SFB 649 Discussion Paper 2006-068

Integral Options in Models with Jumps

Pavel V. Gapeev*

* Weierstrass Institute for Applied Analysis and Stochastics,
Berlin, Germany
and
Russian Academy of Sciences, Institute of Control Sciences,
Moscow, Russia

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin

SFB 649 ECONOMIC RISK BERLIN

Dedicated to the memory of Heiner Zieschang.

Integral options in models with jumps*

Pavel V. Gapeev

We present an explicit solution to the formulated in [17] optimal stopping problem for a geometric compound Poisson process with exponential jumps. The method of proof is based on reducing the initial problem to an integro-differential free-boundary problem where the smooth fit may break down and then be replaced by the continuous fit. The result can be interpreted as pricing perpetual integral options in a model with jumps.

1. Introduction

The main aim of this paper is to present an explicit solution to the optimal stopping problem (2.3) for the process S defined in (2.1)-(2.2). This problem is related to the option pricing theory in mathematical insurance, where the process S can describe the risk process of an insurance company (see, e.g., [12] and [30; Chapter I, Section 3c]). In that case, the value (2.3) can be formally interpreted as a *fair price* of a *perpetual integral option* of American type in a jump market model.

It is known that the change-of-measure theorem allows to reduce the dimension of optimal stopping problems and thereby helps to derive explicit solutions in some particular cases. In the article [27], by means of introducing the so-called *dual* martingale measure, the Russian option problem was reduced to an optimal stopping problem for a one-dimensional Markov reflected diffusion process. By using similar arguments, the perpetual integral option problem in [17] and the early exercise Asian option problem in [26] were reduced to optimal stopping problems for the one-dimensional Markov process called Shiryaev's process, which appears by solving 'disorder' problems (see, e.g., [28]-[29], [14], [22] or [9]). These problems were solved by reducing them to the corresponding free-boundary problems for differential operators and applying the smooth-fit condition. Following the same methodology, in the present paper we solve the problem (2.3) being a discounted optimal stopping problem for an integral of a jump process under some relationships on the parameters of the process S defined in (2.1)-(2.2). We consider the both cases when the process S can have positive or negative jumps, and aiming at closed form expressions, we let the jumps be exponentially distributed. Some other optimal stopping problems for jump processes related to financial and insurance mathematics were earlier considered in the articles [10], [19]-[21], [15]-[16], [3]-[4], and [7]-[8].

---

* This research was supported by Deutsche Forschungsgemeinschaft through the SFB 649 Economic Risk.

Mathematics Subject Classification 2000. Primary 60G40, 34K10, 91B70. Secondary 60J60, 60J75, 91B28.

Key words and phrases: Jump process, stochastic differential equation, optimal stopping problem, integral American option, compound Poisson process, Shiryaev's process, Girsanov's theorem, Itô's formula, integro-differential free-boundary problem, smooth and continuous fit, hypergeometric functions.

The paper is organized as follows. In Section 2, using change-of-measure arguments, for the initial problem (2.3) we construct the equivalent optimal stopping problem (2.10), where the process X defined in (2.7) is an analogue of Shiryaev's process for the jump model (2.1)-(2.2). Analyzing the sample-path behavior of the process X, we give expressions for the optimal stopping boundary under some relationships on the parameters of the model. In Section 3, we formulate the corresponding integro-differential free-boundary problem for the infinitesimal operator of the process X and derive the solution, which is expressed by Gauss' and Kummer's hypergeometric functions and thus admits a representation in closed form. In Section 4, we verify that the solution of the free-boundary problem turns out to be a solution of the initial optimal stopping problem and comment the structure of the solution under different relationships on the parameters of the model.

2. Formulation of the problem

In this section we introduce the setting and notation of the optimal stopping problem which is related to the pricing integral option.

2.1. For a precise formulation of the problem let us consider a probability space $(\Omega, \mathcal{F}, P)$ with a jump process $J = (J_t)_{t \ge 0}$ defined by $J_t = \sum_{i=1}^{N_t} Y_i$, where $N = (N_t)_{t\ge 0}$ is a Poisson process of the intensity $\lambda > 0$, and $(Y_i)_{i \in \mathbb{N}}$ is a sequence of independent random variables exponentially distributed with parameter 1 (N and $(Y_i)_{i\in\mathbb{N}}$ are supposed to be independent). It is assumed that the risk process of an insurance company is described by the process $S = (S_t)_{t\ge 0}$ defined by:

$$S_t = s\, \exp\Big(\big(r - \lambda\theta/(1-\theta)\big)\, t + \theta\, J_t \Big) \tag{2.1}$$

and hence solving the stochastic differential equation:

$$dS_t = rS_{t-}\, dt + S_{t-} \int_0^\infty \Big(e^{\theta y} - 1 \Big) \left(\mu(dt, dy) - \nu(dt, dy) \right) \quad (S_0 = s) \tag{2.2}$$

where $\mu(dt, dy)$ is the measure of jumps of the process J with the compensator $\nu(dt, dy) = \lambda dt I(y > 0) e^{-y} dy$, and $s > 0$ is given and fixed. We recall that here $r > 0$ is the interest rate of a banking account and $\theta < 1$, $\theta \neq 0$ is the volatility coefficient of the jump part of the process S. Note that the assumption $\theta < 1$ guarantees that the jumps of S are integrable and that is not a restriction. The main purpose of the present paper is to find an *explicit solution* to the *integral option* problem which consists of computing the value:

$$V_* = \sup_\tau E \left[e^{-(r+\delta)\tau} \Big(\int_0^\tau S_u\, du + x \Big) \right] \tag{2.3}$$

for some $\delta > 0$ and $x \ge 0$ given and fixed, where the supremum is taken over all finite stopping times τ of the process S (i.e., stopping times with respect to $(\mathcal{F}_t^S)_{t\ge 0}$ denoting the natural filtration of S: $\mathcal{F}_t^S = \sigma\{S_u \,|\, 0 \le u \le t\}$, $t \ge 0$), as well as to determine the finite stopping time τ_* at which the supremum in (2.3) is attained. From the structure of the reward in (2.3) it follows that without loss of generality we can further assume that $s = 1$. Since the initial measure P is a martingale measure for the given jump model (see, e.g., [30; Chapter VII, Section 3g]), the value (2.3) may be interpreted as a rational (fair) price of the integral option in the given model. For the case when S was a geometric Brownian motion the problem (2.3) was formulated and explicitly solved in the paper [17].

2.2. By means of the same arguments as in [31; Section 7], [27; Section 2] and [17; Section 1], it can be shown that there exists a measure $\widetilde{P}$ being locally equivalent to P with respect to the filtration $(\mathcal{F}_t^S)_{t\ge0}$ and such that its density process is given by:

$$\frac{d\widetilde{P}|\mathcal{F}_t^S}{dP|\mathcal{F}_t^S} = \exp\Big(\theta\, J_t - \big(\lambda\theta/(1-\theta)\big)\, t\Big) \tag{2.4}$$

for all $t \ge 0$. In this case, by virtue of Girsanov's theorem for semimartingales (see e.g. [13; Chapter III, Theorem 5.34] or [18; Chapter IV, Theorem 5.3]), we may conclude that the process $J = (J_t)_{t\ge0}$ has the compensator $\widetilde{\nu}(dt, dy) = \lambda dt I(y > 0) e^{-(1-\theta)y} dy$ under the measure $\widetilde{P}$.

Observe that, by using the explicit expression (2.1) as well as the assumption $s = 1$, from (2.4) we obtain:

$$\frac{d\widetilde{P}|\mathcal{F}_\tau^S}{dP|\mathcal{F}_\tau^S} = e^{-r\tau}\, S_\tau \tag{2.5}$$

for all finite stopping times τ of S. It therefore follows that the value (2.3) takes the form:

$$V_* = \sup_\tau \widetilde{E}\big[e^{-\delta\tau}\, X_\tau\big] \tag{2.6}$$

where the process $X = (X_t)_{t\ge0}$ is given by:

$$X_t = \frac{1}{S_t}\Big(\int_0^t S_u\, du + x\Big) \tag{2.7}$$

and hence, by virtue of Itô's formula for semimartingales (see e.g. [13; Chapter I, Theorem 4.57] or [18; Chapter II, Theorem 6.1]), it solves the stochastic differential equation:

$$dX_t = (1 - rX_{t-})\, dt - X_{t-}\int_0^\infty \Big(1 - e^{-\theta y}\Big)\, (\mu(dt, dy) - \widetilde{\nu}(dt, dy)) \quad (X_0 = x) \tag{2.8}$$

with $\widetilde{\nu}(dt, dy)$ defined above. It can be easily verified that X is a time-homogeneous (strong) Markov process under $\widetilde{P}$ with respect to its natural filtration which clearly coincides with $(\mathcal{F}_t^S)_{t\ge0}$. Therefore, the supremum in (2.6) can equivalently be taken over all finite stopping times of the process X playing the role of sufficient statistic in the given optimal stopping problem. We also note that if $\theta < 1$, $\theta \neq 0$ and, in addition, $0 < \lambda\theta/(1-\theta) < r$ holds, then

$$\widehat{B} = 1\Big/\Big(r - \frac{\lambda\theta}{1-\theta}\Big) \tag{2.9}$$

turns out to be a *singularity point* of equation (2.8) in the sense that the drift rate of the continuous part of the process X is positive on the interval $[0, \widehat{B})$, negative on $(\widehat{B}, \infty)$, and equal to zero at the point $\widehat{B}$.

2.3. In order to solve the problem (2.6), let us consider the following optimal stopping problem for the Markov process X given by:

$$V_*(x) = \sup_{\tau} \widetilde{E}_x \big[e^{-\rho \tau} X_\tau \big] \tag{2.10}$$

where $\widetilde{P}_x$ is a probability measure under which the process X defined in (2.7)-(2.8) starts at $x \geq 0$, and the supremum in (2.10) is taken over all finite stopping times τ of X. We will search for an optimal stopping time in the problem (2.10) of the following form:

$$\tau_* = \inf\{ t \geq 0 \,|\, X_t \geq B_* \} \tag{2.11}$$

where B_* is the smallest number from $x \geq 0$ such that $V_*(x) = x$. The point B_* is called an *optimal stopping boundary.* Observe that, by applying Itô's formula to $e^{-\delta t} X_t$ and by using the equation (2.8), it follows that:

$$e^{-\delta t} X_t = x + \int_0^t e^{-\delta u} \, (1 - (r + \delta) X_{u-}) \, du + \widetilde{N}_t \tag{2.12}$$

where $(\widetilde{N}_t)_{t \geq 0}$ is a martingale under the measure $\widetilde{P}_x$ with respect to $(\mathcal{F}_t^S)_{t \geq 0}$. Hence, by the optional sampling theorem (see, e.g., [13; Chapter I, Theorem 1.39]), from (2.12) together with (2.8) we obtain that $\widetilde{E}_x[\widetilde{N}_\tau] = 0$, and thus the equality:

$$\widetilde{E}_x \big[e^{-\delta \tau} X_\tau \big] = x + \widetilde{E}_x \left[\int_0^\tau e^{-\delta u} \, (1 - (r + \delta) X_{u-}) \, du \right] \tag{2.13}$$

holds for any finite stopping time τ. It is seen from (2.13) that one should not stop the process X in the interval $[0, \overline{B})$ with

$$\overline{B} = \frac{1}{r + \delta} \tag{2.14}$$

being a *lower estimation* for the optimal stopping boundary B_* in the sense that $0 < \overline{B} \leq B_*$.

2.4. By using the schema of arguments from [24] and [7] and by analyzing the sample path behavior of the process X, let us now make some conclusions on the optimal stopping boundary B_* under several relationships on the parameters of the model.

Remark 2.1. Observe that if $\theta < 0$ then the process X can have only positive jumps, it can leave $[0, \widehat{B})$ only by jumping and fluctuating in $(\widehat{B}, \infty)$ cannot enter $[0, \widehat{B})$. If X gets into $\widehat{B}$, then it is trapped there until the next jump of J occurs. Moreover, if X is located in $[0, \widehat{B})$ or in $(\widehat{B}, \infty)$, then under the absence of jumps of J the process X will never reach $\widehat{B}$, because while it approaches to $\widehat{B}$ its local drift decreases to zero at the same time with linear order. Hence, if $0 < -\lambda\theta/(1 - \theta) \leq \delta$ also holds, then we have $\overline{B} \leq \widehat{B}$. Recalling that the process X is monotone increasing on $[0, \widehat{B})$, from the representation (2.13) together with (2.14) we may therefore conclude that one should not stop X on $[0, \overline{B})$, but one should stop it immediately after passing through $\overline{B}$, because after leaving $[0, \overline{B})$ the process X never returns back. In other words, in this case for the optimal stopping boundary we have $B_* = \overline{B}$.

Remark 2.2. Note that if $0 < \theta < 1$ and the condition $0 < \lambda\theta/(1 - \theta) < r$ holds, then the process X can have only negative jumps, it is monotone decreasing on $(\widehat{B}, \infty)$, and by virtue of the structure of the value function (2.10), it follows that one should not stop X on $(\widehat{B}, \infty)$. From the expression (2.13) it therefore follows that for the boundary B_* we should have $\overline{B} \le B_* < \widehat{B}$, because otherwise it would not be optimal.

3. Solution of the free-boundary problem

In this section we derive a solution of the free-boundary problem associated with the initial optimal stopping problem.

3.1. By means of standard arguments it is shown that the infinitesimal operator $\mathbb{L}$ of the process $X = (X_t)_{t \ge 0}$ acts on an arbitrary function F from the class C^1 on $[0, \infty)$ according to the rule:

$$(\mathbb{L}F)(x) = (1 - (r + \zeta)x)F'(x) + \int_0^\infty \Big(F\big(xe^{-\theta y}\big) - F(x) \Big) \, \lambda e^{-(1-\theta)y} \, dy \tag{3.1}$$

for all $x \ge 0$ with $\zeta = -\lambda\theta/(1-\theta)$. In order to find explicit expressions for the unknown value function $V_*(x)$ from (2.10) and the boundary B_* from (2.11), using results of the general theory of optimal stopping problems for Markov processes (see, e.g., [11], [29; Chapter III, Section 8] and [25]), we can formulate the following *integro-differential free-boundary problem*:

$$(\mathbb{L}V)(x) = \delta V(x) \quad \text{for} \quad 0 < x < B \tag{3.2}$$

$$V(B-) = B \quad (\textit{continuous fit}) \tag{3.3}$$

$$V(x) = x \quad \text{for} \quad x > B \tag{3.4}$$

$$V(x) > x \quad \text{for} \quad 0 \le x < B \tag{3.5}$$

for some $B \ge \overline{B}$, where (3.3) plays the role of instantaneous-stopping condition. Note that by virtue of the superharmonic characterization of the value function (see [6] and [29]) it follows that $V_*(x)$ is the smallest function satisfying the conditions (3.2)-(3.5). Moreover, we further assume that the condition:

$$V'(B-) = 1 \quad (\textit{smooth fit}) \quad \text{if} \quad 0 < \theta < 1 \quad \text{and} \quad r + \zeta \ge 0 \tag{3.6}$$

is satisfied for $B \ge \overline{B}$ with $\zeta = -\lambda\theta/(1-\theta)$. The latter can be explained by the fact that according to Remark 2.2, leaving the continuation region $[0, B_*)$ the process X can pass through the boundary B_* continuously. This property was earlier observed and explained in [23; Section 2] and [24] by solving some other optimal stopping problems for jump processes (see also [2] for necessary and sufficient conditions for the occurrence of smooth-fit condition and references to the related literature and [25] for an extensive overview).

3.2. By means of straightforward calculations we reduce the equation (3.2) to the form:

$$(1 - (r + \zeta)x)V'(x) + (1 - \alpha)\lambda x^\alpha G(x) = \left(\delta - \frac{\lambda(1-\alpha)}{\alpha} \right) V(x) \tag{3.7}$$

with $\alpha = 1 - 1/\theta$ and $\zeta = -\lambda\theta/(1-\theta)$, where taking into account conditions (3.3)-(3.4) we set:

$$G(x) = -\int_x^B V(z)\,\frac{dz}{z^{\alpha+1}} + \frac{B^{1-\alpha}}{1-\alpha} \quad \text{if} \quad \alpha = 1 - 1/\theta > 1 \tag{3.8}$$

$$G(x) = \int_0^x V(z)\,\frac{dz}{z^{\alpha+1}} \quad \text{if} \quad \alpha = 1 - 1/\theta < 0 \tag{3.9}$$

for all $0 < x < B$. Then, from (3.7) and (3.8)-(3.9) it follows that the function $G(x)$ solves the following (second-order) ordinary differential equation:

$$\begin{aligned} & x(1-(r+\zeta)x)G''(x) \\ & + \left[(\alpha+1)(1-(r+\zeta)x) - \left(\delta - \frac{\lambda(1-\alpha)}{\alpha}\right)x\right] G'(\psi) + (1-\alpha)\lambda G(x) = 0 \end{aligned} \tag{3.10}$$

for $0 < x < B$. Observe that equation (3.7) as well as (3.10) has the singularity point $\widehat{B} \equiv 1/(r+\zeta)$ whenever $r + \zeta > 0$.

3.3. Let us now assume that $r + \zeta > 0$ with $\zeta = -\lambda\theta/(1-\theta)$ holds. In this case, (3.10) is a Gauss' hypergeometric equation, which has the general solution:

$$G(x) = C_1\,A_1(x) + C_2\,x^{-\alpha}\,A_2(x) \tag{3.11}$$

where C_1 and C_2 are some arbitrary constants and the functions $A_1(x)$ and $A_2(x)$ are defined by:

$$A_1(x) = F\Big(\gamma_1, \gamma_2; \alpha+1; (r+\zeta)x\Big) \tag{3.12}$$

$$A_2(x) = F\Big(\gamma_1 - \alpha, \gamma_2 - \alpha; 1-\alpha; (r+\zeta)x\Big) \tag{3.13}$$

for $0 \le x < \widehat{B}$, and γ_i is given by:

$$\gamma_i = \left(\frac{\alpha(\delta+\lambda)-1}{2\alpha(r+\zeta)} + \frac{\alpha}{2}\right) + (-1)^i \sqrt{\left(\frac{\alpha(\delta+\lambda)-1}{2\alpha(r+\zeta)} + \frac{\alpha}{2}\right)^2 + \frac{\lambda(1-\alpha)}{r+\zeta}} \tag{3.14}$$

with $\alpha = 1 - 1/\theta$ and $\zeta = -\lambda\theta/(1-\theta)$ for $i = 1, 2$. Here $F(a, b; c; x)$ denotes Gauss' hypergeometric function, which admits the integral representation:

$$F(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tx)^{-a}\,dt \tag{3.15}$$

for $c > b > 0$ and has the series expansion:

$$F(a, b; c; x) = 1 + \sum_{k=1}^{\infty} \frac{(a)_k (b)_k}{(c)_k}\,\frac{x^k}{k!} \tag{3.16}$$

for $c \neq 0, -1, -2, \ldots$ and $(c)_k = c(c+1)\cdots(c+k-1)$, $k \in \mathbb{N}$, where Γ denotes Euler's Gamma function and the series converges under all $|x| < 1$ (see, e.g., [1; Chapter XV] and [5;

Chapter II]). Therefore, differentiating both sides of the formulas (3.8)-(3.9), by using (3.11) we obtain that in this case the integro-differential equation (3.7) has the general solution:

$$V(x) = C_1\, x^{\alpha+1}\, A_1'(x) + C_2\, [x\, A_2'(x) - \alpha\, A_2(x)] \tag{3.17}$$

for $0 \le x < \widehat{B}$. Hence, applying conditions (3.8), (3.3) and (3.6) to the functions (3.11) and (3.17), respectively, we get that the following equalities:

$$C_1\, B^{\alpha}\, A_1(B) + C_2\, A_2(B) = \frac{B}{1-\alpha} \tag{3.18}$$

$$C_1\, B^{\alpha+1}\, A_1'(B) + C_2\, [B\, A_2'(B) - \alpha\, A_2(B)] = B \tag{3.19}$$

$$C_1\, B^{\alpha}\, [B\, A_1''(B) + (\alpha+1)\, A_1'(B)] + C_2\, [B\, A_2''(B) + (1-\alpha)\, A_2'(B)] = 1 \tag{3.20}$$

hold for some $B \ge \overline{B}$, where condition (3.20) is satisfied when $\alpha = 1 - 1/\theta < 0$.

Note that if, in addition, $\alpha = 1 - 1/\theta > 1$ and $0 < -\lambda\theta/(1-\theta) \le \delta$ holds, then by Remark 2.1 we may conclude that for the optimal stopping boundary we have $B_* = \overline{B} \equiv 1/(r+\delta)$. Hence, solving the system (3.18)-(3.19), by means of straightforward calculations we obtain that the solution of the system (3.2)-(3.4) is given by:

$$\begin{aligned} V(x;B_*) = {} & \frac{B_*^{2-\alpha} A_2'(B_*) - B_*^{1-\alpha} A_2(B_*)}{(1-\alpha) D(B_*)}\, x^{1+\alpha}\, A_1'(x) \\ & + \frac{(1-\alpha) B_* A_1(B_*) - B_*^2 A_1'(B_*)}{(1-\alpha) D(B_*)}\, [x\, A_2'(x) - \alpha\, A_2(x)] \end{aligned} \tag{3.21}$$

where the function $D(x)$ is defined by:

$$D(x) = x\, A_1(x)\, A_2'(x) - x\, A_1'(x)\, A_2(x) - \alpha\, A_1(x)\, A_2(x) \tag{3.22}$$

for all $0 \le x < B_* < \widehat{B}$, and under $B_* = \widehat{B}$ in (3.21) we may set $V(x;B_*) = V(x;B_*-)$. Here the functions $A_1'(x)$ and $A_2'(x)$ are given by:

$$A_1'(x) = \frac{\gamma_1\gamma_2(r+\zeta)}{\alpha+1}\, F\Big(\gamma_1+1, \gamma_2+1; \alpha+2; (r+\zeta)x\Big), \tag{3.23}$$

$$A_2'(x) = \frac{(\gamma_1-\alpha)(\gamma_2-\alpha)(r+\zeta)}{1-\alpha}\, F\Big(\gamma_1-\alpha+1, \gamma_2-\alpha+1; 2-\alpha; (r+\zeta)x\Big) \tag{3.24}$$

for $0 \le x < \widehat{B}$.

Observe that if, in addition, $\alpha = 1 - 1/\theta < 0$ holds, then we have $C_1 = 0$ in (3.11) and (3.17), since otherwise, from expression (3.7) it would follow that $V'(x) \to \pm\infty$ under $x \downarrow 0$ that should be excluded by virtue of the easily proved fact that the value function $V_*(x)$ from (2.10) is convex and increasing on the interval $[0,\infty)$. Thus, solving the system (3.19)-(3.20) with $C_1 = 0$, by using straightforward calculations we obtain that the solution of the system (3.2)-(3.4)+(3.6) is given by:

$$V(x;B_*) = B_*\, \frac{xA_2'(x) - \alpha A_2(x)}{B_* A_2'(B_*) - \alpha A_2(B_*)} \tag{3.25}$$

for all $0 \le x < B_* < \widehat{B}$, where the boundary B_* satisfies the equation:

$$B \, \frac{B A_2''(B) + (1-\alpha) A_2'(B)}{B A_2'(B) - \alpha A_2(B)} = 1. \tag{3.26}$$

Here the function $A_2''(x)$ is given by:

$$\begin{aligned} A_2''(x) = {} & \frac{(\gamma_1-\alpha)(\gamma_1-\alpha+1)(\gamma_2-\alpha)(\gamma_2-\alpha+1)(r+\zeta)^2}{(1-\alpha)(2-\alpha)} \\ & \times F\Big(\gamma_1-\alpha+2, \gamma_2-\alpha+2; 3-\alpha; (r+\zeta)x\Big) \end{aligned} \tag{3.27}$$

for $0 \le x < \widehat{B}$. By virtue of the properties of Gauss' hypergeometric function $F(a,b;c;x)$ defined in (3.15)-(3.16), after some transformations we obtain that the left-hand side of the equality (3.26) is strictly increasing in B on the interval $[0, \widehat{B})$, tends to zero under $B \downarrow 0$, and tends to infinity under $B \uparrow \widehat{B}$. We may therefore conclude that the equation (3.26) admits the unique solution B_* on $[0, \widehat{B})$.

3.4. Let us finally assume that $\alpha = 1 - 1/\theta < 0$ and $r + \zeta = 0$ with $\zeta = -\lambda\theta/(1-\theta)$ holds. In this case, equation (3.10) turns out to be a confluent hypergeometric equation, which has the general solution:

$$G(x) = C_1 \, H_1(x) + C_2 \, H_2(x) \tag{3.28}$$

where C_1 and C_2 are some arbitrary constants and the functions $H_1(x)$ and $H_2(x)$ are defined by:

$$H_1(x) = U\Big(-\lambda(1-\alpha)/\eta, \alpha+1; \eta x\Big) \tag{3.29}$$

$$H_2(x) = M\Big(\lambda(1-\alpha)/\eta, -\alpha-1; \eta x\Big) \tag{3.30}$$

for $x \ge 0$ with $\alpha = 1 - 1/\theta$ and $\eta = \delta + \lambda + \lambda\theta/(1-\theta)$. Here $U(a,b;x)$ is the confluent hypergeometric function, which admits the integral representation:

$$U(a,b;x) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-xt} \, t^{a-1} (1+t)^{b-a-1} \, dt \tag{3.31}$$

for $a > 0$, and $M(a,b;x)$ is Kummer's confluent hypergeometric function, which admits the integral representation:

$$M(a,b;x) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 e^{xt} \, t^{a-1} (1-t)^{b-a-1} \, dt \tag{3.32}$$

for $b > a > 0$ and has the series expansion:

$$M(a,b;x) = 1 + \sum_{k=1}^{\infty} \frac{(a)_k}{(b)_k} \, \frac{x^k}{k!} \tag{3.33}$$

for $b \ne 0, -1, -2, \ldots$ and $(b)_k = b(b+1) \cdots (b+k-1)$, $k \in \mathbb{N}$, where the series converges under all $x > 0$ (see, e.g., [1; Chapter XIII] and [5; Chapter VI] with a different parametrization).

Therefore, differentiating both sides of the formula (3.9), by using (3.28) we get that in this case $\widehat{B} = \infty$ and the integro-differential equation (3.7) has the general solution:

$$V(x) = C_1\, x^{\alpha+1}\, H_1'(x) + C_2\, x^{\alpha+1}\, H_2'(x) \tag{3.34}$$

for $x \geq 0$. Hence, applying conditions (3.3) and (3.6) to the function (3.34), we get that the following equalities:

$$C_1\, B^{\alpha+1}\, H_1'(B) + C_2\, B^{\alpha+1}\, H_2'(B) = B \tag{3.35}$$

$$C_1\, B^{\alpha}\, [(\alpha+1)H_1'(B) + B\, H_1''(B)] + C_2\, B^{\alpha}\, [(\alpha+1)H_2'(B) + B\, H_2''(B)] = 1 \tag{3.36}$$

hold for some $B \geq \overline{B}$.

It thus follows that in (3.28) and (3.34) we have $C_1 = 0$, since otherwise $V(x) \to \pm\infty$ as $x \downarrow 0$, which should be excluded due to the obvious fact that the value function (2.10) is bounded under $x \downarrow 0$. Therefore, solving the system (3.35)-(3.36) with $C_1 = 0$, by using straightforward calculations we obtain that in this case the solution of the system (3.2)-(3.4)+(3.6) is given by:

$$V(x; B_*) = B_*\, \frac{x^{\alpha+1} H_2'(x)}{B_*^{\alpha+1} H_2'(B_*)} \tag{3.37}$$

for all $0 \leq x < B_*$, where the boundary B_* satisfies the equation:

$$B\, \frac{H_2''(B)}{H_2'(B)} = -\alpha. \tag{3.38}$$

Here the functions $H_2'(x)$ and $H_2''(x)$ are given by:

$$H_2'(x) = -\frac{\lambda(1-\alpha)}{\alpha+1}\, M\Big(1 - \lambda(1-\alpha)/\eta, -\alpha; \eta x\Big) \tag{3.39}$$

$$H_2''(x) = \frac{\lambda(1-\alpha)(\eta + \lambda - \lambda\alpha)}{\alpha(\alpha+1)}\, M\Big(2 - \lambda(1-\alpha)/\eta, 1-\alpha; \eta x\Big) \tag{3.40}$$

for $x \geq 0$. By virtue of the properties of Kummer's confluent hypergeometric function $M(a,b;x)$ defined in (3.32)-(3.33), after some transformations we obtain that the left-hand side of the equality (3.38) is strictly increasing in B on the interval $[0,\infty)$, tends to zero under $B \downarrow 0$, and tends to infinity under $B \uparrow \infty$. We may therefore conclude that the equation (3.38) admits the unique solution B_* on $[0,\infty)$.

4. Main result and proof

Taking into account the facts proved above, let us now formulate the main assertion of the paper, which extends the result of the article [17] to the case of some jump processes.

Theorem 4.1. *Suppose that in the model defined in (2.1)-(2.2) we have $r \geq \lambda\theta/(1-\theta)$. Then the value function of the problem (2.10) takes the expression:*

$$V_*(x) = \begin{cases} V(x; B_*), & 0 \leq x < B_*, \\ x, & x \geq B_*, \end{cases} \tag{4.1}$$

and the optimal stopping time τ_* has the structure by (2.11), where the function $V(x; B_*)$ and the optimal stopping boundary B_* are specified as follows:

(i): if $\theta < 0$ and $0 < -\lambda\theta/(1-\theta) < \delta$ then the function $V(x; B_*)$ is given by (3.21), and $B_* = \overline{B} \equiv 1/(r+\delta)$;

(ii): if $\theta < 0$ and $\delta = -\lambda\theta/(1-\theta)$ then the function $V(x; B_*) = V(x; B_*-)$ is also given by (3.21), and $B_* = \overline{B} = \widehat{B} \equiv 1/(r - \lambda\theta/(1-\theta))$;

(iii): if $0 < \theta < 1$ and $0 < \lambda\theta/(1-\theta) < r$ then $V(x; B_*)$ is given by (3.25) and B_* is uniquely determined from the equation (3.26);

(iv): if $0 < \theta < 1$ and $r = \lambda\theta/(1-\theta)$ then $V(x; B_*)$ is given by (3.37) and B_* is uniquely determined from the equation (3.38).

Proof. (i)+(ii) Observe that in this case we have $\overline{B} \le \widehat{B}$. Hence, by Remark 2.1 we get that B_* coincides with $\overline{B}$ from (2.9), and by means of the existence and uniqueness theorem for hypergeometric equations we may conclude that under the assumptions above the value function (2.10) admits the unique representation (4.1) with $V(x; B_*)$ given by (3.21).

(iii)+(iv) Let us show that the function (4.1) coincides with the value function (2.3) and that the stopping time τ_* from (2.11) with the boundary B_* specified above is optimal in (2.11). For this, let us denote by $V(x)$ the right-hand side of the expression (4.1). In this case, by means of straightforward calculations and by construction from the previous section it follows that the function $V(x)$ solves the system (3.2)-(3.4) as well as the condition (3.6) is satisfied. Then, by applying Itô's formula to $e^{-\delta t} V(X_t)$, we obtain:

$$e^{-\delta t} V(X_t) = V(x) + \int_0^t e^{-\delta u} (\mathbb{L}V - \delta V)(X_{u-}) \, du + \widetilde{M}_t \tag{4.2}$$

where the process $(\widetilde{M}_t)_{t \ge 0}$ defined by:

$$\widetilde{M}_t = \int_0^t \int_0^\infty e^{-\delta u} \Big(V\big(X_{u-} e^{-\theta y}\big) - V(X_{u-}) \Big) \left(\mu(du, dy) - \widetilde{\nu}(du, dy) \right) \tag{4.3}$$

is a local martingale under the measure $\widetilde{P}_x$ with respect to $(\mathcal{F}_t^S)_{t \ge 0}$. Observe that the time spent by the process X at the boundary B_* is of Lebesgue measure zero, that allows to extend $(\mathbb{L}V - \delta V)(x)$ arbitrarily to $x = B_*$.

By virtue of the arguments from the previous section we may conclude that $(\mathbb{L}V - \delta V)(x) \le 0$ for all $x > 0$. Moreover, by means of straightforward calculations, it can be shown that the property (3.5) also holds, that together with (3.3)-(3.4) yields $V(x) \ge x$ for all $x \ge 0$. From the expression (4.2) it therefore follows that the inequalities:

$$e^{-\delta \tau} X_\tau \le e^{-\delta \tau} V(X_\tau) \le V(x) + \widetilde{M}_\tau \tag{4.4}$$

hold for any finite stopping time τ of the process X started at $x \ge 0$.

Let $(\sigma_n)_{n \in \mathbb{N}}$ be an arbitrary localizing sequence of stopping times for the process $(\widetilde{M}_t)_{t \ge 0}$. Then, taking in (4.4) expectation with respect to the measure $\widetilde{P}_x$, by means of the optional sampling theorem, we get:

$$\widetilde{E}_x \big[e^{-\delta(\tau \wedge \sigma_n)} X_{\tau \wedge \sigma_n} \big] \le V(x) + \widetilde{E}_x \big[\widetilde{M}_{\tau \wedge \sigma_n} \big] = V(x) \tag{4.5}$$

for all $x \geq 0$. Hence, letting n go to infinity and using Fatou's lemma, we obtain that for any finite stopping time τ the inequalities:

$$\widetilde{E}_x\big[e^{-\delta\tau}\,X_\tau\big] \leq \widetilde{E}_x\big[e^{-\delta\tau}\,V(X_\tau)\big] \leq V(x) \tag{4.6}$$

are satisfied for all $x \geq 0$.

In order to show that the equality in (4.6) is attained at τ_* from (2.11), let us first prove that the property $\widetilde{P}_x[\tau_* < \infty] = 1$ holds. For this, we observe that from (2.8) it follows that the continuous part of the process X in the case (iii) is given by $\widehat{B} - \widehat{B}\exp(-t/\widehat{B})$ and in the case (iv) it is equal to t for all $t \geq 0$. Then, under the absence of jumps, in the case (iii) the process X started at $x < \widehat{B}$ will reach the boundary $\widehat{B} - \varepsilon$ by the time not greater than $\rho(\varepsilon) = -\widehat{B}\log(\varepsilon/\widehat{B})$ for each sufficiently small $\varepsilon > 0$ given and fixed, and in the case (iv) the process X started at $x \geq 0$ will reach the boundary K by the time not greater than K for any $K > 0$ given and fixed. Since from the sample path properties of Poisson processes, by applying the Borel-Cantelli lemma, it follows that the $\widetilde{P}_x$-probability of the event that the time between two jumps of the process N (and thus of J) will never exceed $\rho(\varepsilon)$ in the case (iii) and K in the case (iv) are equal to zero, we may thus conclude that $\widetilde{P}_x[\tau_* < \infty] = 1$.

By virtue of the fact that the function $V(x)$ together with the boundary B_* satisfy the system (3.2)-(3.5), by the structure of the stopping time τ_* in (2.11) and by expression (4.2), it follows that the equality:

$$e^{-\delta(\tau_* \wedge \sigma_n)}\,V(X_{\tau_* \wedge \sigma_n}) = V(x) + \widetilde{M}_{\tau_* \wedge \sigma_n} \tag{4.7}$$

holds. Then, using the expression (4.4), by virtue of the fact that the function $V(x)$ is increasing, we may conclude that the inequalities:

$$-V(x) \leq \widetilde{M}_{\tau_* \wedge \sigma_n} \leq V(B_* \vee x) - V(x) \tag{4.8}$$

are satisfied for all $x \geq 0$, where $(\sigma_n)_{n\in\mathbb{N}}$ is a localizing sequence for $(\widetilde{M}_t)_{t\geq 0}$. Hence, letting n go to infinity in the expression (4.7) and using the conditions (3.3)-(3.4) as well as the proved above properties $V(B_* \vee x) < \infty$ and $\widetilde{P}_x[\tau_* < \infty] = 1$, by means of the Lebesgue dominated convergence theorem we obtain that the equality:

$$\widetilde{E}_x\big[e^{-\delta\tau_*}\,X_{\tau_*}\big] = V(x) \tag{4.9}$$

holds for all $x \geq 0$, which together with (4.6) directly implies the desired assertion. $\square$

Remark 4.1. By means of straightforward calculations, it can be verified that in the conditions of the case (i) of Theorem 4.1 for the function $V(x; B_*)$ from (3.21) we have the equality $V'(B_*-; B_*) = 1$, and by proving the assertions in the cases (iii)-(iv) we have used the equalities (3.26) and (3.38), that means that the smooth-fit condition (3.6) is satisfied. As in [23]-[24] (see also [2] and [25]), this property can be explained by the fact that in the given cases leaving the continuation region $[0, B_*)$ the process X may pass through the boundary B_* continuously.

Remark 4.2. On the other hand, in the conditions of the case (ii) of Theorem 4.1 it can be shown that for the function $V(x; B_*)$ from (3.21) the inequality $V'(B_*-; B_*) < 1$ holds, so that the smooth-fit condition (3.6) breaks down. As in [23]-[24], this property can be explained by the fact that in the given case leaving the continuation region $[0, B_*)$ the process X may pass through B_* only by jumping. According to the results in [2] we may conclude that this property appears because of finite intensity of jumps and exponential distribution of jump sizes of the compound Poisson process J.

Acknowledgments. The author began to work on optimal stopping problems for jump processes during his research visit to the Ruhr University of Bochum. The author is grateful to Heiner Zieschang for the outstanding reception in Bochum and dedicates the paper to his memory.

References

[1] Abramovitz, M. and Stegun, I. A. (1972). *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* National Bureau of Standards. Wiley, New York.

[2] Alili, L. and Kyprianou, A. E. (2005). Some remarks on first passage of Lévy processes, the American put and pasting principles. *Annals of Applied Probability* **15** (2062–2080).

[3] Asmussen, S., Avram, F. and Pistorius, M. (2003). Russian and American put options under exponential phase-type Lévy models. *Stochastic Processes and Applications* **109** (79–111).

[4] Avram, F., Kyprianou, A. E. and Pistorius, M. (2004). *Annals of Applied Probability* **14**(1) (215–238).

[5] Bateman, H. and Erdélyi, A. (1953). *Higher Transcendental Functions.* Mc Graw-Hill, New York.

[6] Dynkin, E. B. (1963). The optimum choice of the instant for stopping a Markov process. *Soviet Math. Dokl.* **4** (627–629).

[7] Gapeev, P. V. (2005). The disorder problem for compound Poisson processes with exponential jumps. *Annals of Applied Probability* **15**(1A) (487-499).

[8] Gapeev, P. V. and Kühn, C. (2005). Perpetual convertible bonds in jump-diffusion models. *Statistics and Decisions* **23**(1) (15-31).

[9] Gapeev, P. V. and Peskir, G. (2006). The Wiener disorder problem with finite horizon. Department of Theoret. Stat., University of Aarhus. *Research Report* **435** (22 pp). To appear in *Stochastic Processes and Applications.*

[10] Gerber, H. U., Michaud, F. and Shiu, E. S. W. (1995). Pricing Russian options with the compound Poisson process. *Transactions of the XXV International Congress of Actuaries* **3**.

[11] Grigelionis, B. I. and Shiryaev, A. N. (1966). On Stefan's problem and optimal stopping rules for Markov processes. *Theory Probab. Appl.* **11** (541–558).

[12] Gummins, J. D. and Geman, H. (1993). An Asian option approach to the valuation of insurance futures contracts. *Review of Futures Markets* **13** (517–557).

[13] Jacod, J. and Shiryaev, A. N. (1987). *Limit Theorems for Stochastic Processes.* Springer, Berlin.

[14] Kolmogorov, A. N., Prokhorov, Yu. V. and Shiryaev, A. N. (1990). Probabilistic-statistical methods of detecting spontaneously occuring effects. *Proc. Steklov Inst. Math.* **182**(1) (1–21).

[15] Kou, S. G. (2002). A jump diffusion model for option pricing. *Management Science* **48** (1086–1101).

[16] Kou, S. G. and Wang, H. (2004). Option pricing under a double exponential jump diffusion model. *Management Science* **50** (1178–1192).

[17] Kramkov, D. O. and Mordecki, E. (1994). Integral opton. *Theory Probab. Appl.* **39**(1) (201–211).

[18] Liptser, R. S. and Shiryaev, A. N. (1989). *Theory of Martingales.* Kluver, Dordrecht.

[19] Mordecki, E. (1999). Optimal stopping for a diffusion with jumps. *Finance and Stochastics* **3** (227–236).

[20] Mordecki, E. (2002). Optimal stopping and perpetual options for Lévy processes. *Finance and Stochastics* **6** (473–493).

[21] Mordecki, E. and Moreira, W. (2001). Russian options for a difussion with negative jumps. *Publicaciones Matemáticas del Uruguay* **9** (37–51).

[22] Peskir, G. (2006). On the fundamental solution of the Kolmogorov-Shiryaev equation. *From Stochastic Calculus to Mathematical Finance.* The Shiryaev Festshrift. Springer, Berlin (535–546).

[23] Peskir, G. and Shiryaev, A. N. (2000). Sequential testing problems for Poisson processes. *Annals of Statistics* **28** (837–859).

[24] Peskir, G. and Shiryaev, A. N. (2002). Solving the Poisson disorder problem. *Advances in Finance and Stochastics.* Essays in Honour of Dieter Sondermann. Sandmann, K. and Schönbucher, P. eds. Springer (295–312).

[25] Peskir, G. and Shiryaev, A. N. (2006). *Optimal Stopping and Free-Boundary Problems.* Bikkhäuser, Basel.

[26] Peskir, G. and Uys, N. (2005). On Asian options of American type. In the Volume *Exotic Options and Advanced Lévy Models.* Wiley, Chichester (217–235).

[27] Shepp, L. A. and Shiryaev, A. N. (1994). A new look at the pricing of Russian options. *Theory Probab. Appl.* **39**(1) (103–119).

[28] Shiryaev, A. N. (1963). About optimal methods in problems of quickest detection. *Theory Probab. Appl.* **8**(1) (26–51).

[29] Shiryaev, A. N. (1978). *Optimal Stopping Rules.* Springer, Berlin.

[30] Shiryaev, A. N. (1999). *Essentials of Stochastic Finance.* World Scientific, Singapore.

[31] Shiryaev, A. N., Kabanov, Y. M., Kramkov, D. O. and Melnikov, A. V. (1994). On the pricing of options of European and American types, II. Continuous time. *Theory Probab. Appl.* **39**(1) (80–129).

Pavel V. Gapeev
Weierstraß Institute
for Applied Analysis and Stochastics (WIAS)
Mohrenstr. 39, D-10117 Berlin, Germany
e-mail: gapeev@wias-berlin.de

(Russian Academy of Sciences
Institute of Control Sciences
Profsoyuznaya Str. 65
117997 Moscow, Russia)

SFB 649 Discussion Paper Series 2006

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

001 "Calibration Risk for Exotic Options" by Kai Detlefsen and Wolfgang K. Härdle, January 2006.
002 "Calibration Design of Implied Volatility Surfaces" by Kai Detlefsen and Wolfgang K. Härdle, January 2006.
003 "On the Appropriateness of Inappropriate VaR Models" by Wolfgang Härdle, Zdeněk Hlávka and Gerhard Stahl, January 2006.
004 "Regional Labor Markets, Network Externalities and Migration: The Case of German Reunification" by Harald Uhlig, January/February 2006.
005 "British Interest Rate Convergence between the US and Europe: A Recursive Cointegration Analysis" by Enzo Weber, January 2006.
006 "A Combined Approach for Segment-Specific Analysis of Market Basket Data" by Yasemin Boztuğ and Thomas Reutterer, January 2006.
007 "Robust utility maximization in a stochastic factor model" by Daniel Hernández–Hernández and Alexander Schied, January 2006.
008 "Economic Growth of Agglomerations and Geographic Concentration of Industries - Evidence for Germany" by Kurt Geppert, Martin Gornig and Axel Werwatz, January 2006.
009 "Institutions, Bargaining Power and Labor Shares" by Benjamin Bental and Dominique Demougin, January 2006.
010 "Common Functional Principal Components" by Michal Benko, Wolfgang Härdle and Alois Kneip, Jauary 2006.
011 "VAR Modeling for Dynamic Semiparametric Factors of Volatility Strings" by Ralf Brüggemann, Wolfgang Härdle, Julius Mungo and Carsten Trenkler, February 2006.
012 "Bootstrapping Systems Cointegration Tests with a Prior Adjustment for Deterministic Terms" by Carsten Trenkler, February 2006.
013 "Penalties and Optimality in Financial Contracts: Taking Stock" by Michel A. Robe, Eva-Maria Steiger and Pierre-Armand Michel, February 2006.
014 "Core Labour Standards and FDI: Friends or Foes? The Case of Child Labour" by Sebastian Braun, February 2006.
015 "Graphical Data Representation in Bankruptcy Analysis" by Wolfgang Härdle, Rouslan Moro and Dorothea Schäfer, February 2006.
016 "Fiscal Policy Effects in the European Union" by Andreas Thams, February 2006.
017 "Estimation with the Nested Logit Model: Specifications and Software Particularities" by Nadja Silberhorn, Yasemin Boztuğ and Lutz Hildebrandt, March 2006.
018 "The Bologna Process: How student mobility affects multi-cultural skills and educational quality" by Lydia Mechtenberg and Roland Strausz, March 2006.
019 "Cheap Talk in the Classroom" by Lydia Mechtenberg, March 2006.
020 "Time Dependent Relative Risk Aversion" by Enzo Giacomini, Michael Handel and Wolfgang Härdle, March 2006.
021 "Finite Sample Properties of Impulse Response Intervals in SVECMs with Long-Run Identifying Restrictions" by Ralf Brüggemann, March 2006.
022 "Barrier Option Hedging under Constraints: A Viscosity Approach" by Imen Bentahar and Bruno Bouchard, March 2006.

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

023 "How Far Are We From The Slippery Slope? The Laffer Curve Revisited" by Mathias Trabandt and Harald Uhlig, April 2006.
024 "e-Learning Statistics – A Selective Review" by Wolfgang Härdle, Sigbert Klinke and Uwe Ziegenhagen, April 2006.
025 "Macroeconomic Regime Switches and Speculative Attacks" by Bartosz Maćkowiak, April 2006.
026 "External Shocks, U.S. Monetary Policy and Macroeconomic Fluctuations in Emerging Markets" by Bartosz Maćkowiak, April 2006.
027 "Institutional Competition, Political Process and Holdup" by Bruno Deffains and Dominique Demougin, April 2006.
028 "Technological Choice under Organizational Diseconomies of Scale" by Dominique Demougin and Anja Schöttner, April 2006.
029 "Tail Conditional Expectation for vector-valued Risks" by Imen Bentahar, April 2006.
030 "Approximate Solutions to Dynamic Models – Linear Methods" by Harald Uhlig, April 2006.
031 "Exploratory Graphics of a Financial Dataset" by Antony Unwin, Martin Theus and Wolfgang Härdle, April 2006.
032 "When did the 2001 recession *really* start?" by Jörg Polzehl, Vladimir Spokoiny and Cătălin Stărică, April 2006.
033 "Varying coefficient GARCH versus local constant volatility modeling. Comparison of the predictive power" by Jörg Polzehl and Vladimir Spokoiny, April 2006.
034 "Spectral calibration of exponential Lévy Models [1]" by Denis Belomestny and Markus Reiß, April 2006.
035 "Spectral calibration of exponential Lévy Models [2]" by Denis Belomestny and Markus Reiß, April 2006.
036 "Spatial aggregation of local likelihood estimates with applications to classification" by Denis Belomestny and Vladimir Spokoiny, April 2006.
037 "A jump-diffusion Libor model and its robust calibration" by Denis Belomestny and John Schoenmakers, April 2006.
038 "Adaptive Simulation Algorithms for Pricing American and Bermudan Options by Local Analysis of Financial Market" by Denis Belomestny and Grigori N. Milstein, April 2006.
039 "Macroeconomic Integration in Asia Pacific: Common Stochastic Trends and Business Cycle Coherence" by Enzo Weber, May 2006.
040 "In Search of Non-Gaussian Components of a High-Dimensional Distribution" by Gilles Blanchard, Motoaki Kawanabe, Masashi Sugiyama, Vladimir Spokoiny and Klaus-Robert Müller, May 2006.
041 "Forward and reverse representations for Markov chains" by Grigori N. Milstein, John G. M. Schoenmakers and Vladimir Spokoiny, May 2006.
042 "Discussion of 'The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions' by Neville Francis and Valerie A. Ramey" by Harald Uhlig, May 2006.
043 "An Iteration Procedure for Solving Integral Equations Related to Optimal Stopping Problems" by Denis Belomestny and Pavel V. Gapeev, May 2006.
044 "East Germany's Wage Gap: A non-parametric decomposition based on establishment characteristics" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.
045 "Firm Specific Wage Spread in Germany - Decomposition of regional differences in inter firm wage dispersion" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
http://sfb649.wiwi.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

046 "Produktdiversifizierung: Haben die ostdeutschen Unternehmen den Anschluss an den Westen geschafft? – Eine vergleichende Analyse mit Mikrodaten der amtlichen Statistik" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.
047 "The Division of Ownership in New Ventures" by Dominique Demougin and Oliver Fabel, June 2006.
048 "The Anglo-German Industrial Productivity Paradox, 1895-1938: A Restatement and a Possible Resolution" by Albrecht Ritschl, May 2006.
049 "The Influence of Information Costs on the Integration of Financial Markets: Northern Europe, 1350-1560" by Oliver Volckart, May 2006.
050 "Robust Econometrics" by Pavel Čížek and Wolfgang Härdle, June 2006.
051 "Regression methods in pricing American and Bermudan options using consumption processes" by Denis Belomestny, Grigori N. Milstein and Vladimir Spokoiny, July 2006.
052 "Forecasting the Term Structure of Variance Swaps" by Kai Detlefsen and Wolfgang Härdle, July 2006.
053 "Governance: Who Controls Matters" by Bruno Deffains and Dominique Demougin, July 2006.
054 "On the Coexistence of Banks and Markets" by Hans Gersbach and Harald Uhlig, August 2006.
055 "Reassessing Intergenerational Mobility in Germany and the United States: The Impact of Differences in Lifecycle Earnings Patterns" by Thorsten Vogel, September 2006.
056 "The Euro and the Transatlantic Capital Market Leadership: A Recursive Cointegration Analysis" by Enzo Weber, September 2006.
057 "Discounted Optimal Stopping for Maxima in Diffusion Models with Finite Horizon" by Pavel V. Gapeev, September 2006.
058 "Perpetual Barrier Options in Jump-Diffusion Models" by Pavel V. Gapeev, September 2006.
059 "Discounted Optimal Stopping for Maxima of some Jump-Diffusion Processes" by Pavel V. Gapeev, September 2006.
060 "On Maximal Inequalities for some Jump Processes" by Pavel V. Gapeev, September 2006.
061 "A Control Approach to Robust Utility Maximization with Logarithmic Utility and Time-Consistent Penalties" by Daniel Hernández–Hernández and Alexander Schied, September 2006.
062 "On the Difficulty to Design Arabic E-learning System in Statistics" by Taleb Ahmad, Wolfgang Härdle and Julius Mungo, September 2006.
063 "Robust Optimization of Consumption with Random Endowment" by Wiebke Wittmüß, September 2006.
064 "Common and Uncommon Sources of Growth in Asia Pacific" by Enzo Weber, September 2006.
065 "Forecasting Euro-Area Variables with German Pre-EMU Data" by Ralf Brüggemann, Helmut Lütkepohl and Massimiliano Marcellino, September 2006.
066 "Pension Systems and the Allocation of Macroeconomic Risk" by Lans Bovenberg and Harald Uhlig, September 2006.
067 "Testing for the Cointegrating Rank of a VAR Process with Level Shift and Trend Break" by Carsten Trenkler, Pentti Saikkonen and Helmut Lütkepohl, September 2006.
068 "Integral Options in Models with Jumps" by Pavel V. Gapeev, September 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
http://sfb649.wiwi.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".