Isotonic Regression in Sobolev Spaces

Michal Pešta

michal.pesta@matfyz.cz

Charles University in Prague
Faculty of Mathematics and Physics

Abstract

We propose a class of nonparametric estimators for the regression models based on least squares over the sets of sufficiently smooth functions. Least squares permit the imposition of additional constraint—isotonia—on nonparametric regression estimation and testing of this constraint.

The estimation takes place over the balls of functions which are elements of a suitable Sobolev space—special types of Hilbert spaces that facilitate calculation of the least squares projection. The Hilbertness is allowing us to take projections and hence to decompose spaces into mutually orthogonal complements. Then we transform the problem of searching for the best fitting function in an infinite dimensional space into a finite dimensional optimization problem. We also generalizes the set-up from [1] into Weighted Least Squares and Total Least Squares.

We prove that the balls of functions in Sobolev space are bounded and have bounded higher order derivatives. It permits us to estimate over rich set of functions with sufficiently low metric entropy and apply Laws of Large Numbers and Central Limit Theorems [2]. We also apply bootstrap techniques to the FEDC data.

Finally, we concentrate on covariance structure of data [3]. Hence we study the dependence in option prices and estimate them using their isotonic character.

Keywords: Isotonic regression, Sobolev spaces, nonparametric, monotonicity.

References

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