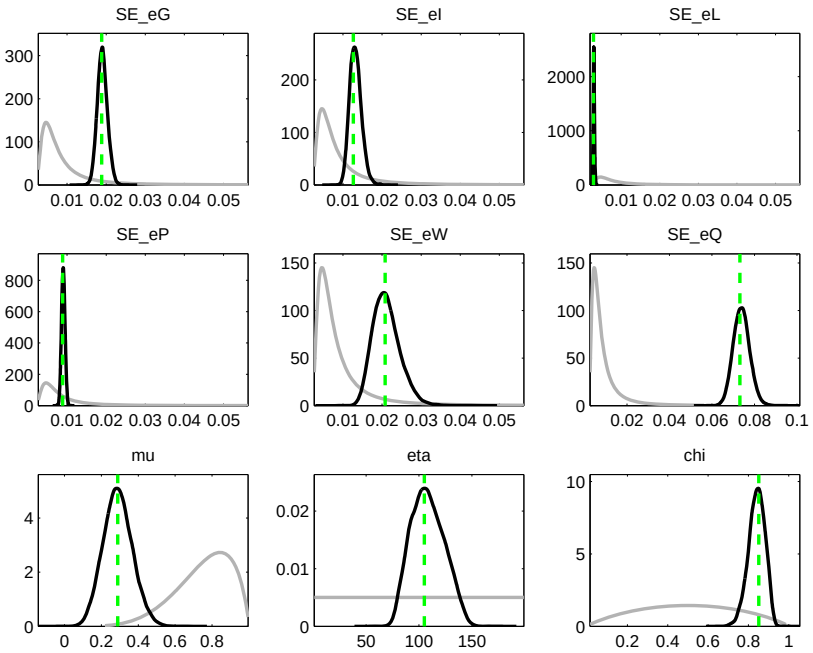

Figure 5: Prior and posterior distribution of the model with augmented prior. Vertical dashed line indicates the posterior mode.

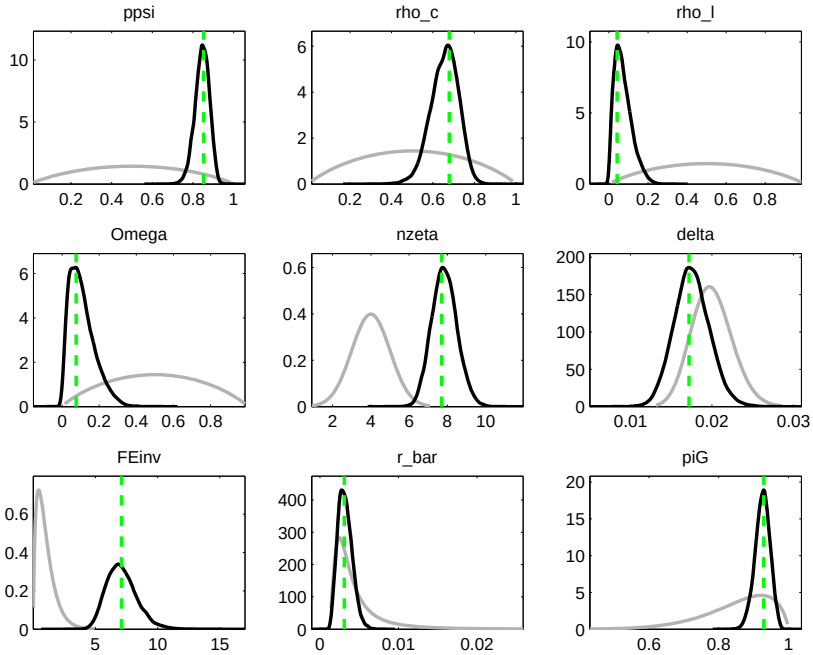

Figure 6: Prior and posterior distribution of the model with augmented prior. Vertical dashed line indicates the posterior mode.

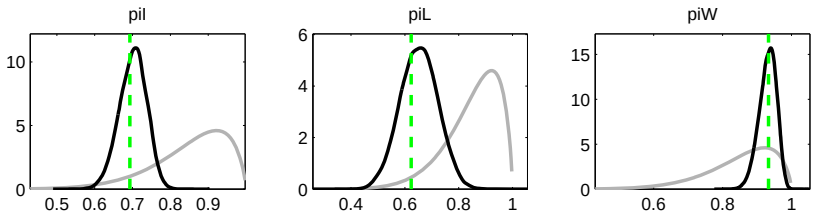

Figure 7: Prior and posterior distribution of the model with augmented prior. Vertical dashed line indicates the posterior mode.

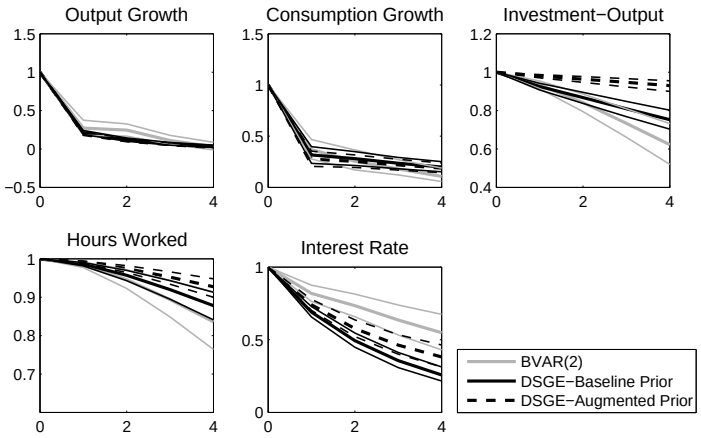

Figure 8: Implied autocorrelation of observable variables of the DSGEs and the BVAR(2).

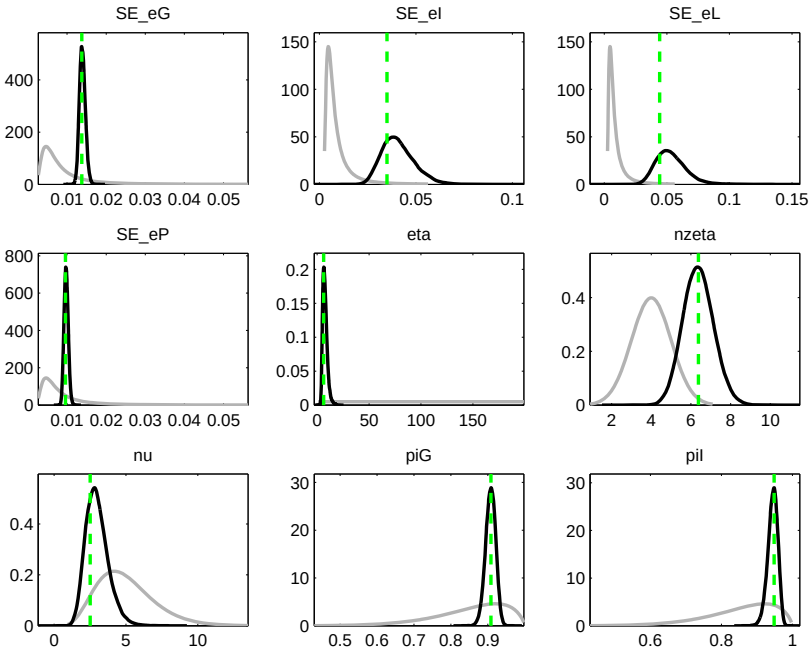

Figure 9: Prior and posterior distribution f the more simple rbc model with baseline prior. Vertical dashed line indicates the posterior mode.

		Benchmark Prior	Augmented Prior	Data
Standard Deviation of Output				
Output	$\hat{\hat{y}}$	0.0129	0.0126	0.0148
		[0.0124;0.0136]	[0.0121;0.0132]	
Relative Standard Deviation to Output				
Consumption	$\hat{\hat{c}}$	1.0415	1.0435	0.5516
		[0.9972;1.0858]	[0.9936;1.1033]	
Investment	$\hat{\hat{x}}$	2.9560	3.1079	3.6632
		[2.8401;3.0858]	[3.0078;3.2211]	
Hours worked	$\hat{\hat{n}}$	1.1570	1.1332	1.2372
		[1.1169;1.1948]	[1.0917;1.1703]	
Correlation with Output				
Consumption	$\hat{\hat{c}}$	0.6387	0.6007	0.8210
		[0.5912;0.6750]	[0.5526;0.6408]	
Investment	$\hat{\hat{x}}$	0.7503	0.7520	0.9226
		[0.7243;0.7725]	[0.7290;0.7768]	
Hours worked	$\hat{\hat{n}}$	0.7829	0.7769	0.8676
		[0.7585;0.8063]	[0.7548;0.7976]	

Table 8: HP-filtered ($\lambda = 1600$) theoretical and empirical moments. The theoretical moments are based on 1200 draws from the posterior. The numbers in brackets indicate 10% and 90% probabilities.

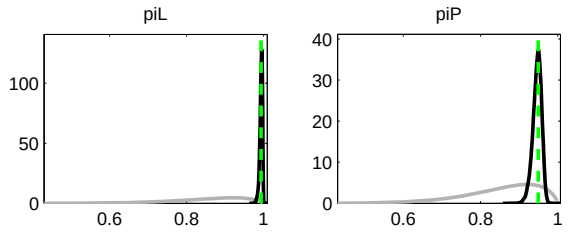

Figure 10: Prior and posterior distribution of the more simple rbc model with baseline prior. Vertical dashed line indicates the posterior mode.

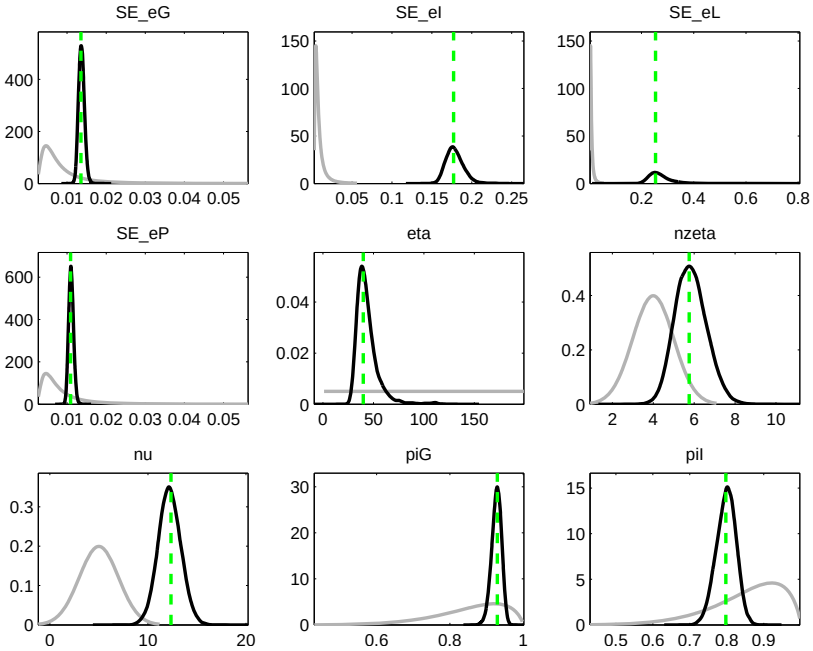

Figure 11: Prior and posterior distribution of the more simple rbc model with augmented prior. Vertical dashed line indicates the posterior mode.

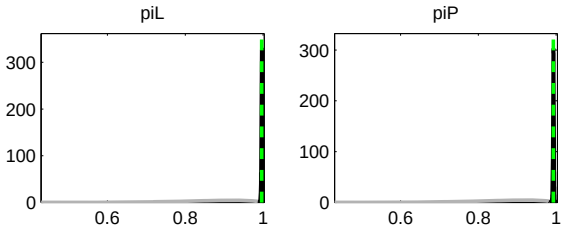

Figure 12: Prior and posterior distribution of the more simple rbc model with augmented prior. Vertical dashed line indicates the posterior mode.

C Model solution (Not for publication)

C.1 FONC

The economy described in the paper follows the trend γ. To write the equilibrium conditions in stationary terms, the set of variables has to be detrended by z_{t-1} as follows:

$$\tilde{c}_t = \frac{c_t}{e^{z_{P,t-1}}}, \qquad \tilde{y}_t = \frac{y_t}{e^{z_{P,t-1}}}, \qquad \tilde{w}_t = \frac{w_t}{e^{z_{P,t-1}}}, \qquad \tilde{w}_t^f = \frac{w_t^f}{e^{z_{P,t-1}}} \tag{C-1}$$

$$\tilde{k}_{t-1} = \frac{k_{t-1}}{e^{z_{P,t-1}}}, \qquad \tilde{x}_t = \frac{x_t}{e^{z_{P,t-1}}}, \qquad \tilde{H}_t = \frac{H_t}{e^{z_{P,t-1}}}, \qquad \widetilde{\lambda}_t = \frac{\lambda_t}{e^{-\eta z_{P,t-1}}}$$

Following, the set of the stationary first order necessary conditions of the equilibrium can be rewritten as:

$$n_t = 1 - l_t \tag{C-2}$$

$$R_t^k q_{t-1} = \frac{\theta \tilde{y}_t}{\tilde{k}_{t-1}} + \left(1 - \delta + g\left(z_{I,t}\frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right) q_t - \frac{\tilde{x}_t}{\tilde{k}_{t-1}} \tag{C-3}$$

$$q_t z_{I,t} = \frac{1}{g'\left(z_{I,t}\frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)} \tag{C-4}$$

$$\frac{1}{R_t^f} = \exp\left(\log \bar{M} + E_t\left[\hat{m}_{t+1}\right] + \frac{\sigma_m^2}{2}\right) \tag{C-5}$$

$$\frac{1}{E_t\left[R_{t+1}^k\right]} = \exp\left(\log \bar{M} + E_t\left[\hat{m}_{t+1}\right] + \frac{\sigma_m^2}{2} + \sigma_{mR^k}\right) \tag{C-6}$$

$$M_t = \beta \frac{\tilde{\lambda}_t}{\tilde{\lambda}_{t-1}} \exp\left(-\eta\left(\gamma + \epsilon_{P,t-1}\right)\right) \tag{C-7}$$

$$\tilde{\lambda}_t = \left(\tilde{c}_t - \tilde{H}_t\right)^{-\eta} \left(A + \left(z_{L,t} l_t - F_t\right)^{\nu}\right)^{1-\eta} \tag{C-8}$$

$$\exp\left(\epsilon_{P,t-1}\right) \tilde{H}_t = \left(1 - \rho_c\right) \chi \tilde{c}_{t-1} + \rho_c \tilde{H}_{t-1} \tag{C-9}$$

$$F_t = \left(1 - \rho_l\right) \psi l_{t-1} + \rho_l F_{t-1} \tag{C-10}$$

$$\tilde{w}_t^f = \frac{z_{L,t} \upsilon\left(\tilde{c}_t - \tilde{H}_t\right)}{A\left(z_{L,t} l_t - F_t\right)^{1-\nu} + z_{L,t} l_t - F_t} \tag{C-11}$$

$$\tilde{w}_t = \frac{\left(1 - \theta\right) \tilde{y}_t}{n_t} \tag{C-12}$$

$$\exp\left(\mu \epsilon_{P,t-1}\right) \tilde{w}_t = \left(\tilde{w}_{t-1}\right)^{\mu} \left(\exp\left(\hat{\varepsilon}_{W,t}\right) \tilde{w}_t^f\right)^{1-\mu} \tag{C-13}$$

$$\tilde{y}_t = \left(\tilde{k}_{t-1}\right)^{\theta} \left(\exp\left(\gamma + \epsilon_{P,t}\right) n_t\right)^{1-\theta} \tag{C-14}$$

$$\exp\left(\gamma + \epsilon_{P,t}\right)\tilde{k}_t = \left(1 - \delta + g\left(z_{I,t}\frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right)\tilde{k}_{t-1} \tag{C-15}$$

$$\tilde{c}_t + \tilde{x}_t + \tilde{g}_t = \tilde{y}_t \tag{C-16}$$

The equilibrium is defined together with the exogenous variables $z_{L,t}$, $z_{I,t}$, $\exp\left(\hat{\varepsilon}_{W,t}\right)$ and $\tilde{g}_t$.

C.2 Steady-state

To calculate the steady state we take the following as given:

$$\bar{z}_L = \bar{z}_I = 1 \quad \text{and} \quad \bar{q} = 1\ , \tag{C-17}$$

as well as that the steady-state ratio of government expenditures to output is 28%:

$$\frac{\bar{\tilde{g}}}{\bar{\tilde{y}}} = 0.28 \tag{C-18}$$

Furthermore, we can calculate the real depreciation rate:

$$\tilde{\delta} = e^{\gamma} + \delta - 1$$

Remembering the previous discussion about the asset pricing implications, we know that the Euler equation has to hold for any asset. This implies that (eq. C-5) is equal to (eq. C-6). Given a value for $\bar{R}^f$ and σ_m^2 we can solve for steady state pricing kernel:

$$\bar{M} = \exp\left(-\log\left(\bar{R}^f\right) - \frac{\sigma_m^2}{2}\right) \tag{C-19}$$

The return on capital is equal to:

$$\bar{R}^k = \frac{1}{\bar{M}\exp\left(\frac{\sigma_m^2}{2} + \sigma_{R^k m}\right)}\ . \tag{C-20}$$

Now, we can also solve for the discount rate:

$$\beta = \bar{m}\exp\left(\eta\gamma\right) \tag{C-21}$$

Now, we can also solve for:

$$\frac{\bar{\tilde{x}}}{\bar{\tilde{y}}} = \frac{\theta\tilde{\delta}}{\bar{R}^k + \delta - 1} \tag{C-22}$$

and

$$\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} = \frac{\bar{R}^k + \delta - 1}{\theta} \tag{C-23}$$

and because $\bar{\tilde{x}}/\bar{\tilde{k}} = \tilde{\delta}$ for:

$$\frac{\bar{\tilde{c}}}{\bar{\tilde{k}}} = \frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} - \frac{\bar{\tilde{x}}}{\bar{\tilde{k}}} - \frac{\bar{\tilde{g}}}{\bar{\tilde{y}}} \cdot \frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}. \tag{C-24}$$

Given the assumption that steady state leisure is twice as high as labor, $\bar{l} = 2/3$ and

$$\bar{n} = 1 - \bar{l}, \tag{C-25}$$

we can solve for the steady state capital:

$$\bar{\tilde{k}} = \left[\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}\right]^{\frac{1}{\theta-1}} \bar{n} e^{\gamma} \ , \tag{C-26}$$

this allows now to solve for steady-state value $\bar{\tilde{y}}$, $\bar{\tilde{x}}$, $\bar{\tilde{g}}$, and $\bar{\tilde{c}}$.

As shown in section 4.2 we use the condition of the Frisch elasticity (τ) to resolve for the remaining steady states and parameters. In the case of wage rigidities, the following steady-state relationship between the market wage and the frictionless wage (marginal rate of substitution) holds:

$$\bar{\tilde{w}} = \bar{\tilde{w}}^{f} e^{\bar{\varepsilon}_W} \ , \tag{C-27}$$

where the market wage is determined by the condition:

$$\bar{\tilde{w}} = (1-\theta)\,\frac{\bar{\tilde{y}}}{\bar{n}} \tag{C-28}$$

Now we define the parameter κ as:

$$\kappa = \frac{\exp \bar{\varepsilon}_W}{1-\theta} \frac{1-\bar{l}}{\bar{l}} \frac{\bar{c}}{\bar{y}} \tag{C-29}$$

Given the Frisch elasticity τ the following has to hold:

$$\Upsilon = \frac{\bar{l}}{1-\bar{l}} \frac{1}{\tau} - \left(2 - \frac{1}{\eta}\right) \frac{1}{(1-\chi)\,\kappa} \tag{C-30}$$

Afterwards, we can resolve for the remaining parameters by solving the equation:

$$\nu = 1 - (1-\psi)\,\Upsilon \tag{C-31}$$

$$\alpha = \frac{\kappa\nu\,(1-\chi)}{1-\psi} - 1 \tag{C-32}$$

$$A = \alpha\,(1-\psi)^{\nu}\,\bar{l}^{\nu} \ . \tag{C-33}$$

Given these remaining parameters we can solve for the steady state values of the remaining variables.

C.3 Log-linearization

$$\hat{l}_t = -\frac{\bar{n}}{1-\bar{n}}\hat{n}_t \tag{C-34}$$

$$\hat{r}_t^k + \hat{q}_{t-1} = \left[\frac{\bar{R}^k - 1 + \delta}{\bar{R}^k}\right]\left(\hat{\tilde{y}}_t - \hat{\tilde{k}}_{t-1}\right) + \frac{e^\gamma}{\bar{R}^k}\hat{q}_t + \frac{\tilde{\delta}}{\bar{R}^k}\hat{z}_{I,t} \tag{C-35}$$

$$\hat{q}_t = \frac{1}{\zeta}\hat{\tilde{x}}_t + \left(\frac{1}{\zeta} - 1\right)\hat{z}_{I,t} - \frac{1}{\zeta}\hat{\tilde{k}}_{t-1} \tag{C-36}$$

$$\hat{\tilde{w}}_t = \hat{\tilde{y}}_t - \hat{n}_t \tag{C-37}$$

$$\hat{\tilde{w}}_t^f = \hat{z}_{L,t} + \hat{c}_t^d + \left[\frac{\nu\alpha}{1+\alpha} - 1\right]\hat{l}_t^d \tag{C-38}$$

$$\hat{\tilde{\lambda}}_t = -\eta\hat{c}_t^d + \left[\frac{\nu\left(1-\eta\right)}{1+\alpha}\right]\hat{l}_t^d \tag{C-39}$$

$$\hat{\tilde{H}}_t + \epsilon_{P,t-1} = \left(1-\rho_c\right)\hat{\tilde{c}}_{t-1} - \rho_c\hat{\tilde{H}}_{t-1} \tag{C-40}$$

$$\hat{F}_t = \left(1-\rho_l\right)\hat{l}_{t-1} - \rho_l\hat{F}_{t-1} \tag{C-41}$$

$$\left(1-\chi\right)\hat{c}_t^d = \hat{\tilde{c}}_t - \chi\hat{\tilde{H}}_t \tag{C-42}$$

$$\left(1-\psi\right)\hat{l}_t^d = \hat{z}_{L,t} + \hat{l}_t - \psi\hat{F}_t \tag{C-43}$$

$$0 = E_t\left[\hat{r}_{t+1}^k + \hat{m}_{t+1}\right] \tag{C-44}$$

$$0 = E_t\left[\hat{m}_{t+1}\right] + \hat{r}_t^f \tag{C-45}$$

$$\hat{m}_t = \hat{\tilde{\lambda}}_t + \hat{\tilde{\lambda}}_{t-1} - \eta\epsilon_{P,t-1} \tag{C-46}$$

$$\hat{\tilde{y}}_t = \theta\hat{\tilde{k}}_{t-1} + \left(1-\theta\right)\hat{n}_t + \left(1-\theta\right)\epsilon_{P,t} \tag{C-47}$$

$$\hat{\tilde{w}}_t = \mu\hat{\tilde{w}}_{t-1} + \left(1-\mu\right)\hat{\tilde{w}}_t^f + \left(1-\mu\right)\hat{\varepsilon}_{W,t} - \mu\epsilon_{P,t-1} \tag{C-48}$$

$$e^\gamma\hat{\tilde{k}}_t = \left(1-\delta\right)\hat{\tilde{k}}_{t-1} + \tilde{\delta}\hat{\tilde{x}}_t + \tilde{\delta}\hat{z}_{I,t} - e^\gamma\epsilon_{P,t} \tag{C-49}$$

$$\bar{\tilde{y}}\hat{\tilde{y}}_t = \bar{\tilde{c}}\hat{\tilde{c}}_t + \bar{\tilde{x}}\hat{\tilde{x}}_t + \bar{\tilde{g}}\hat{\tilde{g}}_t \tag{C-50}$$

$$\hat{\varepsilon}_{W,t} = \pi_W\hat{\varepsilon}_{W,t-1} + \epsilon_{W,t} \tag{C-51}$$

$$\hat{z}_{L,t} = \pi_L\hat{z}_{L,t-1} + \epsilon_{L,t} \tag{C-52}$$

$$\hat{z}_{I,t} = \pi_I\hat{z}_{I,t-1} + \epsilon_{I,t} \tag{C-53}$$

$$\hat{\tilde{g}}_t = \pi_G\hat{\tilde{g}}_{t-1} + \epsilon_{G,t} \tag{C-54}$$

D Model solution of the more simple RBC model (Not for publication)

D.1 FONC

$$n_t = 1 - l_t \tag{D-1}$$

$$R_t^k q_{t-1} = \frac{\theta y_t}{k_{t-1}} + \left(1 - \delta + g\left(z_{I,t}\frac{x_t}{k_{t-1}}\right)\right) q_t - \frac{x_t}{k_{t-1}} \tag{D-2}$$

$$q_t z_{I,t} = \frac{1}{g'\left(z_{I,t}\frac{x_t}{k_{t-1}}\right)} \tag{D-3}$$

$$\frac{1}{R_t^f} = \exp\left(\log \bar{M} + E_t\left[\hat{m}_{t+1}\right] + \frac{\sigma_m^2}{2}\right) \tag{D-4}$$

$$\frac{1}{E_t\left[R_{t+1}^k\right]} = \exp\left(\log \bar{M} + E_t\left[\hat{m}_{t+1}\right] + \frac{\sigma_m^2}{2} + \sigma_{mR^k}\right) \tag{D-5}$$

$$M_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \tag{D-6}$$

$$\lambda_t = c_t^{-\eta} \tag{D-7}$$

$$w_t = \Psi_l z_{L,t} \frac{l_t^{\frac{1}{\tau}}}{\lambda_t} \tag{D-8}$$

$$w_t = \frac{(1-\theta)\, y_t}{n_t} \tag{D-9}$$

$$y_t = (k_{t-1})^{\theta} \left(\exp\left(z_{P,t}\right) n_t\right)^{1-\theta} \tag{D-10}$$

$$k_t = \left(1 - \delta + g\left(z_{I,t}\frac{x_t}{k_{t-1}}\right)\right) k_{t-1} \tag{D-11}$$

$$c_t + x_t + g_t = y_t \tag{D-12}$$

The equilibrium is defined together with the exogenous variables $z_{L,t}$, $z_{I,t}$, $z_{P,t}$ and g_t.

D.2 Steady-state

To calculate the steady state we do the same steps as for the foregoing model with the exception of wages. Given that steady state labor supply is calibrated to $\bar{n} = 1/3$ wages are calculated as before:

$$\bar{w} = (1-\theta)\frac{\bar{y}}{\bar{n}} \tag{D-13}$$

The marginal rate of substitution implies also that

$$\bar{w} = \Psi_l \frac{\bar{l}^{\frac{1}{\tau}}}{\bar{c}^{-\eta}}. \tag{D-14}$$

To clear the labor market we have to solve for the scaling factor

$$\Psi_l = (1-\theta)\,\bar{y}\frac{\bar{c}^{-\eta}}{\bar{n}^{1/\tau}} \tag{D-15}$$

D.3 Log-linearization

$$\hat{l}_t = -\frac{\bar{n}}{1-\bar{n}}\hat{n}_t \tag{D-16}$$

$$\hat{r}_t^k + \hat{q}_{t-1} = \left[\frac{\bar{R}^k - 1 + \delta}{\bar{R}^k}\right]\left(\hat{y}_t - \hat{k}_{t-1}\right) + \frac{1}{\bar{R}^k}\hat{q}_t + \frac{\delta}{\bar{R}^k}\hat{z}_{I,t} \tag{D-17}$$

$$\hat{q}_t = \frac{1}{\zeta}\hat{x}_t + \left(\frac{1}{\zeta} - 1\right)\hat{z}_{I,t} - \frac{1}{\zeta}\hat{k}_{t-1} \tag{D-18}$$

$$\hat{w}_t = \hat{y}_t - \hat{n}_t \tag{D-19}$$

$$\hat{\tilde{w}}_t = \hat{z}_{L,t} + \frac{1}{\tau}\hat{l}_t - \hat{\lambda}_t \tag{D-20}$$

$$\hat{\lambda}_t = -\eta\hat{c}_t \tag{D-21}$$

$$0 = E_t\left[\hat{r}_{t+1}^k + \hat{m}_{t+1}\right] \tag{D-22}$$

$$0 = E_t\left[\hat{m}_{t+1}\right] + \hat{r}_t^f \tag{D-23}$$

$$\hat{m}_t = \hat{\lambda}_t + \hat{\lambda}_{t-1} \tag{D-24}$$

$$\hat{y}_t = \theta\hat{k}_{t-1} + (1-\theta)\,\hat{n}_t + (1-\theta)\,\hat{z}_{P,t} \tag{D-25}$$

$$\hat{k}_t = (1-\delta)\,\hat{k}_{t-1} + \delta\hat{x}_t + \delta\hat{z}_{I,t} \tag{D-26}$$

$$\bar{y}\hat{y}_t = \bar{c}\hat{c}_t + \bar{x}\hat{x}_t + \bar{g}\hat{g}_t \tag{D-27}$$

$$z_{P,t} = \pi_P z_{P,t-1} + \epsilon_{P,t} \tag{D-28}$$

$$\hat{z}_{L,t} = \pi_L\hat{z}_{L,t-1} + \epsilon_{L,t} \tag{D-29}$$

$$\hat{z}_{I,t} = \pi_I\hat{z}_{I,t-1} + \epsilon_{I,t} \tag{D-30}$$

$$\hat{g}_t = \pi_G\hat{g}_{t-1} + \epsilon_{G,t} \tag{D-31}$$