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# Do Factor Shares Reflect Technology?

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# Do Factor Shares Reflect Technology?

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## ABSTRACT

This note demonstrates that it is easily possible to compute technological parameters out of national income accounting data in the presence of bargaining in the labor market. Applying the method to US data, we obtain that the output elasticity with respect to capital exceed 0.5.

[JEL. E23, E25]

KEYWORDS: FACTOR SHARES, NASH BARGAINING

## 1. Introduction

In many macro models, it is standard to associate labor and capital shares of national income with technological parameters of the aggregate production function. In particular, in the Cobb-Douglas formulation, these shares are simply the respective exponents of capital and labor. Since the labor share in national income is approximately 70% in most industrialized nations, it is common to use an exponent of about 0.3 for capital in such a formulation. This association is, of course, based on the assumption that factors are paid their respective marginal products. However, the labor share that emerges from direct estimates of aggregate production functions under the competitive pricing mechanism, tends to be significantly

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lower than the above 70%. For example, the production function estimates of Duffy and Papageorgiou, 2000, imply that labor shares in the developed countries cannot exceed 0.32 under competitive factor markets. This is, of course, the share usually associated with capital.<sup>4</sup>

We show that a model that includes the bargaining feature can reconcile the technological parameters with the factor shares as they are observed. Specifically, we replace the competitive market mechanism of wage determination with a Nash bargaining process as is commonly done in the labor literature (e.g. see Pissarides (2000) and the literature therein), and show that technological parameters can still be extracted from the national income accounts. Not surprisingly however, these will reflect bargaining power and, in general, differ from the factor shares.<sup>5</sup>

## 2. The Model

We consider a small open economy populated by identical workers on a continuum of measure 1. Potential firms are drawn from the real line. Firms own a production technology that employs capital and one unit of labor where both inputs are essential.<sup>6</sup> Let  $f(k)$  denote the output per worker, where  $k$  represents capital per worker, and assume that  $f(k)$  satisfies the Inada conditions. Firms pay a firm

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<sup>4</sup>Duffy and Papageorgiou estimate a production function of the form  $Y = A[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-\frac{1}{\rho}}$ . For this specification, the labor share is  $\frac{(1-\delta)k^\rho}{\delta+(1-\delta)k^\rho}$ , where  $k$  is the capital labor ratio. The implication follows from the parameter values estimated for developed countries,  $\rho = -0.08$  (with marginal significance) and  $\delta = 0.68$  (highly significant, see Table 3, page 109).

<sup>5</sup>We are not the first to observe that bargaining may cause deviations from marginal value of product pricing. For example, Bentolila and Saint-Paul (2003), as well as Blanchard (2004) invoke bargaining power as one of the explaining factors for movements in labor shares in Europe (see also Spector, 2004).

<sup>6</sup>For parsimony, we assume that every firm can employ at most one worker. One could relax the requirement as long as there is an upper bound on the number of workers a firm can hire.

specific setup cost of  $z(i)$  units of unmeasured effort and time upon entering the market where  $i$  indexes the firm.<sup>7</sup> Without loss of generality, we order the potential entrants according to their setup costs in an ascending order, i.e.  $z'(i) > 0$ . Moreover, we assume  $z(0) = 0$  and  $z'' > 0$ .<sup>8</sup>

The sequence of events is as follows. Upon entering each firm pays the setup costs. Next, it hires capital at a given rental rate  $r$  (which includes depreciation allowance). Third, firms are matched with workers. For parsimony, we assume that every worker is matched with a firm, but not vice et versa. We denote by  $p$  the fraction of firms that are matched. This fraction is taken as given from the point of view of potential entrants, but is determined in the aggregate by the measure of firms that enter.

Concerning the modelling strategy, three remarks are in order. First, we introduce increasing setup costs in order to limit entry. Specifically, the marginal entrant will have zero profits. Second, the sequence of events is different from the usual one in the MP literature. In particular, firms invest in capital before they hire labor which generates a hold up problem at the later wage negotiations stage.<sup>9</sup> Third, the matching technology reconciles the zero profit condition of the marginal entrant with the deviation from the competitive wage determination mechanism presented below. More general matching technologies, where not all workers are matched, would also work, as long as  $p$  remains decreasing in the measure of entering firms.

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<sup>7</sup>Our notion of unmeasured effort can be interpreted in the spirit of McGrattan and Prescott (2005), who have recently suggested that there is a sizable amount of "sweat equity" that does not get measured by NIPA.

<sup>8</sup>The essential feature is that setup costs increase with the number of firms.

<sup>9</sup>A notable exception is Acemoglu and Shimer (1999).

We solve the model by backward induction. We start in the third stage. In the case of a match between a worker and a firm, the two parties bargain over the quasi-rent. We assume that the outcome of the bargaining process can be represented by the Nash-bargaining solution where the respective disagreement points are zero. We parametrize the bargaining power of labor by  $\alpha$ , which we assume to be exogenously given.<sup>10</sup> Since at the bargaining stage the capital/labor ratio,  $k$ , is given, the bargaining problem between a firm and its worker amounts to solving

$$\max_w [f(k) - w]^{1-\alpha} w^\alpha. \quad (2.1)$$

Accordingly, the worker is paid a fixed proportion of output:

$$w = \alpha f(k). \quad (2.2)$$

Moving back to the second stage of the game, firms that have entered decide on the capital/labor ratio anticipating the expected outcome of the matching and bargaining stage. The problem of the firm is:

$$\pi = \max_k p(1 - \alpha)f(k) - rk, \quad (2.3)$$

implying the following decision rule for capital:

$$p(1 - \alpha)f'(k) - r = 0. \quad (2.4)$$

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<sup>10</sup>This parameter is thought to represent *inter alia*, legal institutions governing wage setting and work conditions.

In the first stage firm  $i$  enters if

$$\pi - z(i) \geq 0 \tag{2.5}$$

**Definition 2.1.** An equilibrium is a vector  $(p^*, i^*, k^*, w^*, \pi^*)$  satisfying the following conditions:

*At the firm level:*

- i) The solution to the Nash-bargaining problem from (2.2).
- ii) The capital optimization condition (2.4).
- iii) The entry decision from (2.3) and (2.5).

*At the aggregate level:*

- iv) Zero profit determining the marginal entrant,  $\iota^*$ .
- v) The probability of a match  $p^* = 1/i^*$ .

Given the Inada condition, it is easily verified that an equilibrium exists as long as  $z(1)$  is not too large.<sup>11</sup> Thus, we have shown that the above game is consistent with profit maximizing, zero profit condition for the marginal entrant, and the deviation from the competitive wage determination mechanism described above.

### 3. Factor shares and technology parameters

In our model, the labor share is simply  $\alpha$ . In particular, it does not reflect any technological parameters, thus, no contradiction between implied factor shares

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<sup>11</sup>This condition simply guarantee that at  $i = 1$  firms are still attracted to the market. Otherwise, the model is not consistent with the assumption that every worker is matched.

and technology can arise. Nevertheless, we show next that it is still possible to elicit technology parameters from standard macro-data.

To do so, we proceed by using the Cobb-Douglas specification so that  $f(k) = k^\gamma$ . Using (2.2) and (2.4), we get:

$$\gamma = \frac{r \frac{k}{y}}{p \left[ 1 - \frac{w}{y} \right]} \quad (3.1)$$

where  $y = f(k)$ .

We apply this result to the corporate sector in the US. This is done in order to isolate capital from other types of assets such as residential and government capital. We use the computations of McGrattan and Prescott (2000). They report (in table 1) that the corporate sector was responsible in the period 1990-1999 for 59.2% of GNP. The capital GNP ratio of that sector amounted to 0.821. Accordingly, the capital output ratio in the corporate sector amounts to 1.38. Next, we compute  $r$  using the capital consumption (6.9% of GDP) and net interest payments (1.5% of GDP), thus,  $r = 14.2\%$ . We compute  $\alpha$  from labor compensation (37.8% of GDP) to be 0.63. Finally, using an average vacancy rate of 2.7% from the BLS, we set  $p = 0.973$ .<sup>12</sup> Altogether, we obtain  $\gamma = 0.54$ .

Furthermore, the model allows us to compute for profit in the economy. To do so, we interpret the  $iz(i)$  as the sum of profit. Specifically, we assume that the entry costs,  $z(\cdot)$ , are not reflected by the NIPA and therefore all reported profits are gross of the entry cost. Accordingly, total profits in the economy amount to:

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<sup>12</sup>For the data, see Job Openings and Labor Turnover Survey at <http://data.bls.gov/cgi-bin/surveymost>.

$$\int_0^i \pi(z) dz = (1 - \alpha)f(k) - irk \quad (3.2)$$

Simple manipulations show that the share of profit out of output is  $(1 - \alpha)(1 - \gamma)$ . Using the above numbers this share amounts to 17%. In McGrattan and Prescott, (2000), the corporate profit to GNP ratio is 0.093. Therefore, in the data we get the share of profit out of corporate output to be 15.7%.<sup>13</sup>

#### 4. Concluding remarks

Clearly a higher value of  $\gamma$  reduces the contribution to growth attributed to total factor productivity. As a matter of fact, high values of  $\gamma$  have already been invoked by Romer (1986) and Bental and Peled (1996) in order to explain growth phenomena. We have shown that introducing bargaining into a standard macroeconomic model may help justify such estimates.

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<sup>13</sup>Observe that we did not take indirect business taxes of 5.7% of GNP into account.



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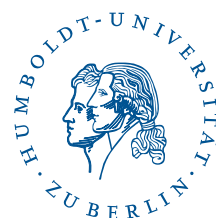
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