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Are Correlations Constant Over Time? Application of the CC-TRIG$_t$-test to Return Series from Different Asset Classes.

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ABSTRACT

A new test for constant correlation is proposed. Based on the bivariate Student-t distribution, this test is derived as Lagrange multiplier (LM) test. Whereas most of the traditional tests (e.g. Jennrich, 1970, Tang, 1995 and Goetzmann, Li & Rouwenhorst, 2005) specify the unknown correlations as piecewise constant, our model-setup for the correlation coefficient is based on trigonometric functions. Applying this test to assets from different financial markets (stocks, exchange rates, metals) there is empirical evidence that many of the correlations vary over time.

1. INTRODUCTION

Realistic models for the dependence structure of financial returns is one of the current issues in financial statistics. Even though there are more powerful measures of dependence (e.g. Drouet-Mari & Kotz, 2001), the focus of this work is solely on correlations as underlying dependence concept. This means that linear dependence is entirely appropriate which in turn is justified if the joint distribution of the data is multivariate normal, or more generally, multivariate elliptical. Indeed, empirical investigations give reason to assume that the joint distributions of many asset returns are frequently found to being multivariate elliptical (see, e.g. Zhou, 1993).

Agreeing on correlation as adequate dependence measure, it might next be of particular interest whether correlations are constant over time. Stability of correlation is a crucial point: Unstable correlations make it difficult to price and hedge derivatives whose payoffs depends on more than one asset. Similarly, portfolio managers rely on stable correlations in order to reduce or even eliminate their portfolio risk.

This motivated numerous empirical studies trying to shed light on the constancy of correlations of financial return data. Kaplanis (1988), for instance, investigates stock indices of 10 countries from 1967 to 1982 and arrives at the conclusion that correlation structure is stable over time. Considering monthly returns from 1973 to 1989, Meric & Meric (1989) found evidence that that international stock market co-movements are stable in the September-May period, but relatively unstable in the May-September period. Similarly, Koch & Koch (1991) analyzed the stock indices of 8 countries and concluded that the market interdependence within the same geographical region is growing over time. Similarly, Erb, Harvey & Viskanta (1994) found evidence of unstable correlations on the basis of monthly stock indices of G7-countries from 1970 to 1993. Increasing correlations in bear markets but not in bull markets is postulated by Longin & Solnik (1995) for excess returns of stock indices of 7 countries from 1960 to 1990. In contrast, exploring daily stock index returns from 1999 to 2002, Ragea (2002) states that correlation remains stable during volatile periods. Recently, Goetzmann, Li & Rouwenhorst (2005) claim that the correlation structure varies significantly, using worldwide monthly return series from 1872 to 2000.

All of these studies rely on a few statistical tests which assume that correlations of financial returns are piecewise constant over time. Above that, the points of time where regimes (i.e. periods with constant correlation) change are commonly unknown. In contrast, we make use of a test which allows correlations to vary over time according to certain trigonometric functions. Hence, no assumptions on "change points" are necessary and "smooth transitions" between two consecutive regimes are admitted. This allows to detect time-varying dependencies, where "conventional" tests might fail.

The outline of this work is as follows: Section 2 briefly reviews standard tests for constant correlation. A test based on trigonometric functions is introduced in section 3. Section 4 is dedicated to the description of the data set and to the discussion of the empirical results. Finally, section 5 concludes.

2. TESTS FOR CONSTANT CORRELATION: A SHORT REVIEW

Though all of the following tests for constant correlation are designed for the multivariate case, we restrict discussion to the bivariate case, henceforth. In general, these tests are rooted on Bartlett's (1937) test on equal variances, say σ_1^2 and σ_2^2, of two *iid*-normally distributed random samples with possibly different lengths N_1 and N_2. Denoting the sample variance of group j by S_j^2 and defining a pooled sample variance $S^2 = \sum_{j=1}^{2} \frac{N_j-1}{N_1+N_2-2} S_j^2$, Bartlett's test statistic is given by

$$\mathcal{T}_{Bartlett} = (N_1 + N_2 - 2)\ln(S^2) - \sum_{j=1}^{2}(N_j - 1)\ln(S_j^2) \quad \approx \quad \chi^2(1). \tag{1}$$

Box (1949) extended Bartlett's proposal to a test for homogeneity of covariance matrices, say $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$, of two subperiods. Equation (1) generalizes to

$$\mathcal{T}_{Box} = (N_1 + N_2 - 2)\ln(\det(\mathbf{S})) - \sum_{j=1}^{2}(N_j - 1)\ln(\det(\mathbf{S}_j)) \text{ with } \mathbf{S} \equiv \sum_{j=1}^{2} \frac{N_j - 1}{N_1 + N_2 - 2}\mathbf{S}_j,$$

where $\mathbf{S}_j$ denotes the sample covariance matrix of subperiod j. Assuming independent and bivariate normally distributed random samples, Box (1949) proposes both a χ^2- and an $F-$approximation to $\mathcal{T}_{Box}$. Finally, Kullback (1967) and Tang (1995) deal with the application of Box's test to correlation matrices rather than covariance matrices (by substituting the covariance matrices by the corresponding correlation matrices in the last formula). In particular, Kullback (1967) asserts that if all populations have the same non-singular correlation matrix, then the distribution of the test statistics is asymptotically chi-squared with certain degrees of freedom. However, Jennrich (1970, p. 905) presented a counterexample where Kullback's assertion fails. Jennrich (1970) himself suggested a test for equality of correlation matrices. Under the assumption of independent samples from two k-variate normal populations, the vector $\mathbf{d}$ – which contains all $k^* = k(k-1)/2$ dissimilar element-by-element differences of the two sample correlation matrices in lexographic order – is asymptotically normal with mean zero and non-singular covariance matrix $\mathbf{\Gamma}$. Therefore,

$$\mathcal{T}_{Jennrich} = \frac{N_1 N_2}{N_1 + N_2} \cdot \mathbf{d}'\widehat{\mathbf{\Gamma}}^{-1}\mathbf{d} \quad \overset{a}{\sim} \quad \chi^2(k^*),$$

where $\widehat{\mathbf{\Gamma}}$ is a consistent estimator of $\mathbf{\Gamma}$. Jennrich's main contribution was to derive a representation for the inverse of $\widehat{\mathbf{\Gamma}}$ which also applies to high dimensions in a simple way. In order to get rid off the normality assumption, Goetzmann, Li & Rouwenhorst (2005) utilize the asymptotic distribution of the correlation matrix from Browne & Shapiro (1986) and Neudecker & Wesselman (1990). Their proposal only requires that the observation vectors are independent and identically distributed according to a multivariate distribution with finite fourth moments, the corresponding test statistic reads

$$\mathcal{T}_{GLR} = \frac{N_1 N_2}{N_1 + N_2} \cdot \mathbf{d}'\widehat{\mathbf{\Omega}}^{-1}\mathbf{d} \quad \overset{a}{\sim} \quad \chi^2(k^+),$$

with a certain matrix estimator $\widehat{\mathbf{\Omega}}$ and suitable degrees of freedom k^+ (For more details see Goetzmann, Li & Rouwenhorst, 2005). Note that all of these tests presume that correlation is piecewise constant over time. In contrast, the TC_t-test which is introduced next section allows correlation to vary over time according to certain trigonometric functions. In particular, our model includes smooth transitions between different regimes rather than abrupt changes.

3. DERIVATION OF THE CC-TRIG$_t$ TEST

Undoubtedly, the multivariate Student-t distribution can be considered as one of the most popular models for financial returns (see, e.g., Aas & Haff, 2006). A random variable $\mathbf{X}$ with mean vector μ and dispersion matrix $\mathbf{\Sigma}_*$ is said to follow a d-variate Student-t distribution with ν degrees of freedom if its density has the form

$$f(\boldsymbol{x};\boldsymbol{\mu},\mathbf{\Sigma}_*,\nu) = \frac{\Gamma(\frac{\nu+d}{2})}{\Gamma(\frac{\nu}{2})(\nu\pi)^{d/2}}|\mathbf{\Sigma}_*|^{-1/2}\left[1 + \frac{(\mathbf{x}-\boldsymbol{\mu})'\mathbf{\Sigma}_*^{-1}(\mathbf{x}-\boldsymbol{\mu})}{\nu}\right]^{-(\nu+d)/2}. \tag{2}$$

The corresponding covariance matrix is $\mathbf{\Sigma} \equiv Cov(\mathbf{X}) = \frac{\nu}{\nu-2}\mathbf{\Sigma}_* \neq \mathbf{\Sigma}_*$, $\nu > 2$. For a better interpretation, we re-scale the Student-t density such that

$$f(\boldsymbol{x};\boldsymbol{\mu},\mathbf{\Sigma},\nu) = \frac{\Gamma(\frac{\nu+d}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{d/2}\sqrt{|\mathbf{\Sigma}|}}\left[1 + \frac{(\mathbf{x}-\boldsymbol{\mu})'\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}{\nu-2}\right]^{-(\nu+d)/2}. \tag{3}$$

Defining the unknown parameter vector $\theta^* \equiv (\mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \nu)$ and the standardized observations $\overline{x}_i \equiv \frac{x_i - \mu_i}{\sigma_i}$, $i = 1, 2$, equation (3) can be re-written for $d = 2$ as

$$f(x_1, x_2; \theta^*) = \frac{\nu}{(\nu-2)2\pi\,\sigma_1\,\sigma_2\sqrt{1-\rho^2}}\left[1 + \frac{\overline{x}_1^2 - 2\overline{x}_1\overline{x}_2\rho + \overline{x}_2^2}{(1-\rho^2)(\nu-2)}\right]^{-\frac{\nu+2}{2}}. \tag{4}$$

noting that

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \quad \text{and} \quad \rho = \frac{\sigma_{12}}{\sigma_1\sigma_2}.$$

For a given random sample $\mathbf{X}_t$, $t = 1, \ldots, T$ from a bivariate Student-t distribution let us further assume that the correlation coefficient ρ_t at time t evolves according to the trigonometric equation

$$\rho_t \equiv \beta_0 + \beta_1 \sin(2f\pi t/T) + \beta_2 \cos(2f\pi t/T), \quad t = 1, \ldots, T, \quad f \in \mathbb{R}, \tag{5}$$

where β_0, β_1 and β_2 have to be chosen such that $\rho \in [-1, 1]$ and f denotes an appropriate though in general unknown frequency. Possible curves of ρ_t are plotted in figure 1, below.

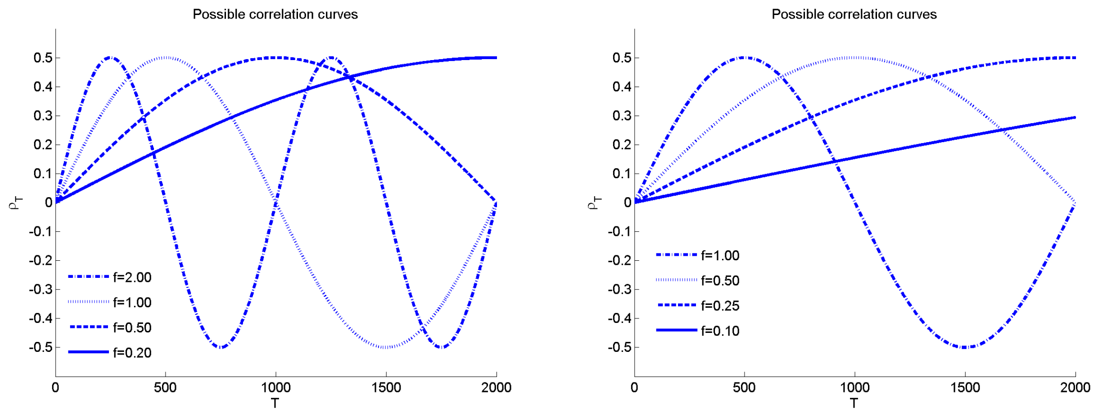

Figure 1: $\beta_0 = 0, \beta_1 = 0.2, \beta_2 = 0.3$ and $\beta_0 = 0, \beta_1 = -0.1, \beta_2 = 0.6$.

In order to demonstrate that correlation curves with these patterns may inherent to real-life data sets, consider the plot of the rolling correlations between two assets (where the correlation coefficient is successively calculated on the basis of the last 150 days over the whole time-period) as a proxy to the time-varying correlation. For the exchange rate pairs Canadian Dollar/US-Dollar versus Japanese Yen/US-Dollar and Japanese Yen/US-Dollar versus Britisch pound/US-Dollar we obtain (for a period of about 10 years) the running correlation curves in figure 2, below. Additionally, the solid lines represent fitted correlation curves of the form

$$\widehat{\rho}_t \equiv \widehat{\beta}_0 + \widehat{\beta}_1 \sin(2\widehat{f}\pi t/T) + \widehat{\beta}_2 \cos(2\widehat{f}\pi t/T),$$

where the unknown parameters have been estimated by means of non-linear least-square methods. More details about that point are provided in the empirical section of this paper.

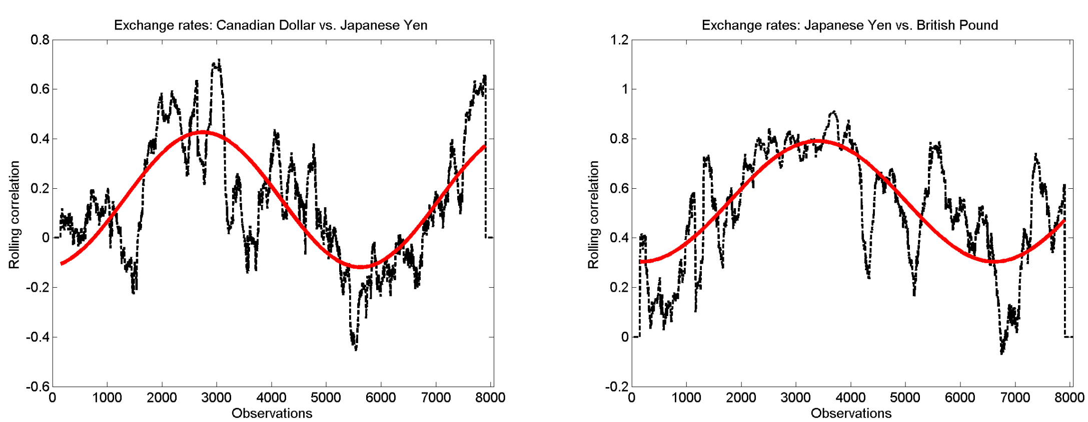

Figure 2: Left panel: $\widehat{\beta}_0 = 0.14$, $\widehat{\beta}_1 = 0.047$, $\widehat{\beta}_2 = 0.023$ and $\widehat{f} = 1.4$, right panel: $\widehat{\beta}_0 = 0.54$, $\widehat{\beta}_1 = -0.04$, $\widehat{\beta}_2 = -0.24$ and $\widehat{f} = 1.25$.

Assuming that the *iid*-sample $\mathbf{X}_t,\ t = 1, \ldots, T$ stems from a bivariate Student-t distribution and that the correlation coefficient at time t evolves according to the extended model in equation (5), we next derive a test whether $\mathcal{H}_0 : \beta_1 = \beta_2 = 0$, i.e. whether correlations are constant over time. On the basis of the log-likelihood function,

$$\begin{aligned}
\ell(\theta) &= \sum_{t=1}^{T} \log f(x_{1t}, x_{2t}; \theta) \quad \text{with} \quad \theta \equiv (\mu_1, \mu_2, \sigma_1, \sigma_2, \beta_0, \beta_1, \beta_2, \nu) \\
&= T\log(\nu) - T\log(\nu-2) - T\log(2\pi) - T\log(\sigma_1) - T\log(\sigma_2) \\
&\quad -\frac{1}{2}\sum_{t=1}^{T}\log(1-\rho_t^2) - \left(\frac{\nu+2}{2}\right)\sum_{t=1}^{T}\log\left[1 + \frac{\overline{x}_{1t}^2 - 2\overline{x}_{1t}\overline{x}_{2t}\rho_t + \overline{x}_{2t}^2}{(1-\rho_t^2)(\nu-2)}\right]
\end{aligned}$$

one might consider a likelihood ratio (LR) test, where $\mathcal{LR} = -2[\ell(\widehat{\theta}) - \ell(\widehat{\theta}_0)]$ has to be calculated. Note that $\widehat{\theta}$ and $\widehat{\theta}_0$, respectively, denote the maximum likelihood (ML) estimators of θ for the unrestricted and the restricted model, respectively. Within this work we make use of the Lagrange multiplier (LM) test (A comprehensive treatment of LM tests can be found, for instance, in Godfrey, 1988) which only requires estimates under the constant-correlation (i.e. under the restricted) model. It is essentially based on the score vector

$$\boldsymbol{s}(\boldsymbol{\theta}) \equiv \sum_{t=1}^{T}\left(\frac{\partial \ell_t}{\partial \mu_1}, \frac{\partial \ell_t}{\partial \mu_2}, \frac{\partial \ell_t}{\partial \sigma_1^2}, \frac{\partial \ell_t}{\partial \sigma_2^2}, \frac{\partial \ell_t}{\partial \beta_0}, \frac{\partial \ell_t}{\partial \beta_1}, \frac{\partial \ell_t}{\partial \beta_2}\right)'$$

whose particular elements derive as

$$\begin{aligned}
\frac{\partial \ell_t}{\partial \mu_1} &= \frac{\nu+2}{(\nu-2)\sigma_1}\frac{\overline{x}_{1t} - \overline{x}_{2t}\rho_t}{(1-\rho_t^2)A_t}, \qquad \frac{\partial \ell_t}{\partial \mu_2} = \frac{\nu+2}{(\nu-2)\sigma_2}\frac{\overline{x}_{2t} - \overline{x}_{1t}\rho_t}{(1-\rho_t^2)A_t}, \\
\frac{\partial \ell_t}{\partial \sigma_1} &= -\frac{1}{\sigma_1} + \frac{\nu+2}{(\nu-2)\sigma_1}\frac{\overline{x}_{1t}^2 - \overline{x}_{1t}\overline{x}_{2t}\rho_t}{(1-\rho_t^2)A_t}, \qquad \frac{\partial \ell_t}{\partial \sigma_2} = -\frac{1}{\sigma_2} + \frac{\nu+2}{(\nu-2)\sigma_2}\frac{\overline{x}_{2t}^2 - \overline{x}_{1t}\overline{x}_{2t}\rho_t}{(1-\rho_t^2)A_t} \\
\frac{\partial \ell_t}{\partial \nu} &= \frac{1}{\nu} - \frac{1}{\nu-2} - \left[\frac{1}{2}\log(A_t) - \frac{\nu+2}{2(\nu-2)^2}\cdot\frac{\overline{x}_{1t}^2 - 2\overline{x}_{1t}\overline{x}_{2t}\rho_t + \overline{x}_{2t}^2}{A_t(1-\rho_t^2)}\right] \quad \text{and} \\
\frac{\partial \ell_t}{\partial \beta_i} &= \frac{\rho_t}{1-\rho_t^2}\cdot\frac{\partial \rho_t}{\partial \beta_i} + \frac{\nu+2}{\nu-2}\cdot\frac{\overline{x}_{1t}\overline{x}_{2t}(1-\rho_t^2) - (\overline{x}_{1t}^2 - 2\overline{x}_{1t}\overline{x}_{2t}\rho_t + \overline{x}_{2t}^2)\rho_t}{A_t(1-\rho_t^2)^2}\cdot\frac{\partial \rho_t}{\partial \beta_i}
\end{aligned}$$

$$\text{noting that } \frac{\partial\partial \rho_t}{\partial \beta_0} = 1, \quad \frac{\partial\partial \rho_t}{\partial \beta_1} = \sin(2\pi f t/T), \quad \frac{\partial\partial \rho_t}{\partial \beta_2} = \cos(2\pi f t/T).$$

In the context of unrestricted estimates the score function is zero by definition. Now if the restricted estimates are close to the unrestricted estimates, the evaluated score should be close to zero. With $\widehat{\boldsymbol{s}} = \boldsymbol{s}(\widehat{\theta}_0)$ and

$$\widehat{\boldsymbol{S}} \equiv \left\{\widehat{s}_{ti} \equiv \frac{\partial \ell_t}{\partial \theta_i}(\widehat{\theta}_0)\right\}_{t=1,\ldots,T,i=1,\ldots,7}$$

and under the temporary assumption that the frequency f is known, the LM-type test statistics reads $\mathcal{LM}(f) \equiv \widehat{\boldsymbol{s}}'\left(\widehat{\boldsymbol{S}}'\widehat{\boldsymbol{S}}\right)^{-1}\widehat{\boldsymbol{s}}$. Being the sum of cross products of the first derivatives of ℓ_t, $\widehat{\boldsymbol{S}}'\widehat{\boldsymbol{S}}$ estimates Fisher's information matrix $V = E(\partial^2\ell/\partial\theta\partial\theta')$.

Unfortunately, the frequency f is unknown in practice and has to be estimated from the data set. Following a proposal of Beckers, Ender & Hurn (2004) – who consider functions as in equation (5) in order to detect structural breaks in regression models within a likelihood ratio framework – one might consider a finite set Υ of K different frequencies and consider the test statistics

$$\mathcal{LM}_m \equiv \max_{f\in\Upsilon} \mathcal{LM}(f), \quad \mathcal{LM}_a \equiv \sum_{f\in\Upsilon} \frac{\mathcal{LM}(f)}{K} \quad \text{and} \quad \mathcal{LM}_e \equiv \log \sum_{f\in\Upsilon} \frac{\exp\left(\mathcal{LM}(f)/2\right)}{K}$$

which arise from the maximum, the average and the exponentially weighted average of the $\mathcal{LM}(f)$-statistics. Whereas $\mathcal{LM}(f)$ is asymptotically chi-squared with 2 degrees of freedom under certain regularity conditions and for known f, $\mathcal{LM}_m$, $\mathcal{LM}_s$ and $\mathcal{LM}_a$ are no longer. For this reason, critical values are obtained employing parametric bootstrap techniques as follows:

1. For $i = 1, \ldots, J$ repeat the following two steps:
2. Generate *iid*-samples $\mathbf{X}_1^{(i)}, \ldots, \mathbf{X}_T^{(i)}$ from a bivariate Student-t with $\theta = \widehat{\theta}_0$.
3. For a given set Υ, calculate $\mathcal{LM}_m^{(i)}$, $\mathcal{LM}_s^{(i)}$ and $\mathcal{LM}_a^{(i)}$.
4. The proportion of J bootstrapped test statistics which exceed the test statistics from the observed data is then an estimate of the p-values of the test(s).

4. EMPIRICAL RESULTS

To cover a broad range of financial assets, we applied the CC-Trig$_t$-test to foreign exchange rate data from leading currencies (Yen/USD, Swiss Franc/USD, Britisch pound/USD, Canadian Dollar/USD) as well as to exchange rates from Asian emerging markets (Yen/EUR, Singapore Dollar/EUR, Won/EUR, Taiwan Dollar/EUR, Baht/EUR). Above that, emphasis was put on stock returns from the telecommunication market (Telecom Austria, France Telekom, Deutsche Telekom, Telefonica, T.I.M) and from the automobile sector (BMW, VW, DaimlerChrysler, Porsche, Renault, Peugeot). Finally, empirical investigations were performed for assets from the metal market (lead, tin, nickel, zinc). A first impression on the aforementioned data is provided by table 1, below, which summarizes estimated means, estimated variances, empirical skewness and kurtosis coefficients (measured by the third and fourth standardized moments) and the results of the Ljung-Box test on serial correlation and Engle's LM test (for $p = 10$ lags). Obviously, most of the data sets are highly leptokurtic (in particular the Asian exchange rates) and skew to some extend. Moreover, the results of the Ljung-Box test and of Engle's LM test indicate the presence of minor serial correlation and of significant (G)ARCH effects. In order to eliminate the possible influence of time-dependencies on the results of our test, we additionally fitted univariate ARMA-GARCH models to each of the series and considered standardized residuals (from now on briefly "GARCH residuals") in addition to the original returns (from now on briefly "plain returns").

Asset	Abbr.	N	$\widehat{\mu}$	$\widehat{\sigma}$	$\widehat{m}_3$	$\widehat{m}_4$	$\mathcal{LB}(10)$	$\mathcal{LM}(10)$
	Exchange rates, Jan 1973 to Dec 2004 (per US-Dollar)							
Canadian Dollar	CAD	8055	0.002	0.09	−0.004	6.75	12.65	912.18*
Japanese Yen	JPY	8055	−0.015	0.56	−0.002	6.11	12.50	429.24*
Swiss Franc	CHF	8055	0.003	0.36	0.132	6.84	55.79*	485.26*
British Pound	BRP	8055	−0.013	0.44	−0.723	13.33	34.48*	176.20*
	Asian exchange rates, Jun 1996 to Aug 2006 (per Euro)							
Japanese Yen	JPY	2642	0.051	4.53	0.818	48.42	72.91*	692.84*
Singapur Dollar	SGD	2642	0.026	0.39	−1.307	68.07	67.42*	48.84*
South Korean Won	KRW	2642	0.004	0.15	−0.991	19.14	40.28*	343.52*
Taiwan Dollar	TWD	2642	0.008	1.46	−2.349	109.27	262.90*	483.05*
Thai Baht	THB	2642	0.007	0.11	0.578	27.51	13.75	266.99*
	Stocks: Telecommunication, Nov 2000 to Aug 2006							
Telecom Austria	TA1	1489	−0.006	6.23	−0.081	9.10	27.93*	438.96*
France Telecom	FTE	1489	−0.015	2.07	−0.952	10.50	34.25*	146.04*
Deutsche Telekom	DTE	1489	0.055	3.37	−1.622	24.50	16.90	15.57
Telefonica	TEF	1489	−0.083	4.61	−0.923	11.86	35.10*	129.01*
T.I.M	TQI	1489	−0.082	40.64	1.131	85.56	115.66*	410.27*
	Stocks: Automobile, Aug 1989 to Aug 2006							
BMW	BMW	3056	0.039	4.37	0.073	7.24	19.47*	219.93*
VW	VW	3056	0.032	4.32	−0.154	6.66	30.11*	247.29*
DaimlerChrysler	DCX	3056	0.002	3.71	−0.101	6.15	24.13*	413.65*
Porsche	POR	3056	0.103	5.73	−0.021	6.64	10.74	252.84*
Renault	RNL	3056	−0.005	0.41	0.350	5.66	8.88	46.92*
Peugeot	PEU	3056	0.037	5.53	−0.011	5.63	22.81*	352.56*
	Metals, Nov 2001 to Aug 2006 (US-Dollar per tonne)							
Lead	LE	1093	0.034	1.22	−0.555	8.72	29.74*	161.56*
Tin	TI	1093	0.084	4.21	−0.368	5.59	30.11*	100.37*
Nickel	NI	1093	0.142	2.38	−0.139	5.13	12.95	127.66*
Zinc	ZI	1093	0.130	4.97	−0.618	7.90	10.72	21.45*

Table 1: Data statistics.

Figure 3 displays the time series for each of the five markets. To guarantee a better graphical comparison between the series of each market we re-scaled all series to 100 at the starting point of the data collection. This highlights, for instance, the tremendous growth of Porsche compared to other companies from the car industry.

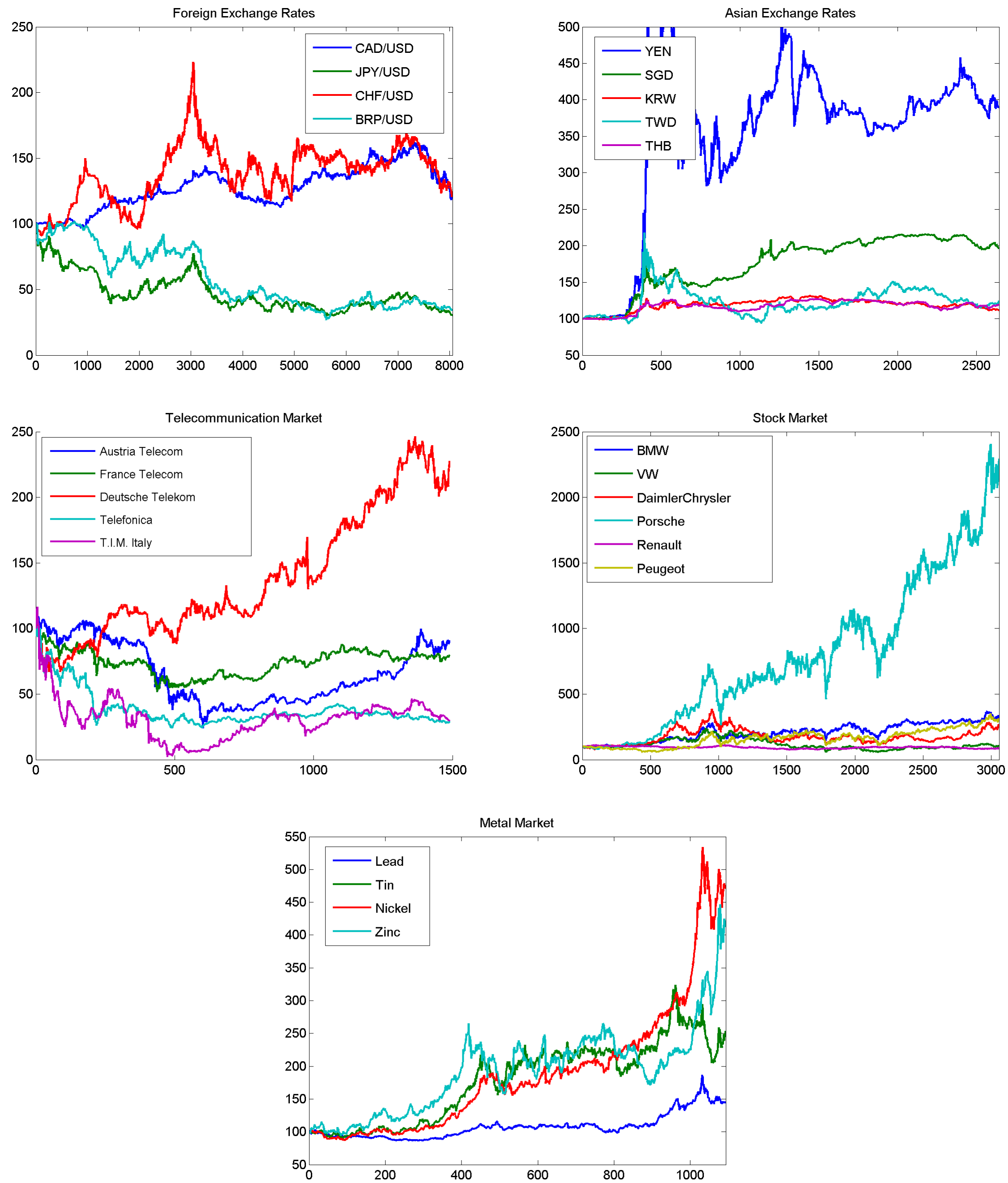

Figure 3: Re-scaled time series for all markets.

One of the crucial points of our test is to determine the set Υ which contains the frequencies underlying our test. On the one hand, too many frequencies may result in unnecessary computational burden, wrong or too less frequencies may produce misleading results. Against this background we suggest the following procedure to identify the relevant frequencies:

In a first step, the rolling correlations rc_t, $t - \tau, \ldots, T$ between two asset returns (based on the last τ observations) are calculated. In a second step, for given frequencies $0 = f_1$, f_2, ..., $f_N = U$, functions of the form

$$\rho_t \equiv \beta_0 + \beta_1 \sin(2f\pi t/T) + \beta_2 \cos(2f\pi t/T)$$

are fitted to the rolling correlations such that the squared differences

$$K(\beta_0, \beta_1, \beta_2) = \sum_t (rc_t - \rho_t)^2$$

are minimal. Let $M(f)$ denote the corresponding minimum which belongs to frequency f. Plotting f against $M(f)$ for all frequencies under consideration can be used as a graphical instrument to determine the interval. Exemplarily, plots for different stock returns are subject to figure 4, below. The minimal frequencies are given by $f \approx 1.25$ and $f \approx 2.00$, respectively.

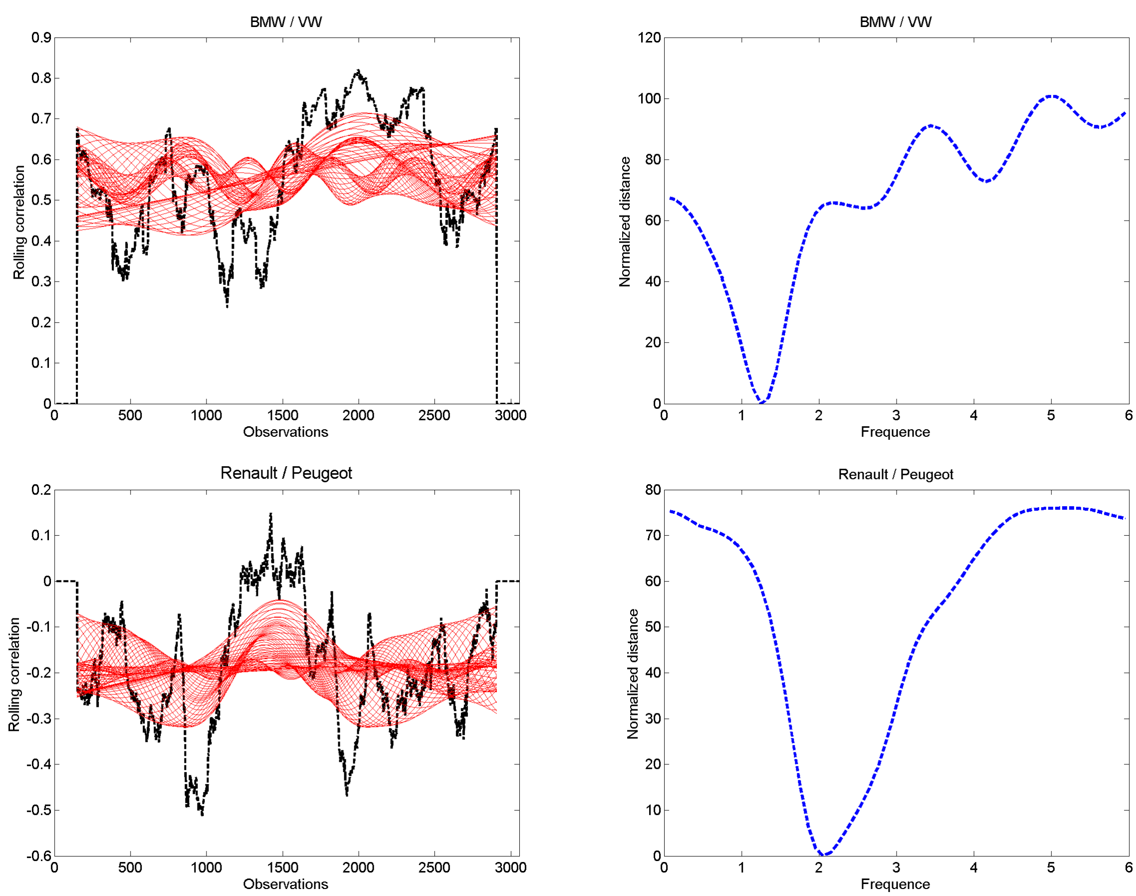

Figure 4: Choosing the interval for f ($U = 6$, $\tau = 150$, $N = 60$).

The empirical results of our new test are documented in table 2 to 6. Concerning the FX rates in table 2, there seems to be strong evidence for time-varying correlation for both plain returns and GARCH residuals. Critical values are printed in brackets, below.

		Plain returns			GARCH-residuals		
Asset 1	Asset 2	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$
CAD	JPY	103.5439	246.0640	242.6129	115.4731	292.1115	288.9490
		(4.6281)	(8.6212)	(6.8616)	(4.6632)	(9.0096)	(7.4392)
CAD	CHF	57.0160	127.9684	124.6356	73.3458	188.0232	184.6896
		(4.2749)	(8.9648)	(7.1185)	(4.4053)	(8.6535)	(6.9843)
CAD	GPB	75.0485	170.0469	166.6209	85.9070	219.7406	216.3305
		(4.4398)	(8.9023)	(7.0615)	(4.6617)	(8.9784)	(7.0360)
JPY	CHF	76.4903	134.6122	131.3341	135.5781	265.0354	261.8274
		(4.4167)	(8.7623)	(6.8567)	(4.5883)	(9.5074)	(7.5057)
JPY	GPB	224.8565	454.8221	451.1984	262.6755	524.5010	521.1735
		(4.7420)	(9.5445)	(7.6867)	(4.3955)	(9.1284)	(7.2191)
CHF	GPB	126.0475	212.6770	209.4634	160.9184	316.1506	312.5655
		(4.4918)	(9.1534)	(7.2703)	(4.6560)	(9.0342)	(7.1173)

Table 2: FX returns.

The situation for the Asian exchange rate returns is quite different. While the null hypothesis is rejected for the plain returns, there is less evidence for correlations to vary over time if the time-serial dependencies are removed. This does not apply to the exchange rates of Korea and Taiwan.

		Plain returns			GARCH-residuals		
Asset 1	Asset 2	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$
JPY	SGD	214.6688	226.2212	223.8829	6.8189	15.3512	12.9907
		(5.3966)	(9.8919)	(7.9425)	(4.4894)	(8.8712)	(7.0696)
JPY	KRW	97.3095	115.0183	113.7123	16.1599	24.9849	23.6628
		(5.6687)	(9.4488)	(7.7439)	(4.6384)	(9.3952)	(7.6126)
JPY	TWD	65.2014	69.5137	67.6327	5.2571	8.6171	6.8537
		(6.9925)	(10.9047)	(9.0861)	(5.9887)	(9.8794)	(8.3755)
JPY	THB	125.5002	131.1136	129.6513	7.7395	16.3925	14.6583
		(5.8615)	(10.1463)	(8.2883)	(4.3946)	(8.9015)	(6.9980)
SGD	KRW	14.1800	15.7647	14.5730	1.7656	3.0429	2.0735
		(5.4788)	(10.1921)	(8.3345)	(4.3662)	(8.8144)	(6.9692)
SGD	TWD	3.9471	4.8421	4.0042	0.2031	0.3916	0.2126
		(5.6581)	(10.1231)	(8.0469)	(4.5643)	(9.0082)	(7.1839)
SGD	THB	71.8024	83.0011	80.5351	7.7482	14.4696	12.4812
		(5.5774)	(9.4376)	(7.6371)	(4.6258)	(8.8686)	(7.1149)
KRW	TWD	28.2941	48.9142	46.2857	17.2255	34.1560	31.6199
		(5.8784)	(10.3788)	(8.5078)	(4.5745)	(9.0889)	(7.3838)
KRW	THB	11.3578	16.8097	14.6937	8.5802	13.1685	11.0267
		(5.3217)	(9.6466)	(8.0823)	(4.5663)	(8.9945)	(7.2895)
TWD	THB	2.4646	6.8926	4.0968	3.5768	11.5692	7.9922
		(4.7045)	(9.1225)	(7.4439)	(4.4779)	(8.7159)	(6.9577)

Table 3: Exchanges rates: Asian Markets.

For the data sets from the automobile industry, correlation seems to be non-constant over time, even if GARCH-residuals are considered rather than plain returns.

		Plain returns			GARCH-residuals		
Asset 1	Asset 2	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$
BMW	VW	127.6182	212.4633	209.4241	45.4953	89.0968	85.8959
		(4.7173)	(9.4346)	(7.5516)	(4.8523)	(9.3660)	(7.6172)
BMW	DCX	200.5955	335.2793	331.9755	72.4488	129.1108	125.8327
		(4.5666)	(9.0758)	(7.1477)	(4.5597)	(9.1869)	(7.3543)
BMW	POR	99.6223	184.6362	182.3042	55.6603	114.3281	112.0828
		(4.5569)	(9.4141)	(7.4055)	(4.5417)	(9.1590)	(7.3205)
BMW	RNL	14.0591	42.8074	40.2622	15.0432	44.6061	42.1099
		(4.6057)	(9.3431)	(7.5654)	(4.8690)	(9.7535)	(7.6944)
BMW	PEU	90.5562	187.9338	185.4617	60.2558	123.0475	120.2055
		(4.5848)	(9.1851)	(7.3258)	(4.4666)	(8.7668)	(7.1233)
VW	DCX	88.9277	162.0190	158.6811	37.9808	77.0180	73.9289
		(4.0545)	(8.5025)	(6.7604)	(4.3566)	(8.9752)	(7.2113)
VW	POR	71.7736	136.1786	134.2285	46.4044	95.1935	93.2933
		(4.4712)	(8.7086)	(7.0186)	(4.3115)	(8.8574)	(7.0191)
VW	RNL	12.7337	32.7761	29.8995	11.5033	28.7465	26.4179
		(4.2568)	(8.6653)	(7.0924)	(5.1055)	(10.164)	(8.2904)
VW	PEU	66.5393	155.6894	153.6036	51.5981	109.1128	107.0299
		(4.7294)	(9.5197)	(7.5642)	(4.6324)	(9.4440)	(7.4704)
DCX	POR	73.2158	135.1195	132.8649	50.4921	97.0667	94.7815
		(4.5668)	(9.5383)	(7.5630)	(4.3711)	(8.9952)	(7.0508)
DCX	RNL	10.3433	28.3432	25.1021	9.5658	30.9925	27.6805
		(4.7183)	(9.1379)	(7.4684)	(4.5572)	(9.3041)	(7.4820)
DCX	PEU	82.4998	177.6023	175.2209	62.0517	127.8215	125.0783
		(4.6203)	(8.9406)	(7.2553)	(4.6370)	(9.7432)	(7.8390)
POR	RNL	7.7898	25.7745	22.9932	8.2506	24.9472	22.3784
		(4.3079)	(8.7719)	(6.9053)	(4.1977)	(8.1309)	(6.2960)
POR	PEU	57.0277	135.1566	132.9208	45.7259	100.1991	98.2859
		(4.7608)	(9.7691)	(7.9325)	(4.4566)	(9.0866)	(7.0940)
RNL	PEU	18.4628	30.8887	28.3690	10.9015	18.3849	16.3949
		(4.4029)	(9.0245)	(7.1396)	(4.6168)	(9.4743)	(7.4755)

Table 4: Stock returns: Automobiles.

Metal returns, in contrast, are not suspected to feature changes in correlation over time, at least after eliminating GARCH-effects.

		Plain returns			GARCH-residuals		
Asset 1	Asset 2	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$
LE	TI	5.7723	12.1817	10.3145	7.2776	13.7303	11.7270
		(4.4539)	(8.9237)	(6.9544)	(4.5262)	(9.6047)	(7.7467)
LE	NI	8.9925	17.7502	15.4934	19.2794	42.4652	40.7679
		(4.6612)	(9.2126)	(7.3049)	(4.8175)	(9.3530)	(7.5905)
LE	ZI	2.8521	5.1534	3.8227	1.5607	3.6657	2.5955
		(4.6192)	(9.1057)	(7.4266)	(4.6103)	(9.1347)	(7.3307)
TI	NI	15.9406	23.9646	21.5118	5.0515	8.1373	5.8778
		(4.7786)	(9.0998)	(7.3519)	(4.8373)	(9.4455)	(7.9777)
TI	ZI	12.5654	16.7924	14.8412	4.3095	5.5260	4.6571
		(4.5835)	(9.0724)	(7.2402)	(4.6448)	(9.5511)	(7.6388)
NI	ZI	12.3726	19.4806	17.9455	4.8251	8.4573	6.9361
		(4.4955)	(8.9634)	(7.1328)	(4.8934)	(9.9313)	(8.1484)

Table 5: Metals.

Surprisingly, the correlations between different stocks from the telecommunication market appear rather stable in the time period which we considered, except for $TA1/TEF$, FTE/TEF and TEF/TQI. In particular, the stock returns of Deutsche Telekom and that of its competitors show no time-varying pattern.

		Plain returns			GARCH-residuals		
Asset 1	Asset 2	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$	$\mathcal{LM}_a$	$\mathcal{LM}_m$	$\mathcal{LM}_e$
TA1	FTE	21.5897	51.2293	48.9153	14.1013	25.8216	23.3035
		(4.7944)	(9.5541)	(7.7627)	(4.4825)	(8.3040)	(6.6872)
TA1	DTE	4.8833	9.6227	7.5495	2.8162	6.5803	4.5544
		(4.7924)	(9.7740)	(7.7495)	(4.5709)	(9.1759)	(7.3454)
TA1	TEF	33.6799	66.6899	64.7945	15.5119	45.2180	41.4879
		(4.6308)	(8.9103)	(6.7734)	(4.7959)	(9.4852)	(7.9944)
TA1	TQI	27.5244	36.1989	34.3343	7.3520	10.4698	9.3944
		(5.2854)	(9.8352)	(8.0526)	(4.7042)	(9.2295)	(7.3093)
FTE	DTE	3.1854	7.7643	5.5116	2.8652	8.5911	6.6070
		(4.3458)	(8.8536)	(7.0210)	(4.7874)	(9.7024)	(7.7817)
FTE	TEF	12.2554	32.4945	29.7269	5.8253	14.2057	11.8020
		(4.5769)	(8.8507)	(7.1415)	(4.4715)	(9.0315)	(7.0240)
FTE	TQI	10.1478	13.9656	12.0456	5.3928	8.7289	7.0345
		(5.6197)	(9.8894)	(8.0794)	(4.9305)	(9.4033)	(7.6470)
DTE	TEF	6.3027	13.9836	11.9523	4.2362	7.4210	5.8086
		(4.5129)	(9.2707)	(7.2868)	(4.4610)	(8.9845)	(7.1056)
DTE	TQI	6.1464	9.3595	7.7409	3.5301	6.0384	4.4345
		(5.6924)	(9.9619)	(8.3504)	(5.2216)	(9.4431)	(7.7931)
TEF	TQI	23.5632	35.2809	32.9010	8.8052	14.1831	12.3405
		(5.6456)	(9.9207)	(8.0327)	(4.4004)	(9.0969)	(7.3630)

Table 6: Stock returns: Telecommunication.

We conclude this section providing the results for the (competitive) tests of section 2. Due to its design for heavy-tailed data, only the test of Goetzmann, Li & Rouwenhorst (2005) for GARCH residuals was taken into account. Table 7 summarizes the empirical results. In many cases, results are identically between the GLR test and the CC-Trig$_t$ test. However, in some cases (e.g. tin and zinc or nickel and zinc), empirical evidence for time-varying correlation is found using the CC-Trig$_t$ test while the GLR test fails. This might be due to the construction of the GLR test, where only piecewise constant correlation is allowed.

5. SUMMARY

This paper proposes a new test for constant correlation. Based on the bivariate Student-t distribution and correlation coefficients which are allowed to change according to certain trigonometric functions, three Lagrange multiplier-type test statistics are proposed. We approximate the critical values of these statistics by means of bootstrapping. Secondly, the test is applied to asset returns, resp. GARCH residuals from different markets. The results show that stock returns from the automobile industry exhibit time-varying correlations, whereas the correlations observed on the telecommunication market tend to be constant over time. This also applies to Asian exchange rate returns and some of the assets from the commodity market.

Asset 1	Asset 2	GLR	Asset 1	Asset 2	GLR	Asset 1	Asset 2	GLR
BMW	VW	30.5865 (3.8415)	JPY	SGD	1.1555 (3.8415)	TA1	FTE	0.8952 (3.8415)
BMW	DCX	29.3089 (3.8415)	JPY	KRW	0.0031 (3.8415)	TA1	DTE	2.0026 (3.8415)
BMW	POR	37.5042 (3.8415)	JPY	TWD	10.6249 (3.8415)	TA1	TEF	3.9206 (3.8415)
BMW	RNR	0.2552 (3.8415)	JPY	THB	11.1625 (3.8415)	TA1	TQI	7.0756 (3.8415)
BMW	PEU	85.0125 (3.8415)	SGD	KRW	0.2008 (3.8415)	FTE	DTE	0.0877 (3.8415)
VW	DCX	19.7082 (3.8415)	SGD	TWD	0.1168 (3.8415)	FTE	TEF	0.0019 (3.8415)
VW	POR	53.5808 (3.8415)	SGD	THB	6.8121 (3.8415)	FTE	TQI	2.9143 (3.8415)
VW	RNR	1.7018 (3.8415)	KRW	TWD	3.0181 (3.8415)	DTE	TEF	7.9740 (3.8415)
VW	PEU	60.0430 (3.8415)	KRW	THB	25.7978 (3.8415)	DTE	TQI	7.6441 (3.8415)
DCX	POR	34.2926 (3.8415)	TWD	THB	6.2924 (3.8415)	TEF	TQI	6.3126 (3.8415)
DCX	RNR	1.1368 (3.8415)	CAD	JPY	27.4309 (3.8415)			
DCX	PEU	53.2693 (3.8415)	CAD	CHF	10.5284 (3.8415)			
POR	RNR	0.0003 (3.8415)	CAD	BRP	6.7731 (3.8415)			
POR	PEU	48.9124 (3.8415)	JPY	CHF	15.5098 (3.8415)			
RNR	PEU	0.3393 (3.8415)	JPY	BRP	31.5584 (3.8415)			
LE	TI	10.2082 (3.8415)	CHF	BRP	10.2658 (3.8415)			
LE	NI	27.5722 (3.8415)	CAD	JPY	27.4309 (3.8415)			
LE	ZI	9.0261 (3.8415)	CAD	CHF	10.5284 (3.8415)			
TI	NI	0.9648 (3.8415)	CAD	BRP	6.7731 (3.8415)			
TI	ZI	1.5681 (3.8415)	JPY	CHF	15.5098 (3.8415)			
NI	ZI	0.0979 (3.8415)	JPY	BRP	31.5584 (3.8415)			
			CHF	BRP	10.2658 (3.8415)			

Table 7: Results of the GLR test for $\alpha = 0.05$.

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