Lectures on Choosing and Processing Information

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Why Study Information Choice?

- Every expectation, mean, variance, covariance is conditioned on some information set. Information assumptions pervade every model.

- Preferences, technologies and budget constraints have been exhaustively studied. Information has not.

- Problem: We can’t observe information. Which information set do we use?
Solution: Information choice. Information differs from preferences because we have some control over what we learn.
Types of learning models

• Adaptive (non-Bayesian) learning (Evans and Honkapohja 2005)

• Passive learning

• Active learning
  this lecture
Recurrent Themes of this Class

- Information is non-rival and has increasing returns to scale.
- Information choices determine covariances of actions and states. Changes in information choices create regime changes.
- Information choice facilitates empirical work.
- Information-based theories can explain some of the most spectacular and seemingly-irrational events in modern macroeconomics.
Part II:

Strategic Motives in Information Acquisition
Main idea

Coordination (substitutability) in actions produces coordination (substitutability) in information choices.

Why is this result important? Does it get us back to square one?

1. Conditions for multiple equilibria change. Depends on:
   - Are information choices discrete or continuous?
   - Is information public or private?
2. Equilibrium switches move covariances, not levels.
   - Actions can only covary with what agents observe.
3. It explains why some applied models find information herding, while others predict specialization.
Strategic Motives in Information Acquisition

Definition 1 An action $a_i$ by agent $i$ is a strategic complement if the optimal choice of $a_i$ rises when $a = \int_j a_j dj$ rises.

Examples of strategic complementarity: speculative attacks in financial markets, banks runs, price-setting with monopolistic competition and investment with increasing returns.

Definition 2 An action $a_i$ by agent $i$ is a strategic substitute if the optimal choice of $a_i$ falls when $a = \int_j a_j dj$ rises.

Example of substitutability: portfolio choice.
A Beauty Contest Model

- A quadratic loss model that is a quadratic approximation to many models of strategic interactions.
- Work through the model with information choice.
The Main Result: Strategic Information Acquisition

- The marginal value of private information:
  \[ B(\tau_w) = -\frac{\partial}{\partial \tau_w} EL(\tau_w, \tau_z; \tau_w^*, \tau_z^*). \]

- The marginal value of public information:
  \[ B(\tau_z) = -\frac{\partial}{\partial \tau_z} EL(\tau_w, \tau_z; \tau_w^*, \tau_z^*). \]

**Proposition:**

\[
\begin{align*}
r &> 0 \iff \frac{\partial}{\partial \tau_w^*} B(\tau_w), \quad \frac{\partial}{\partial \tau_z^*} B(\tau_z) > 0 \\
r &= 0 \iff \frac{\partial}{\partial \tau_w^*} B(\tau_w), \quad \frac{\partial}{\partial \tau_z^*} B(\tau_z) = 0 \\
r &< 0 \iff \frac{\partial}{\partial \tau_w^*} B(\tau_w), \quad \frac{\partial}{\partial \tau_z^*} B(\tau_z) < 0
\end{align*}
\]
Intuition for the Main Result

- Complementarity ($r > 0$): High $\tau_w + \tau_z$ raises $\text{cov}(a,s)$, creates more payoff uncertainty, raises information value.

- Substitutability ($r < 0$): High $\tau_w + \tau_z$ raises $\text{cov}(a,s)$, creates less payoff uncertainty, lowers information value.
What Does This Teach Us?

• Why do agents want information that others not have in portfolio investment models and want information that others do have in price-setting models?

• Gives you intuition for what answers to expect before you even write the model down.

• If actions are complements, information can add excess volatility, regime changes in covariances and multiple equilibria.

• If actions are substitutes, seemingly identical agents may learn different information. Looks like market segmentation.

• Information choice can change strategic motives in richer models. Example: media frenzies.
A Simple Investment Example

- A continuum of profit-maximizing firms chooses investment $k_i$.
- Profit is output - cost
  \[
  \pi_i = [(1 - r) s + rK] k_i - \frac{1}{2} k_i^2 \tag{1}
  \]
  where $K = \int k_i \, di$ and $r$ measures investment externality.
- Nature draws $s$ with mean $y$ and variance $\sigma^2$.
- Firms can pay cost $C$ to observe $s$ or not.
A Simple Investment Example

- first-order condition for optimal investment is
  \[ k_i = (1 - r) E_i(s) + r E_i(K). \]

- Expected equilibrium profits
  \[ E_i(\pi_i) = \frac{1}{2} [(1 - r) E_i(s) + r E_i(K)]^2 \] (2)

- Work out payoffs of being informed and uninformed when other firms are informed and uninformed.
A Simple Investment Example

<table>
<thead>
<tr>
<th>Expected Net Profit</th>
<th>Others are Informed</th>
<th>Others are Uninformed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Become Informed</td>
<td>( \frac{1}{2} y^2 + \frac{1}{2} \sigma^2 - C' )</td>
<td>( \frac{1}{2} y^2 + \frac{1}{2} (1 - r)^2 \sigma^2 - C' )</td>
</tr>
<tr>
<td>Remain Uninformed</td>
<td>( \frac{1}{2} y^2 )</td>
<td>( \frac{1}{2} y^2 )</td>
</tr>
</tbody>
</table>

Others being informed raises the payoff to information.
Related Work

- Excess volatility in asset prices: Ozdenoren and Yuan (2007)
Part III:  
Inattentiveness
Applying theory results to price-setting

- Coordination motives in price setting make information acquisition a coordination game.
- Complementarity helps to slow down price adjustment. But it creates two problems:
  - When agents learn at the same time, prices jump.
  - Because public information is better for coordinating on, unrestricted information choice causes multiple equilibria. Model predictions are indeterminate.
Motivation for inattentiveness in price-setting

- Why don’t firms continuously adjust prices to macroeconomic news?
- Why isn’t money neutral? What effects does monetary policy have on the real economy?
- Can information choice explain the slow reaction of aggregate activity to monetary shocks?
A Costly Planning Model (Reis 2006)

- Firms choose price $p_t^i$ to minimize losses:
  \[
  \sum_{t=0}^{\infty} \beta^t \left[ (p_t^i - p_t^*)^2 + D_t^i C \right]
  \]

- $p_t^*$ is “target price” (a full-info optimum):
  \[
  p_t^* = (1 - r) m_t + r p_t
  \]

- $p_t$ is average price of all firms, $m_t$ is nominal money supply.

- $m_t$ is a random walk:
  \[
  m_t = m_{t-1} + \varepsilon_t; \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)
  \]

- To plan $(D_t^i)$: Pay $C$ to observe $\{m_{\tau}\}_{\tau=0}^t$, $\{p_{\tau}\}_{\tau=0}^t$. 

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An equilibrium is a sequence of planning choices by every firm $i$ \( \{D_t^i\} \), and prices \( \{p_{it}\} \), that are $I_{i,t}$-measurable and maximize the firm’s objective, taking as given the choices of all other firms.
Equilibria With Strategic Planning

- *Staggered planning* - Agents plan every $T$ periods. A fraction $1/T$ of agents plans each period.
  - In continuous time, there is a unique staggered equilibrium.

- *Synchronized planning* - Agents plan every $T$ periods, all in the same period.
  - Synchronized planning delivers unrealistic price paths.
  - There are multiple synchronized planning equilibria.
Solving Inattentiveness Model

- At $t$, $\lambda_{t,\hat{\tau}}$ is the measure of firms who last planned at $\hat{\tau} \leq t$. $\Lambda_{t,\hat{\tau}} = \sum_{\tau=\hat{\tau}}^{t} \lambda_{t,\tau}$ is firms who last planned between $\hat{\tau}$ and $t$.

- Aggregate price is a weighted sum of target prices from past periods,

$$p_t = \sum_{\tau=0}^{\infty} \lambda_{t,t-\tau} E(p_t^* | I_{t-\tau}).$$

- Lemma: The average price is a weighted sum of all past demand innovations,

$$p_t = \sum_{\tau=0}^{\infty} \frac{\Lambda_{t,t-\tau}(1-r)}{1-r\Lambda_{t,t-\tau}} \varepsilon_{t-\tau}. \quad (3)$$
Solving Inattentiveness Model

• The target price process is

\[ p_t^* = \sum_{\tau=0}^{\infty} \frac{1 - r}{1 - r\Lambda_{t,t-\tau}} \varepsilon_{t-\tau}. \]

• Agents who last planned at date \( \hat{\tau} \) set price

\[ E(p_t^*|I_{\hat{\tau}}) = \sum_{\tau=t-\hat{\tau}}^{\infty} \frac{1 - r}{1 - r\Lambda_{t,t-\tau}} \varepsilon_{t-\tau}. \]

• Their expected 1-period loss is

\[ L_{t,\hat{\tau}} = E\left( E(p_t^*|I_{\hat{\tau}}) - p_t^*|I_{\hat{\tau}} \right)^2 = \sum_{\tau=0}^{t-\hat{\tau}-1} \left( \frac{1 - r}{1 - r\Lambda_{t,t-\tau}} \right)^2 \sigma^2 \]
• Plug one-period loss into Bellman equation. Guess and verify a staggered equilibrium.

• Proposition: There exists a staggered planning equilibrium with horizon $T$, if and only if

$$\sigma^2 \sum_{s=1}^{T-1} \frac{1 - \beta^s}{1 - \beta} \left( \frac{1 - r}{1 - rs/T} \right)^2 \leq C \leq \sigma^2 \sum_{s=1}^{T} \frac{1 - \beta^s}{1 - \beta} \left( \frac{1 - r}{1 - rs/T} \right)^2.$$ 

• Left side: marginal value of planning one period earlier.
  Right side: marginal value of not delaying one more period.
**Information Complementarity**

- When prices are complements \( r > 0 \), there is complementarity in planning.

\[
\frac{\partial L_t,\tau}{\partial \Lambda_{t,t-\tau}} > 0 \text{ if and only if } r > 0.
\]

- Higher \( r \) reduces both sides of inequality (last slide). The same planning horizon with higher \( r \) requires a smaller planning cost \( C \).

- With a fixed \( C \), stronger complementarity makes the equilibrium planning horizon longer \( \rightarrow \) Less frequent price adjustment.
Information Complementarity

- Complementarity delays in price adjustment 2 ways:
  - If other firms have prices based on old information, firms that do plan temper their reactions to recent information.
  - longer planning horizons → less frequent price adjustment (new effect).

- Complementarity and covariance are mutually reinforcing. Incomplete information ↓ $cov(p, Z)$. Reduces incentives to plan.

- Complementarity is the key result of the price-setting model that allows it to match the data.
Practical Advice: Avoid Multiple Equilibria

- Complementarity in actions means complementarity in information choices. Likewise for substitutability.

- Private information in priors does not eliminate multiplicity. Information choices depend on second moments. These are common knowledge. Morris-Shin refinement does not work here.

- 2 solutions to restore uniqueness:
  - Force choices to be staggered. When no two agents learn at the same time, information is private (Reis 2006).
  - Limited channel capacity adds private noise to all signals (Mackowiack and Wiederholt 2006).
Public information, in excess of what others observe, is private.
Jump in marginal value → multiplicity.
Mankiw and Reis (2002) popularized this idea, but information is exogenous.

Lots of subsequent work by Ricardo Reis.

Abel, Eberly and Panageas have used this kind of learning technology to solve a dynamic optimal portfolio problem.
Part IV:

Returns to Scale in Information
Returns to Scale in Information

- Information is different from physical goods because it can be replicated freely. One signal can be used to decide how to invest one unit or one million units of capital.

- This fundamental difference allows information choice to explain phenomena that models of physical production cannot explain.

- A simple model by Wilson (1975) shows how adding information choice can overturn standard competitive equilibrium results. Let’s work through it.
Conclusions from Wilson’s Model

- Holding production fixed, information has decreasing returns.
- Holding information fixed, production has decreasing returns.
- The more the agent learns, the more he wants to produce and the more he produces, the more valuable information becomes. *endogenous increasing returns to information.*
- Wilson’s paper has other examples/models. Either risk aversion with an additive precision or risk neutrality with an entropy-based learning → increasing returns.
- Many applications.
Growth theory builds on these insights (Romer 1988).

Radner and Stiglitz (1984) generalize these ideas in a paper on the non-concave value of information.

We’ll see some other applications in a bit.
Part V:
Information Choice in an Equilibrium Model
Motivation

- So far, information has been an endowment or a choice. But market provide another important source of information through prices.
- Market interactions also create strategic motives in learning.
- Grossman and Stiglitz (AER, 1980) is the canonical model of information choice in an equilibrium model. We’ll work through that model now.
Conclusions: What do we learn?

- the value of information is related to the square root of the ratio of payoff variance with the information to the payoff variance without the information. Lots of things don’t matter.

- Strategic substitutability reappears. It creates a “fundamental conflict between the efficiency with which markets spread information and the incentives to acquire information.”

- This class of models are called *noisy rational expectations* models.
Related Work

- In the 1980’s, this was extended by Verricchia (1982) and Hellwig (1980).
- Joel Peress is actively using Grossman-Stiglitz-style models to answer questions in macro and finance.
- A market microstructure literature in finance (Easley and O’Hara) also uses frameworks of Kyle (1985) and Glosten and Milgrom (1985)
Part VI: Measuring Multi-Dimensional Information Flows
Questions

- What is the feasible set of information choices? What is the cost of observing a set of signals?
- Like asking about physical production technology. But we can’t observe the outputs as easily. Market prices for information are harder to interpret.
- Should a signal with one unit less variance cost 1 cent less? Should a signal with one unit more precision cost 1 cent more?
- Some examples of constraints used in the literature.
Entropy and Rational Inattention

- With multi-dimensional, normally distributed random variables, such a constraint takes the form

\[
\frac{|\hat{\Sigma}|}{|\Sigma|} = \exp(-2K)
\]

\(\hat{\Sigma}\) is the posterior variance-covariance matrix, \(\Sigma\) is the prior variance-covariance matrix and \(K\) is a measure of information capacity, or mutual information.

- Adding one unit of capacity reduces the generalized standard deviation \(|\hat{\Sigma}|^{1/2}\) by a factor of \(e \approx 2.7\).
Like Wilson’s model.

Suppose risks are independent ($\Sigma$, $\hat{\Sigma}$ diagonal)

$$\sum_{i=1}^{N} \Sigma_{ii} \geq \tilde{K}'$$

is linear, while entropy would be

$$\prod_{i=1}^{N} \Sigma_{ii} \geq \tilde{K}$$

for some constant $\tilde{K}$. 
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Linear vs. Entropy Measure

- Entropy: A process of $K$ consecutive bisections of a sample space.
- Linear: Drawing consecutive independent signals.
- Does initial information cause us to narrow our search?
- Does entropy build in increasing returns to scale in learning? Should returns to scale be measured in precision units or variance units?
Diminishing Returns to Learning

- Diminishing returns from unlearnable risk.
- Eliminating all learnable risk (reducing \( \hat{\Sigma} \) to \( \alpha \Sigma \)) requires infinite capacity. When \( \hat{\Sigma} = \Sigma \), the investor is not learning anything, and no capacity is required.

\[
\frac{|\Sigma - \alpha \Sigma|}{|\hat{\Sigma} - \alpha \Sigma|} \leq e^{2K}.
\]
Inattentiveness

- Inattention and inattentiveness sound almost the same, but are completely different learning technologies.

- Rational inattention: constant, noisy flow of information. Inattentiveness: no information flow then occasional perfect information.

- Merton (1987) calls this “recognition.”

- A cost to looking up the right answer, not information processing.
What is the Right Technology?

• There is no right answer.

• Different technologies produce different results.

• How to figure out which one to use?
  – Experiments
  – Adopt a standard with some realistic properties.
  – Justify the assumptions by testing the results.
Part VII:

Learning and Portfolio Allocation
Choosing and Processing Information

Multiple Risky Assets and Fixed Information

• Admati (Econometrica, 1985) solved an equilibrium portfolio model with exogenous, heterogeneous information.

• We will begin by working through her model and then add information choice (entropy-based) on to it.

• Questions: How does information heterogeneity affect investment decisions and asset prices?
What do we learn?

What assets command high returns (low prices)?

- Assets about which agents are uncertain $\Psi$ is low.
- Assets that are abundant. (Large $\bar{x}$).
- Assets with large asset supply shocks (big $\sigma^2_x$).

What assets does an investor want to hold? (Recall $q_i = \frac{1}{\rho_i} \hat{\Sigma}_i^{-1}(\hat{\mu}_i - pr)$)

- Hold assets you are optimistic about (high $\hat{\mu}$)
- you know lots about (high $\hat{\Sigma}_i^{-1}$)
- and that command high returns (low $pr$)
What Is Information About?

- Why choose only the variance?
- Choosing signal variance = choosing posterior variance
- Independent assets case
- Correlation assets
- Interpreting orthogonal risk factors
- Does this assumption matter?
Van Nieuwerburgh and Veldkamp (2007) asks: If an investor has limited capacity to learn before choosing his portfolio, what should he learn about?

Should investors specialize or learn about many assets?

Does specialization cause under-diversification?
Or, do all investors study the same assets?

How do investors balance gains to specialization and diversification?

Can we predict which assets will be learned about?
Results

- Work through the model.

- In partial equilibrium:

**Result 1** The optimal information allocation with N correlated assets uses all capacity to learn about one linear combination of asset payoffs. The linear combination coefficients are given by the eigenvector $\Gamma_i$, with the highest factor learning index $\theta_i^2 = ((\mu - pr)'\Gamma_i)^2 \Lambda_i^{-1}$. 
Proposition: The optimal information portfolio with $N$ correlated assets uses all capacity to learn about one linear combination of asset payoffs $F'\Gamma_i$, associated with the highest learning index:

$$\left(\Gamma'_i(\mu - pr)\right)^2 \Lambda_i^{-1} + \Lambda_{pi} \Lambda_i^{-1}.$$

- Learn about risks with high: (factor Sharpe ratio)$^2 = \text{expected factor return } \Gamma'_i(\mu - pr) \times \text{factor portfolio share } \Gamma_i E[q]$.
- Learn about risks with high noise in prices: $\Lambda_{pi}$ is exploitable pricing error.
- Fully specialize in that risk factor: Increasing returns to information.
Objective is: $\max_{\Lambda} \sum_i (\Lambda_{pi} + (\Gamma_i' E[\hat{\mu} - pr])^2) (\hat{\Lambda}_i)^{-1}$. 

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Waterfilling Equilibrium

Investors with Low Capacity Learn One Factor

With $K > K_1$, Investors Learn Two Factors

With $K > K_2$, Investors Learn Three Factors
Portfolio allocation: What do we learn?

- Information choice can overturn basic ideas about optimal investment.
- Information choice matters for asset prices.
- Small initial differences in beliefs can be amplified in settings with strategic substitutability in actions.
Increasing Returns to Information and Home Bias

- Learn about assets you expect to hold.
- Hold more of assets you’ve learned about.
Increasing Returns to Information and Home Bias

- Learn about assets you expect to hold.
- Hold more of assets you’ve learned about.
Evaluating the model - Testable Predictions

- Forecast precision
- Survey data
- Local bias
- Industry bias
- Concentrated portfolio out-performance
- Under-diversified foreign investment
- Cross-sectional asset prices
Expected utility agents are not averse to the risk they face at date 2 because it will be resolved at time 3.
Learning and Risk Preferences

- Mean-variance preferences are
  
  \[ U = E \left[ - \log (E [\exp (-\rho W) | \mu, \mu]) | \mu \right] \]

  where \( W = rW_0 + q'(f - pr) \) is the budget constraint and payoffs \( f \) are normally distributed.

- Related to Epstein and Zin (1989) preference for early resolution of uncertainty.
Related papers

- Peng and Xiong (2006) - Focus on dynamics of asset price.
- Berhardt and Taub (2005) - Large strategic investors.
Part VIII:
Markets for Information
Motivation

• Information is an interesting good because
  – it is non-rival. We can’t all hold 50% shares of IBM. We can all read a story about IBM.
  – It is cheap to replicate.

• Key to long-run growth.

• Creates hidden complementarities in short-run models. Explains herding, asset price anomalies, business cycle comovement across sectors.

• Let’s work through that model.
Choosing and Processing Information

Net Benefit of Information

\[ B(\lambda), \text{Net Benefit of Information Purchase} \]

\[ \lambda, \text{Fraction of Population that is Informed} \]

\[ 0 0.2 0.4 0.6 0.8 1 \]

\[ -0.15 -0.1 -0.05 0 0.05 0.1 0.15 \]

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Cost and Benefit of Information

![Graph showing cost and benefit of information]

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Simulated asset prices without (red) and with information.
Coordination or Correlation?

- Coordination is where agents want to take the same action because other agents are taking that action.
- Correlated behavior is when agents end up taking the same actions because they see the same or similar information.
- What looks like coordination may be correlation.
- Complementarities in information are much easier to generate than complementarity in asset markets.
An Application to Business Cycles

Empirical Motivation

• Fact: Output is more correlated than technology. \( \text{avg corr}(GDP_i, GDP_a) = 0.51, \text{ avg corr}(TFP_i, TFP_a) = 0.17. \)

• Where do aggregate business cycle shocks come from? Sectoral TFP is not highly correlated. Other shocks not enough.

• Standard business cycle models do not generate more correlation in output than in productivity.
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Business Cycle Comovement

Comovement of Output and Productivity

Dashed line shows 45-degrees

Correlation of Industry Output with Aggregate Output (both value-added)

Correlation of Industry TFP with Aggregate TFP

Dashed line shows 45-degrees

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Main Idea

• Agents who observe only aggregate information have similar beliefs and make similar decisions. A firm that only observes 1 aggregate shock only reacts to 1 aggregate shock.

• Aggregate information is cheap because its discovery cost is shared among many purchasers. Idiosyncratic information is expensive because its discovery cost must be borne alone.

• Based on Veldkamp and Wolfers (JME 2007)
What Creates Aggregate Business Cycles?

• Lucas (1977): A central feature of business cycles is that sectors grow and contract together → aggregate shocks

• 2 problems:
  1. Even with an aggregate shock, consumption and investment sectors covary negatively (Christiano and Fitzgerald 1998).
     - Possible solutions: Home production, habit persistence and limits on labor mobility, complementarity.
       Benhabib, Rogerson and Wright (91), Christiano and Fisher (98)
  2. Where do aggregate shocks come from? Sectoral TFP is not highly correlated. Other shocks not enough. Cochrane (94)

• We propose an answer to the second question.
• Island economies with industry-specific shocks share only a common information market.
  – $\text{corr}(\text{output}) > \text{corr}(\text{TFP})$.

• Typical problem: Market undoes information asymmetry.
  – We add a labor market that reveals all signals.

• Labor market creates Labor trade-off problem
  – Solution: Add a home production sector.
    (As in Benhabib, Rogerson Wright, 1991)

• Empirical support.
Island Economies with an Information Market

- Objective is exponential (CARA): $-E[\exp(-\rho(c_i - \psi n_i))]$.
- Consume output of island-specific labor: $y_i = z_i n_i$.
- Productivity has aggregate and idiosyncratic, learnable and unlearnable components: $z_i = \mu z + \beta_i \bar{z} + \eta_i + e_i$.
- 2 types of signals:
  - Aggregate: $s_0 = \bar{z} + e_0$,
  - Island-specific: $s_i = \beta_i (\bar{z} + e_0) + \eta_i$.
- Information production: Fixed cost $\chi$ for discovery. Zero cost to replicate.
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**Equilibrium**

Agents choose the following to maximize their objective, taking others’ actions as given.

1. **Info production** - Agents announce signal prices $\tilde{\tau}_j$. Then they choose whether to produce each signal at cost $\chi$. Profit on signal $j = \pi_j$

2. **Info purchase** - Each agent chooses what signals to purchase.

3. **Goods production** - Agents choose $c$ and $n$, given all observed signals, s.t. budget constraint:

   $$c_i = z_i n_i + \sum_{j} (\pi_j - \tau_j L_{ij}).$$

   (5)

   where $L_{ij} = 1$ if agent $i$ buys signal $j$ and 0 otherwise.
Result: Market Filters Out Industry Information

- Equilibrium information price is

\[ \tau_j = \chi / \lambda_j \]

where \( \lambda_j \) is the number of agents who buy signal \( j \).

- For industry-specific information, industry must pay \( \chi \).

- Market supplies lots of aggregate information at a low price, and little industry-specific information.
**Result: Aggregate Shocks to Choice Variables**

- Firms that observe the aggregate signal (common info) have perfectly correlated beliefs $E[z_i|s_0]$.

- Labor is linear in $E[z_i|s_0] \rightarrow \text{corr}(n_i(s_0), n_j(s_0)) = 1$ or $-1$.
  - Cochrane (94): “Shocks to consumption, output, or other endogenous variables... account for the bulk of business cycle variations.”
  - High output correlation.

- For firms that observe industry-specific signal (heterogeneous info), $\text{corr(labor input)} \approx \text{corr(TFP)}$.
  - No excess comovement
Who Buys Aggregate Information?

A firm will purchase the aggregate signal $s_0$ if two conditions are satisfied:

1. Buying the aggregate signal at price $\chi/\lambda$ yields higher utility than buying the industry signal, at the higher cost $\chi$:

$$\frac{1}{2\rho} \log \left( \frac{Var(z_i)}{Var(z_i|s_0)} \right) - \frac{\chi}{\lambda} \geq \frac{1}{2\rho} \log \left( \frac{Var(z_i)}{Var(z_i|s_i)} \right) - \chi.$$

2. Buying the aggregate signal yields higher utility than not purchasing any information:

$$\frac{1}{2\rho} \log \left( \frac{Var(z_i)}{Var(z_i|s_0)} \right) - \frac{\chi}{\lambda} \geq 0.$$
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Calibration

- \( z_i \) and \( \bar{z} \) processes match std and corr of industry and aggregate TFP.

- Of each \( z_i, \bar{z} \) shock, 1/2 is (un)observable: \( \sigma_\eta = \sigma_e \) and \( \sigma_{e0} = \sigma_z \).

- Disutility of labor: \( \psi = 0.96 \) matches \( \text{std}(n)/\text{std}(y) = 0.8 \) in data.

- Risk aversion and information cost ensure that all 3 possible information choices are made by \( \geq 1 \) industry: \( \rho = 4, \chi = 0.2 \).

- Normalization: \( \mu_z = 1 \)
Result: Information Markets $\rightarrow$ Aggregate Shocks

Comovement of Output and Total Factor Productivity

Industry consumes:
- No information
- Aggregate information
- Specialized information

Dashed line is 45-degree line

Model Fit: RMSE=0.61
Labor Markets: Price and Information Effects

• Change the model: Make labor not island-specific and tradeable.

• Input prices and quantities reveal ALL information.
  – Free-riding collapses the information market. All info purchased is firm-specific.
  – Few firms pay for information. Non-learners can make inference from others’ signals. $\rightarrow$ common shocks to beliefs.
Labor Markets: Belief comovement persists

- Firms that do not buy information observe $S$ signals \{${s_1, \ldots, s_S}$\}. They form identical beliefs about aggregate: $E[\bar{z}|s_1, \ldots, s_S]$.

- With no sector-specific information, beliefs are perfectly correlated: $E[z_i] = \beta_i E[\bar{z}]$

- Labor depends on $E[z_i|s_1, \ldots, s_S]$, and the wage. 
  \[
  \text{corr}(n_i(s_0), n_j(s_0)) = 1 \text{ if } \beta_i = \beta_j. \text{ Otherwise, correlation is high, but not perfect.}
  \]
Success: Looks like the 1-shock multi-sector RBC model.

Failure: 1-shock RBC → no comovement (labor trade-off).
A Model with Home Production

- We apply Benhabib, Rogerson and Wright’s (1991) solution to the labor-trade-off problem: a home production sector.

- This sector is modelled like all the others.
  - It is nearly acyclical: $\beta_h = 0.05$.
  - It is large: We choose the industry-specific productivity variance $\phi^2_h$ to match BRW’s finding that hours in home production and in market work are about equal.
Conclusion: To get $\text{corr}(\text{GDP}) > \text{corr}(\text{TFP})$, you need to solve both ‘labor trade-off’ and ‘aggregate shock’ problems.
Home Production & Costly Information

Our sector-specific shocks produce same effect as BWR’s aggregate shock.
A second aggregate shock is already present: home prod TFP. Raising info cost makes it more dominant. Solves $\beta < 0$ problem.

Punchline: Costly information can amplify other aggregate shocks.
Conclusions

• Information is costly to discover but cheap to replicate.
  → Learning aggregate information, but not industry-specific information is efficient (minimizes cost).

• Transmitting aggregate information generates aggregate shocks to choice variables.

• Works even when markets fully reveal private information.

• The model can generate $\text{corr}(\text{GDP}) > \text{corr}(\text{TFP})$.

• Suggestive evidence supports our explanation.
Main Ideas from Information Choice

• Is information a complement or a substitute?
• Are agents coordinating their actions or do they act on similar information?
• Information is fundamentally different from physical goods because it has increasing returns to scale.
• Information choice determines covariance of actions and states.
• Next: Research ideas and testing information models.
Part IX:

Research Ideas
Firm and Price Dynamics

- Why do sectors choose such similar outputs and firms choose such different prices?
- Are the factors that matter for each of these decisions so different?
- Is there an information model that can reconcile the cyclical behavior of both prices and quantities?
Central Bank Transparency

• Morris and Shin (02) argue that agents overweight public information relative to private information in situations where they want to coordinate.

• In Amador and Weill (2007), agents learn from observing the actions of others. With lots of public information, actions reveal little private information and others learn little new from observing them.

• Another possibility: Public information reduces the incentive to acquire private information. How big an effect might this be?
Business Cycles

- Information-driven models of business cycles (Beaudry and Portier (2004), Jaimovich and Rebelo (2007)) show how information about future productivity can create realistic business cycles.

- What are the incentives to acquire information? Is information more valuable at some times than others? In some sectors more than others?

- Maybe information choice can help this model address more business cycle facts.
Trade and Globalization

- How might trade in goods and trade in information interact?
- Can predictions about information patterns help explain otherwise puzzling patterns of production and trade?
- Work in progress: Use specialization to explain why small differences in distance make big differences in the amount countries trade.
Choosing and Processing Information

Exchange Rates

- Why don’t macroeconomic fundamentals (inflation, interest rate differentials, growth rates) predict exchange rate movements?

- Cheung and Chinn (2001) survey professional currency traders. Most important macro variable changes over time.

- Suggests regime shifts in information demand. Which piece of information should we coordinate on? How do transitions between coordinated equilibria occur?
Exchange Rates and the Carry Trade

- Carry trade: holding high-interest-rate currencies and shorting low-interest-rate currencies. Small payoff with a high probability and a small probability of a rapid decline.

- Why is there skewness in exchange rates? Why do some currencies have systematic negative skewness and others positive and what determines which is which?
When the average investor learns about an asset, its risk and return fall. Can this explain pricing errors of the CAPM?

Result 2 If the market payoff is defined as $f_m = \sum_{k=1}^{N} (\bar{x} + x_k) f_k$, the market return is $R_m = f_m \left( \sum_{k=1}^{N} (\bar{x} + x_k) p_k \right)^{-1}$, and the return on an asset $i$ is $R_i = f_i / p_i$, then the equilibrium price of asset $i$ can be expressed as $p_i = \frac{1}{r} (E_a[f_i] - \rho \text{Cov}_a[f_i, f_m])$. The equilibrium return is $E_a[R_i] - r = \beta^i_a (E_a[R_m] - r)$, where $\beta^i_a \equiv \text{Cov}_a[R_i, R_m] / \text{Var}_a[R_m]$. 
Understanding the Financial Industry

- What if a portfolio manager can process information for many investors?
- Portfolio managers will have the same incentives to specialize learning.
- How to prevent information leakage?
- How to price information services?
- Will under-diversification still be optimal for investors?
Understanding the Financial Industry

- What is the optimal distribution of funds?
- Are there externalities that cause too many funds to enter the market?
- A large literature on mutual funds, their investment styles, their returns and fees. Very little theory here.
- Challenge: Large actors in a market will have price impact. The Admati framework does not handle that. See Bernhardt and Taub (2005).
Geography and Economic Transitions

- Transitions do not happen uniformly across space. Lots of geographic heterogeneity.

- Spatial learning models can explain this. They generate both a positive externality (similarity with neighbors) and a friction (differences across regions).

- See Fogli and Veldkamp (2007) for an example with female labor force participation. Many other events could be modeled this way.
Wilson (1975) shows that zero information acquisition and zero investment can be an equilibrium because of increasing returns.

Endogenous increasing returns is a form of development trap. It could help explain why poor countries stay poor.
Income Distributions

• If wealthier individuals acquire more information, they earn higher returns.

• Poor individuals may stay poor while the rich get richer.

• See Peress (RFS 2004) for a related paper.
Part X:
Evaluating Information Models Empirically
Information Choice as a Substitute for Information Data

- Problem with information-based theories - If information is unobservable are the theories falsifiable? Is this all a waste of time?

- Information choice is a solution to the problem of unobservable information.

- Using observable variables to predict information sets is a substitute for information data.

Estimating the learning index

1. Compute the eigen-decomposition of asset payoffs:
\[ f_t = d_t + p_{t+1} \]. Post-multiply by the eigenvector matrix \( \Gamma \), to form risk factor prices and payoffs.

2. Construct prior risk factor Sharpe ratios: Divide return by standard deviation.

3. Estimate \( \Lambda_B \). Regress risk factor prices (\( \Gamma'p \)) on a constant and risk factor payoffs (\( \Gamma'f \)). Residual variance is \( \Lambda_C^2\sigma_x^2 \).
\[ 1 - R^2 = \Lambda_{pi}/\Lambda_i. \]

4. Learning index = squared Sharpe ratio + \( (1 - R^2) \)

5. Pre-multiply risk factor indices by \( \Gamma \). The resulting vector contains learning indices for each asset.
Choose and Processing Information

How to Use Learning Indices

- They should predict information-related variables such as analyst coverage.

- Countries, regions or firms with higher learning indices should have lower returns, relative to CAPM.

- A country or region’s learning index should be related to the home bias of its residents’ portfolios (non-monotonic).
Other Examples of Using Observables


- In Brennan and Hughes’ (1991) model, brokers have an incentive to produce research reports on firms with low share prices. This pattern of information provision can explain stock splits.
Choosing and Processing Information

Count news stories or analyst coverage


- Beware of endogeneity. Use instruments whenever possible.
Count news stories: Results

<table>
<thead>
<tr>
<th>Equation I</th>
<th>Coefficient on $\text{news}_{mt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td></td>
</tr>
<tr>
<td>$P_{mt}$</td>
<td>1.35 (0.03)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation II</th>
<th>Coefficient on $(\Delta \log(R_{mt}))^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable:</td>
<td></td>
</tr>
<tr>
<td>$\text{news}_{mt}$</td>
<td>211.98 (21.63)</td>
</tr>
</tbody>
</table>

3SLS estimation. Instruments for $(\Delta \log(R_{mt}))^2$: $\text{div}_{mt}$, $(\Delta \text{div}_{mt})^2$, and $(\Delta \log(R_{m(t-1)}))^2$. 

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Forecast precision

- Macro forecasts - Survey of professional forecasters panel data, quarterly from 1968:4, 9-76 analysts per quarter.

- 2 measures of precision
  - Dispersion
  - Mean squared error from realized value

- Bae, Stultz and Tan (2005) measure information asymmetry using earnings forecast precision. Home analysts in 32 countries make 8% more precise earnings forecasts than foreigners.
Using covariances to infer information sets

- Hong and Stein (2007) - What performance metric are Amazon’s stock prices most correlated with?
- Biais, Bossaerts and Spatt (2004) - price-contingent strategies generate annual returns (Sharpe ratios) that are 3% (16.5%) higher than the indexing strategy.
- Durnev et.al. (2003) measure which assets’ prices contain more and less information about future earnings.
One of the most commonly used measures of information in financial markets is probability of informed trading (PIN). Due to Easley, Kiefer, O’Hara and Paperman (1996).

Data on the sequence of buy and sell orders used to estimate information and predict the bid-ask spread.

We’ll work through the PIN model and derive their information estimator.
Estimating Information

• Most applied theories will not get far without empirical evidence. Facts can be your own or can come from others’ empirical work.

• It’s important to evaluate theories empirically for this area of the literature to gain widespread acceptance.

• Good news: There are lots of ways to measure information.
Conclusions

• This is an area that is taking off. Lots of different applications.

• How to get started?
  – Look for facts that are puzzles to existing theories. Behavioral economics papers are good sources of puzzles.
  – Ask yourself, might information explain that? Some effects we’ve seen include: amplification, hidden complementarities, regime shifts, changing covariances and excess volatility, inertia.
  – Once you have a theory model, back it up with evidence.

• Support other people in the field.

• Be creative, be persistent and good luck!