Adaptive Forecasting of the EURIBOR Swap Term Structure

Oliver Blaskowitz*
Helmut Herwartz**

* Humboldt-Universität zu Berlin, Germany
** Christian-Albrechts-Universität zu Kiel, Germany

BERLIN

ECONOMIC RISK

SFB 649

Adaptive Forecasting of the EURIBOR Swap Term Structure. [*]

Oliver Blaskowitz

Humboldt–Universität zu Berlin

Institute of Statistics and Econometrics

Phone: $+49 - 30 - 2093 - 5705$

Email: blaskowitz@wiwi.hu–berlin.de

Helmut Herwartz

Christian–Albrechts–Universität zu Kiel

Institute of Statistics and Econometrics

Phone: $+49 - 431 - 880 - 2417$

Email: herwartz@stat–econ.uni–kiel.de

Abstract

In this paper we adopt a principal components analysis (PCA) to reduce the dimensionality of the term structure and employ autoregressive models (AR) to forecast principal components which, in turn, are used to forecast swap rates. Arguing in favor of structural variation, we propose data driven, adaptive model selection strategies based on the PCA/AR model. To evaluate ex-ante forecasting performance for particular rates, different forecast features such as mean squared errors, directional accuracy and big hit ability are considered. It turns out that relative to benchmark models, the adaptive approach offers additional forecast accuracy in terms of directional accuracy and big hit ability.

Keywords: Principal components, ex–ante forecasting, EURIBOR swap rates, term structure, directional accuracy, big hit ability.

JEL classification: C32, C53, E43, G29.

1 Introduction

While term structure modelling has undergone extensive improvements, advances in term structure forecasting are comparatively small. Yet, the latter is particularly important for purposes of managing risk or hedging derivatives. Diebold & Li (2006) point out that the empirical performance of model based out–of–sample forecasts is rather poor. Reformulating the Nelson & Siegel (1987) model, they use autoregressive models for the factors to obtain encouraging results for long horizon ex–ante forecasts. While Diebold & Li (2006) find that forecasts based on vector autoregressive models (VARs) outperform forecasts implied by the random walk model, Duffee (2002) concludes that the random walk model is superior to standard affine term structure models.

Ang & Piazzesi (2003) model yield curves by means of traditional latent yield factors and observable macroeconomic variables. Forecast error variance decompositions show that macro factors explain up to 85% of the variation in bond yields. Taking a dynamic factor approach Diebold, Rudebusch & Aruoba (2006) model the yield curve by means of latent level, slope and curvature factors as well as macroeconomic variables as real activity, inflation and the federal funds rate. They find convincing evidence of the macroeconomic effects on the yield curve. Mönch (2007) forecasts the yield curve in a data rich environment. He uses a factor–augmented VAR jointly with an affine term structure model with parameter restrictions implied by a no–arbitrage condition. The model turns out to outperform different benchmark models such as a random walk, standard VAR and the Diebold & Li (2006) approach among others.

Though a large part of the term structure literature is concerned with factor models, a uniform conclusion with regard to the appropriate number of factors has not been achieved yet. Nelson & Siegel (1987) introduce a parsimonious three factor model for term structures and conclude that it is able to capture important yield curve characteristics. Numerous extensions of the Nelson–Siegel model exist. Inter alia, a two factor version is applied by Diebold, Piazzesi and Rudebusch (2005) and the four factor version from Svennson (1994) is frequently used by central banks (BIS 2005). Empirical support is provided by Litterman & Scheinkman (1991) and Steeley (1990). Their factor, respectively, principal component analysis (PCA) suggests that most of the term structure variation can be explained by three factors, interpreted as level, slope and curvature. Examining money market returns, Knez,

Litterman & Scheinkman (1994) present a four factor model to find that the additional factor is related to private issuer credit spreads. Duffie & Singleton (1997) advocate a multi factor model for interest rate swaps that accommodates counterparty default risk and liquidity differences between Treasury and Swap markets. They conclude that credit and liquidity factors are important sources to explain swap term structure dynamics. Within this framework Liu, Longstaff & Mandell (2006) estimate a five factor model to analyze swap spreads.

To explain forecast failures of macroeconomic models, Clements & Hendry (2002), among others, argue that economies evolve and are subject to changes, e. g., in institutions or technology. Neglecting the change of economic relations is a potential reason for the poor performance of model based out–of–sample term structure forecasts. To admit for dynamic heterogeneity, data based adaptive forecasting procedures appear to be useful alternatives. Swanson and White (1997a,b) find that an adaptive approach yields promising results in forecasting macroeconomic variables. A particular issue in dynamic ex–ante forecasting is the stability of model parameters. Splitting a sample of US government interest rates covering the period January 1970 to December 1995 into three parts, Bliss (1997) concludes that factor loadings varied only slightly. Yet, factor volatilities turned out to be relatively stable. For US zero coupon bond yields Audrino, Barone–Adesi & Mira (2005) find that loadings are unstable over the period from January 1986 to May 1995 in a three factor model allowing for conditional heteroscedasticity.

A large fraction of the term structure literature is concerned with the US treasury market. However, Remolona & Wooldridge (2003) point out that the EURO swap market has become one of the largest and most liquid markets world wide. The enormous increase in hedging and positioning activity tripled the turnover in Euro denominated interest rate swaps between 2000 and 2006 (ECB 2007).

Due to the huge size of swap markets and the neglected attention paid to forecasting the term structure, we focus on forecasting the EURIBOR (European interbank offered rate) swap term structure. Employing a purely statistical factor model approach, we decompose the term structure of swap rates by means of PCA and apply AR models to compute (adaptive) forecasts. Using various combinations of the number of factors, AR orders and time windows, our analysis includes a set of 100 model specifications. The latter are evaluated in

terms of mean squared forecast errors, directional accuracy and big hit ability.

Similar to Härdle, Herwartz & Spokoiny (2003) we argue in favor of dynamic variation of the term structure and motivate an adaptive procedure relying on local homogeneity of the term structure. By means of several data driven model selection algorithms, we analyze the relative performance of an adaptive approach and particular 'unconditional' PCA/AR forecasting schemes. Compared with standard benchmark models an adaptive approach offers additional forecast accuracy in terms of directional accuracy and big hit ability.

The remainder of the paper is organized as follows. The factor model approach is presented in the next Section. In Section 3 we introduce the loss measures used to evaluate forecasting performance. Sections 4 and 5 describe the data and characterize the unconditional approach to motivate adaptive model selection procedures. Section 6 proposes particular adaptive strategies. Moreover, we compare the adaptive strategies to unconditionally implemented factor models as well as to some benchmark models. Section 7 concludes.

2 A forecast model for the swap rate term structure

The investigated EURIBOR swap term structure consists of daily swap rates for $M = 10$ maturities (3 months (3m), 6m, 1 year, 2years (2yr), 3yr, 5yr, 7yr, 10yr, 12yr and 15yr). Let $\tilde{y}_t = (\tilde{y}_{1,t}, \tilde{y}_{2,t}, \ldots, \tilde{y}_{M,t})'$ denote the 10 dimensional vector of observed swap rates measured in terms of deviations from their unconditional mean, $\tilde{y}_t = y_t - \bar{y}_{T^*}$, $\bar{y}_{T^*} = 1/\tau \sum_{t=T^*-\tau+1}^{T^*} y_t$. To generate rolling swap rate forecasts we summarize the dynamic variation of the term structure in a time window of size τ by a few underlying factors. More precisely, equations (2.1) and (2.2) below constitute the local description of the term structure

$$\tilde{y}_t = \Gamma_K F_t + \xi_t, \quad t = T^* - \tau + 1, \ldots, T^*, \tag{2.1}$$

$$\Delta F_t = \nu + \Phi_1 \Delta F_{t-1} + \ldots + \Phi_p \Delta F_{t-p} + \eta_t . \tag{2.2}$$

In (2.1) the K–dimensional vector F_t consists of factors $f_{k,t}$ that govern the term structure. In (2.2) the first differences for each $f_{k,t}$, $k = 1, \ldots, K$, are assumed to follow 'cross sectionally' uncorrelated AR(p) processes. Hence, $\Phi_1, \ldots, \Phi_p$ are diagonal matrices and η_t is a K–dimensional zero mean error term with a diagonal covariance matrix. Moreover, the error terms ξ_t and η_t are assumed to be 'cross sectionally' and serially uncorrelated. The matrix Γ_K in (2.1) is obtained by means of PCA which decomposes the unconditional covariance

matrix of $\tilde{y}_t$, i.e.

$$\hat{\Sigma}_{T^*} = \frac{1}{\tau} \sum_{t=T^*-\tau+1}^{T^*} \tilde{y}_t \tilde{y}_t', \quad \hat{\Sigma}_{T^*} = \Gamma \Lambda \Gamma'. \tag{2.3}$$

In (2.3) the eigenvectors of $\hat{\Sigma}_{T^*}$ constitute the matrix Γ and the diagonal matrix Λ contains the corresponding eigenvalues in decreasing order. To account for the variation explained by the K most important principal components, the matrix Γ_K consists of the first K columns of Γ. Note that even if PCA and factor analysis are conceptually different, they are closely related (see e.g. Johnson & Wichern 2002). Thus, we do not distinguish between factors and principal components.

To implement ex–ante forecasting of swap rates the K most important factors $F_t = (f_{1t}, \ldots, f_{Kt})'$ are estimated as $\widehat{F}_t = \Gamma'_K \tilde{y}_t$ for $t = T^* - \tau + 1, \ldots, T^*$. First differences of the factors are modelled by univariate AR(p) processes

$$\Delta \widehat{f}_{k,t} = \gamma_{k0} + \gamma_{k1} \Delta \widehat{f}_{k,t-1} + \ldots + \gamma_{kp} \Delta \widehat{f}_{k,t-p} + u_{k,t}, \ k = 1, \ldots, K.$$

Iterated h–step forecasts for the first differences, $\Delta \widehat{f}_{k,T^*+h|T^*}$, are computed as

$$\Delta \widehat{f}_{k,T^*+h|T^*} = \widehat{\gamma}_{k0} + \widehat{\gamma}_{k1} \Delta \widehat{f}_{k,T^*+h-1|T^*} + \ldots + \widehat{\gamma}_{kp} \Delta \widehat{f}_{k,T^*+h-p|T^*},$$

where $\widehat{\gamma}_{k0}, \ldots, \widehat{\gamma}_{kp}$ are OLS estimates of $\gamma_{k0}, \ldots, \gamma_{kp}$ and $\Delta \widehat{f}_{k,T^*+j|T^*} = \Delta \widehat{f}_{k,T^*+j}$ if $j \leq 0$. Then, factor 'level' forecasts are

$$\widehat{f}_{k,T^*+h|T^*} = \widehat{f}_{k,T^*} + \sum_{j=1}^{h} \Delta \widehat{f}_{k,T^*+j|T^*}.$$

Finally, the h–step ahead forecast of the swap rate term structure conditional on information available at time T^* is

$$\widehat{y}_{T^*+h|T^*} = \Gamma_K \widehat{F}_{T^*+h|T^*} + \bar{y}_{T^*},$$

where $\bar{y}_{T^*}$ readjusts for the unconditional in–sample mean.

Note that we compute principal components from centered swap rate levels. If swap rates are non stationary then some eigenvectors of Γ may be interpreted as (unidentified) cointegration parameters (Johansen 1995), and PCA yields at least some non stationary factors. For our forecasting procedures it turns out that results for AR models specified in first differences of the factors F_t are more stable than for AR models in levels. Finally, in the

local model the drift parameter ν in (2.2) implies a linear trend in the levels of interest rates. While such a feature contradicts theoretical and empirical long run interest rate properties, in the presence of local trends, however, including the parameter ν might be beneficial for ex–ante forecasting procedures.

To generate h–step forecasts for a particular swap rate m in time T^* by means of the local model given in (2.1) and (2.2) an analyst has to choose the parameters τ, K and p. In this study, we consider a set of 100 competing model specifications implied by combining a variety of choices for τ, K, p. Then, an adaptive model selection approach is based on out–of–sample forecast performance evaluated with particular loss functions such as quadratic loss, directional accuracy and big hit ability.

3   Loss functions

Before motivating the loss functions considered in this study, we briefly introduce some notation. Let a general loss function depend on the h–step ahead swap rate forecast, $\hat{y}_{m,T^*+h|T^*}$, the current swap rate, y_{m,T^*} and the future (true, realized) swap rate, y_{m,T^*+h}, with maturity m, i.e.

$$L_{T^*}^{h,m} = L(\hat{y}_{m,T^*+h|T^*}, y_{m,T^*}, y_{m,T^*+h}).$$

A common loss function is the quadratic loss:

$$L_{1,T^*}^{h,m} = (y_{m,T^*+h} - \hat{y}_{m,T^*+h|T^*})^2. \tag{3.1}$$

Diebold & Mariano (1995) point out that in light of the variety of economic decision problems relying on forecasts, statistical loss functions such as the quadratic loss do not necessarily conform to economic loss functions. In an interest rate setting, Swanson & White (1995) show that the mean squared forecast error (MSFE) and profit measures are not closely linked. Similarly, Leitch & Tanner (1991) find that, opposite to MSFE, the directional accuracy (DA) of forecasts, i. e. the ability of correctly predicting directions, is highly correlated with profits in a term structure analysis. Lai (1990) points out that an investor can still gain profits even with statistically biased forecasts if they are characterized by significant DA. Ash, Smith & Heravi (1998) indicate that qualitative statements on the change of the economy in the near future are important pre–requisites for the appropriate implementation of monetary and fiscal policy. Similarly, Öller & Barot (2000) emphasize the

importance of DA for central banks as a forecast of increased inflation (above target) would prompt central banks to raise interest rates. With $I(\bullet)$ denoting an indicator function a loss function for DA is:

$$
\begin{aligned}
L_{2,T^*}^{h,m} &= I\left((\hat{y}_{m,T^*+h|T^*} - y_{m,T^*})(y_{m,T^*+h|T^*} - y_{m,T^*}) > 0 \right) \\
&\quad -I\left((\hat{y}_{m,T^*+h|T^*} - y_{m,T^*})(y_{m,T^*+h|T^*} - y_{m,T^*}) < 0 \right).
\end{aligned}
$$

Hatzmark (1991) investigates forecast ability by looking at DA and 'Big Hit Ability'. To motivate the latter, it might occur that a profit seeking trader is better able to predict big price changes rather than small changes. In this case, forecast performance could depend on a small number of correct directional forecasts generating large profits, and a large number of incorrect directional forecasts associated with negligibly small losses. A loss function for Big Hit Ability (BH) is:

$$
L_{3,T^*}^{h,m} = L_{2,T^*}^{h,m}|y_{m,T^*+h} - y_{m,T^*}|
$$

The BH measure generalizes the quadratic loss and DA statistics in that it takes the sign as well as the magnitude of the movement into account. If y_{m,T^*} is a swap rate, $L_{3,T^*}^{h,m}$ is only approximately a profit function. The profit/loss from closing a swap position in $T^* + h$ is given by the swap value in $T^* + h$ since in T^* a swap with a (fair value) fixed rate y_{m,T^*} has a value of zero. However, as a swap is a financial derivative, in $T^* + h$ the value of a swap with rate y_{m,T^*} is a non linear function of y_{m,T^*+h} (Miron & Swannell 1991). Yet, as the second derivative of the swap value function with respect to y_{m,T^*+h}, is often very small, most traders and risk managers consider swaps as linear instruments even if they are actually non linear. That is to say upward/downward movements in y_{m,T^*+h} are almost proportional to changes in the profit/losses from closing the corresponding swap position.

4   Data

We investigate closing rates for Eurozone interest rate swaps with maturity 1yr, 2yr, 3yr, 5yr, 7yr, 10yr, 12yr and 15yr and the 3m resp. 6m Euribor rate as obtained from the database EcoWin (http://www.ecowin.com/). The sample period comprises 2100 daily observations (Mon through Fri) from February 15, 1999, to March 2, 2007. Figure 1 shows the evolution of the swap term structure. It displays the variability of the term structure shape over time. For example, the level of the swap term structure is higher in October/November

2000 (around week 100) than in March 2004 (around week 280). Similarly, the slope of the term structure is higher in March 2003 (around week 210) as e. g. in November 2007 (around week 400). Moreover, the curvature in November 2003 (around week 250) exceeds the corresponding measure in October 2001 (around week 140).

Table 1 documents that all observed term structures increase with a minimum slope measure of 0.01. Consequently, the average term structure is also increasing. Swap rates at long maturities exhibit less variation than those at the short end. For the curvature the evidence is mixed. The sample mean (median) of this quantity indicates a slightly concave curvature of -0.00543 (-0.00325) with minimum and maximum values between -0.0825 and 0.084, respectively. Hence, on average the curvature of the term structure is not very pronounced, although Figure 1 uncovers locally concave/convex patterns.

5   Unconditional forecast models

We consider 4 forecast horizons ($h = 1, 5, 10, 15$ days) and focus on h–step forecasts of 2yr, 5yr and 10yr swap rates. Hence, overall there are 12 distinct forecast 'exercises' $FE_j = \{m^j, h^j\}$, $j = 1, \ldots, 12$, where FE_j is a tuple from the cartesian set defined by $\{2, 4, 8\} \times \{1, 5, 10, 15\}$. To define the adaptive strategies let a particular model specification be denoted by $M_s = \{\tau^s, K^s, p^s\}$, where

$$
\begin{aligned}
\tau^s &\in \Omega_\tau = \{42, 63, 126, 189, 252\} \ , \\
K^s &\in \Omega_K = \{1, 2, 3, 4, 5\} \ , \\
p^s &\in \Omega_p = \{0, 1, 2, 3\} \ .
\end{aligned}
$$

M_s is a three dimensional tuple from the cartesian set $\Omega_\tau \times \Omega_K \times \Omega_p$ the cardinality of which is 100. A forecast for a specification s at time T^* is $\hat{y}^s_{m, T^*+h|T^*}$. For a particular loss function $L_i^{h,m,s}, i = 1, 2, 3$, and each model specification M_s an average out–of–sample forecast performance over the time interval $[T_1^*; T_2^*]$ is

$$
\frac{1}{T_2^* - T_1^* + 1} \sum_{T^*=T_1^*}^{T_2^*} L_{i,T^*}^{h,m,s} = \frac{1}{T_2^* - T_1^* + 1} \sum_{T^*=T_1^*}^{T_2^*} L_i(\hat{y}^s_{m, T^*+h|T^*}, y_{m,T^*}, y_{m,T^*+h}) \ .
$$

We refer to the average loss associated with $L_i^{h,m,s}, i = 1, 2, 3$, respectively, as MSFE, mean directional accuracy (MDA) and mean big hit ability (MBH). The average losses of model

specification s in forecasting rate m at horizon h are denoted by $\text{MSFE}_s^{h,m}$, $\text{MDA}_s^{h,m}$ and $\text{MBH}_s^{h,m}$.

To motivate an adaptive model selection approach we first consider the 'unconditional' forecast performance, i. e. the average forecast performance of the 100 models M_s for the period $T_1^* = 308$ (April 19, 2000) to $T_2^* = 2085$ (February 9, 2007). Table 2 shows the MSFEs obtained when forecasting the 2yr swap rate one day ahead ($h = 1$). The best model is approximately 8 times better than the worst model, the 10th best model is still more than 3 times better than the 90th best model. For MDA and MBH the overall picture is similar. Hence, choosing the wrong model may provide poor forecasts. Moreover, for MDA and MBH the latter conclusion holds throughout for all forecast exercises FE_j. For the MSFE criterion, however, the 'spread' between the best and worst models diminishes for forecast horizons $h > 5$.

[Insert Table 2 about here]

In addition to marked differences in relative model performance, forecasting accuracy of a particular factor model might vary over time. In case of structural variation each factor model specification might be seen as an approximation of the true data generating process and the approximation accuracy of particular models depends on 'local' term structures. To describe time varying model performance we consider transition probability matrices as in Camba–Mendez, Kapetanios & Weale (2002). Each of the 100 models is mapped to performance quartiles conditional on the first and second half of the sample period. The transition probabilities are obtained from counting the models that move from one quartile in the first to a particular quartile in the second subsample. While a diagonal transition matrix indicates performance stability, large off–diagonal entries hint at performance instability. Table 3 shows the transition probabilities for the accuracy measures MSFE, MDA and MBH for the 2yr rate and $h = 1$. The upper left panel of Table 3 refers to the MSFE criterion. While there are transitions within the two upper and the two lower quartiles, there are not so many transitions crossing the subsample medians. The respective patterns for the MDA and MBH measures are remarkably different, and indicate much more transitions across quartiles. Off–diagonal elements take values between 0.083 and 0.417, respectively, 0.04 and 0.48. Again, the results are similar over all horizons $h = 1, 5, 10, 15$ and swap rates 2yr, 5yr, 10yr (see the average transition matrices in Table 8 given in the appendix). For a similar data

set, Blaskowitz, Herwartz & de Cadenas (2005) conclude that model parameters τ, K and p do not have a uniform impact on the forecasting performance. In summary, we diagnose marked heterogeneity of model specifications in terms of MDA and MBH performance while with respect to model implied MSFE measures model choice appears less crucial.

[Insert Table 3 about here]

In light of time dependent forecast accuracy it is desirable to have a strategy at hand that ex-ante identifies the locally best model. In the next Section we describe and evaluate data driven model selection strategies.

6   Adaptive strategies

6.1   Data driven model selection

An unconditional model approach is inherently subjected to misspecification under changing relations between economic variables. The rolling window strategy allows the parameters of a model to evolve over time. Yet, if parameter values are exposed to variation one may conjecture that the quality of a model approximation is time specific as well. An adaptive selection/estimation strategy is a promising means to account for distinct relative forecasting performance.

Our data adaptive model selection approach is based on a further time window of $\widetilde{\tau} = 42$ days in which the 'local' out–of–sample performance of specifications M_s, $s = 1, \ldots, 100$, is evaluated. Choosing evaluation windows of length $\widetilde{\tau} = 42$ is thought to balance between the needs of modeling flexibility under local heterogeneity on the one hand and statistical precision of parameter estimates on the other hand.

At each time point T^* the most recent $\widetilde{\tau}$ $h-$step forecast errors for swap rate m and model specification M_s are known. A local MSFE measure is

$$\text{MSFE}_{T^*}^{h,m,s} = \sum_{t=T^*-h-\widetilde{\tau}+1}^{T^*-h} L_{1,t}^{h,m,s}/\widetilde{\tau}.$$

The adaptive strategy, denoted MinMSFE, chooses the local MSFE minimizing specification:

$$\hat{y}_{m,T^*+h|T^*}^{\text{MinMSFE}} = \hat{y}_{m,T^*+h|T^*}^{s^*}, \quad s^* = \underset{s=1,\ldots,100}{\operatorname{argmin}} \left\{\text{MSFE}_{T^*}^{h,m,s}\right\}.$$

10

Another strategy to adaptively select a particular model specification is based on an ANOVA regression of the local MSFEs of the 100 factor models M_s on dummy variables representing the model parameters τ, K and p. The AnoMSFE forecast is given by

$$\hat{y}_{m,T^*+h|T^*}^{\text{AnoMSFE}} = \hat{y}_{m,T^*+h|T^*}^{s^*} \; ,$$

where $s^* = \{\tau^*, K^*, p^*\}$ is the locally best model specification as indicated by the smallest estimated (dummy variable) coefficients for τ, K and p (see also Blaskowitz, Herwartz and de Cadenas 2005).

Among others, Diebold & Pauly (1987) argue that in the presence of structural shifts composite forecasts can improve forecast precision. Numerous combining procedures have been proposed in the literature. We focus on both an equal weight scheme and a combination procedure that assigns different weights to individual forecasts. The Av10MSFE forecast is

$$\hat{y}_{m,T^*+h|T^*}^{\text{Av10MSFE}} = \frac{1}{10} \left(\hat{y}_{m,T^*+h|T^*}^{s_1^*} + \ldots + \hat{y}_{m,T^*+h|T^*}^{s_{10}^*} \right) ,$$

where $M_{s_1^*}, \ldots, M_{s_{10}^*}$ refer to the 10 best models in terms of $\text{MSFE}_{T^*}^{h,m,s}$.

Conditional on $M_{s_1^*}, \ldots, M_{s_{10}^*}$ the BunnMSFE forecast is given by

$$\hat{y}_{m,T^*+h|T^*}^{\text{BunnMSFE}} = \hat{\theta}_{s_1^*} \hat{y}_{m,T^*+h|T^*}^{s_1^*} + \ldots + \hat{\theta}_{s_{10}^*} \hat{y}_{m,T^*+h|T^*}^{s_{10}^*} \; ,$$

where the weights $\hat{\theta}_{s_q^*}$, $q = 1, ..., 10$, are proportional to the number of times (out of $\tilde{\tau}$ forecast realizations) that model s_q^* outperforms all other 9 models in terms of smaller squared error (Bunn 1975).

Along similar lines as described for the MSFE criterion we also use the loss functions MDA and MBH for adaptive forecasting.

Finally, we employ two combining strategies that have found support in the empirical literature (Clemen 1989). The AvStrat resp. MedStrat take the average resp. median forecast of the 100 forecast models irrespective of past performance. At time T^* these forecasts are given by

$$
\begin{aligned}
\hat{y}_{m,T^*+h|T^*}^{\text{AvStrat}} &= \frac{1}{100} \sum_{s=1}^{100} \hat{y}_{m,T^*+h|T^*}^{s} , \\
\hat{y}_{m,T^*+h|T^*}^{\text{MedStrat}} &= \underset{s=1,\ldots,100}{\text{Median}} \left\{ \hat{y}_{m,T^*+h|T^*}^{s} \right\} .
\end{aligned}
$$

In summary, the set of adaptive strategies is

$$\Omega_{AS} = \{\text{MinMSFE, Av10MSFE, AnoMSFE, BunnMSFE, MaxMDA, Av10MDA,}$$
$$\text{AnoMDA, BunnMDA, MaxMBH, Av10MBH, AnoMBH, BunnMBH, AvStrat,}$$
$$\text{MedStrat}\}.$$

All forecast comparisons are performed over the same sample period comprising 1778 time instances. Hence, accounting for the largest estimation window ($\tau = 252$), the highest forecast horizon ($h = 15$) and the model evaluation window ($\widetilde{\tau} = 42$), the rolling forecasting analysis starts in time point $T_1^* = 252 + 15 + 42 - 1 = 308$. Average losses of a particular adaptive strategy AS and forecast exercise FE_j are denoted by $\text{MSFE}_{AS}^{h,m}$, $\text{MDA}_{AS}^{h,m}$ and $\text{MBH}_{AS}^{h,m}$. To compare the performance of the adaptive strategies we provide respective normalized average losses. Normalization is accomplished with respect to the best and worst unconditional models in terms of average loss:

$$\text{nMSFE}_{AS}^{h,m} = 1 - \frac{\text{MSFE}_{AS}^{h,m} - \min_s\left\{\text{MSFE}_s^{h,m}\right\}}{\max_s\left\{\text{MSFE}_s^{h,m}\right\} - \min_s\left\{\text{MSFE}_s^{h,m}\right\}},$$

$$\text{nMDA}_{AS}^{h,m} = \frac{\text{MDA}_{AS}^{h,m} - \min_s\left\{\text{MDA}_s^{h,m}\right\}}{\max_s\left\{\text{MDA}_s^{h,m}\right\} - \min_s\left\{\text{MDA}_s^{h,m}\right\}},$$

$$\text{nMBH}_{AS}^{h,m} = \frac{\text{MBH}_{AS}^{h,m} - \min_s\left\{\text{MBH}_s^{h,m}\right\}}{\max_s\left\{\text{MBH}_s^{h,m}\right\} - \min_s\left\{\text{MBH}_s^{h,m}\right\}}.$$

For a given forecast horizon the sum of normalized losses for forecasts of the 2yr, 5yr and 10yr rates for the six best strategies are provided in Table 4. For the MSFE criterion it can be seen that the MedStrat strategy produces for all horizons superior normalized losses. The AvStrat strategy performs slightly worse for horizons $h = 5, 10, 15$ and is overall the 2nd best performing strategy. The Av10MSFE strategy is for all horizons among the best 3 adaptive strategies. In terms of MDA and MBH the MedStrat is again overall the best strategy. For $h = 1, 10, 15$ normalized losses are always better than the normalized losses of at least all but one adaptive strategy. In contrast to the MSFE criterion the Av10MDA resp. BunnMDA strategies are overall the second resp. third best competitor strategies.

6.2 Unconditional models vs. adaptive strategies

Having identified the overall best adaptive strategies we analyze in this Section how the forecasts from these adaptive strategies perform relative to unconditional models. Table 5 shows for each forecasting exercise normalized average loss estimates. Moreover, it provides for a given adaptive strategy the number of unconditional model specifications M_s performing worse (columns labelled $\succ$):

$$\sum_{s=1}^{100} \mathrm{I}(\mathrm{MSFE}_{AS}^{h,m} < \mathrm{MSFE}_{s}^{h,m}), \sum_{s=1}^{100} \mathrm{I}(\mathrm{MDA}_{AS}^{h,m} > \mathrm{MDA}_{s}^{h,m}), \sum_{s=1}^{100} \mathrm{I}(\mathrm{MBH}_{AS}^{h,m} > \mathrm{MBH}_{s}^{h,m}) \ .$$

From the upper panel of Table 5 it can be seen that the adaptive strategies perform well in terms of MSFE. The MedStrat strategy is always better than at least 68 unconditional models. The AvStrat strategy is in 9 forecast exercises better than 62, and the Av10MSFE strategy is still in 3 cases better than 63 unconditional models. No adaptive strategy is worse than the 40th best unconditional model. The relative performance in terms of MDA and MBH is provided in the two lower panels of Table 5. It is documented that the MedStrat strategy is always better than 66 unconditional models (except for the 5yr rate and $h = 5$ in terms of MDA). For the 10yr rate and $h = 10$ it is even better than the best unconditional model both in terms of MDA and MBH. For six forecasting exercises (the 2yr rate for $h = 5, 10, 15$, the 5yr rate, $h = 1, 15$ and the 10yr rate for $h = 5$) all three adaptive strategies are better than at least 60 unconditional models in terms of MDA. Regarding the MBH measure all the three adaptive strategies are at least better than 65 unconditional models in terms of MBH, except for forecasting the 2yr and 10yr rate for $h = 1$ and the 10yr rate for $h = 10$. These results can be viewed as an indication for the robustness of adaptive model selection in terms of MDA and MBH as compared to the MSFE criterion. Indeed, an analysis of all adaptive strategies considered in Section 6.1 (not reported), reveals that 'on average' adaptive model selection is more succesfull in terms of MDA and MBH than in terms of MSFE.

[Insert Table 5 about here]

We conclude that adaptive model selection approaches offer a promising forecast performance within the class of models introduced in Sections 2 and 5. Furthermore, it is of interest how the adaptive procedures compare to some standard benchmark models. We

remark that the adaptive approach does not lead to additional forecast accuracy in terms of MSFE when compared to the benchmark models. Hence, further results for the MSFE measure are not reported.

6.3 Adaptive forecasts vs benchmark approaches

We compare the adaptive strategies with naive forecasts, an autoregressive time series model and the Diebold & Li (2006) approach. The naive forecast is

$$\hat{y}_{m,T^*+h|T^*}^{\text{Naive}} = y_{m,T^*} \ .$$

For the purpose of measuring DA and BH accuracy, the naive forecast is always a downward movement. Average losses of the naive strategy are denoted by $\text{MDA}_{\text{Naive}}^{h,m}$ and $\text{MBH}_{\text{Naive}}^{h,m}$.

Next, time series forecasts for swap rate levels are based on a univariate AR(1) model for the first differences of the 2yr, 5yr and 10yr swap rates. This model is fitted recursively to a sample of size 42 respectively 252 days. For the latter benchmark forecasts average performance measures are denoted by $\text{MDA}_{\bullet}^{h,m}$ and $\text{MBH}_{\bullet}^{h,m}$, $\bullet = \text{AR42, AR252}$.

Using a decay parameter of $\lambda_t = 0.0609$ the Diebold & Li (2006) model is implemented by recursively fitting independent AR(1) processes for the first differences of factors using sample sizes of 42 days respectively 252 days. Average losses of iterated forecasts are denoted by $\text{MDA}_{\bullet}^{h,m}$ and $\text{MBH}_{\bullet}^{h,m}$, $\bullet = \text{DL42, DL252}$.

The last two columns of Table 6 show that the adaptive strategies MedStrat, Av10MDA and BunnMDA outperform the benchmark strategies in a comparison over all forecast exercises. In particular, the MedStrat strategy is overall best in terms of MDA and MBH. For the latter measure it is in 8 forecast exercises (2yr rate for $h = 1, 5, 10$, 5yr for all horizons, 10yr rate for $h = 10$) better than all other strategies. Regarding the losses in terms of MDA, in 8 forecasting exercises (2yr rate for $h = 1, 5, 15$, 5yr rate for all horizons, 10yr rate for $h = 10$) at least one of the three adaptive strategies outperforms all benchmark models. In terms of MBH, this is the case for 10 forecast exercises (all but 10yr rate for $h = 1, 5$).

[Insert Table 6 about here]

A summary of bilateral model comparisons is provided in Table 7. Furthermore, it is formally tested if the expected loss of a particular adaptive strategy is significantly larger

than the expected loss of the naive resp. AR benchmark strategy (which outperform the Diebold–Li model). The number of forecast exercises $FE_j, \ j = 1, \ldots, 12$, in which adaptive strategy $AS \in \{\text{MedStrat, Av10MDA, BunnMDA}\}$ is (significantly) better than the benchmark model $BM \in \{\text{Naive, AR42, AR252}\}$ can be found in the left hand side panels of Table 7. The right hand panels show how often is benchmark model BM (significantly) better than adaptive strategy AS. As can be verified from the left hand panels of Table 7, any of the three adaptive strategies is better than a given benchmark model in more than 6 (out of 12) forecast exercises in terms of MDA. For the MBH measure the results are even more compelling, each adaptive strategy outperforms a given benchmark model in more than 8 forecast exercises. In particular, the MedStrat strategy is better than the naive, AR42 resp. AR252 benchmark in 11, 10 resp. 12 forecast exercises. In 5, 3 resp. 9 cases it is also significantly better. On the other hand, the benchmark models are rarely significantly better than the adaptive strategies. For example, neither the naive nor the AR benchmark model significantly outperform the MedStrat strategy in any forecast exercise. Hence, we conclude that adaptive model selection/estimation within the class of models considered in this paper is preferable to standard benchmark models with MedStrat being the most convincing adaptive approach.

[Insert Table 7 about here]

7 Conclusions

Based on a factor model characterized by a dynamic autoregressive factor representation we forecast 2yr, 5yr and 10yr swap rates one day, resp., one, two and three weeks ahead. We compare a set of 100 unconditional model specifications to a variety of adaptive model selection strategies. Additionally, the latter procedures undergo a comparison with a naive, a standard time series and the Nelson–Siegel/Diebold–Li term structure model.

Building the comparison on out–of–sample forecast performance measured by quadratic loss, directional accuracy and big hit ability, we analyze the suitability of a standard PCA factor model approach for ex–ante forecasting. We find that an adaptive model selection approach leads to additional gains in directional accuracy and big hit ability. In particular, the MedStrat strategy turns out to consistently produce highly accurate forecasts for distinct

swap rates and forecast horizons. This result can be interpreted as evidence for an evolving economy characterized by changing underlying relations in economic variables (which is in line with the conclusions from Swanson & White (1997a,b) or Clements & Hendry (2002), for example). Hence, we show that an adaptive approach represents a promising and costless candidate for ex–ante forecasting that merits further consideration.

Moreover, the big hit measure as defined in this paper may also be used to evaluate the profitability of trading systems. For basic financial instruments such as stocks it represents cash flows from an elementary buy/sell strategy. For quasi linear financial derivatives, such as swaps, it is proportional to cash flows of a buy/sell strategy. Our definition of BH can be easily generalized using the cash flow function based on the 'exact' pricing function of the financial instrument or portfolio under consideration. Hence, in this framework it is possible to test for significant differences in profitability between two or more trading systems, see also Diebold & Mariano (1995) and West (2006), for example.

References

Ang, A. and M. Piazzesi, 'A No–Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables', *Journal of Monetary Economics*, **50**, 745–787 (2003).

Ash J. C. K., D. J. Smith and S. M. Heravi, 'Are OECD Forecasts Rational and Useful?: A Directional Analysis' *International Journal of Forecasting*, **14**, 381–391 (1998).

Audrino, F., G. Barone–Adesi and A. Mira, 'The Stability of Factor Models of Interest Rates', *Journal of Financial Econometrics*, **3**, 422–441 (2005).

BIS, 'Zero–Coupon Yield Curves: Technical Documentation', *BIS Papers No. 25, Bank for International Settlements* (2005).

Blaskowitz, O., H. Herwartz and G. de Cadenas, 'Modeling the FIBOR/EURIBOR Swap Term Structure: An Empirical Approach' *SFB 649 Discussion Paper No. 24, Humboldt–Universität zu Berlin* (2005).

Bliss, R., 'Movements in the Term Structure of Interest Rates', *Federal Reserve Bank of Atlanta, Economic Review* (1997).

Bunn, D., W., 'A Bayesian Approach to the Linear Combination of Forecasts', *Operational Research Quarterly*, **3**, 325–329 (1975).

Camba–Mendez, G., G. Kapetanios and M. R. Weale, 'The Forecasting Performance of the OECD Composite Leading Indicators for France, Germany,Italy and the UK', *In Clements, M. P., and D. F. Hendry (eds.), A Companion to Economic Forecasting*, Oxford, Blackwell Publishing (2002).

Clemen, R. T., 'Combining Forecasts: A Review and Annotated Bibliography', *International Journal of Forecasting*, **5**, 559–583, (1989).

Clements, M. P. and D. F. Hendry, 'Explaining Forecast Failure in Macroeconomics', *In Clements, M. P., and D. F. Hendry (eds.), A Companion to Economic Forecasting*, Oxford, Blackwell Publishing (2002).

Diebold, F. X. and C. Li, 'Forecasting the Term Structure of Government Bond Yields', *Journal of Econometrics*, **130**, 337–364, (2006).

Diebold, F. X. and R. Mariano, 'Comparing Predictive Accuracy', *Journal of Business & Economic Statistics*, **13**, 253–263, (1995).

Diebold, F. X. and P. Pauly, 'Structural Change and the Combination of Forecasts', *Journal of Forecasting*, **6**, 21–40, (1987).

Diebold, F. X., M. Piazzesi and G. D. Rudebusch, 'Modeling Bond Yields in Finance and Macroeconomics', *American Economic Review*, **95**, 415–420, (2005).

Diebold, F. X., G. D. Rudebusch and S. B. Aruoba, ''The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach', *Journal of Econometrics*, **131**, 309–338, (2006).

Duffee, G. R., 'Term Premia and Interest Rate Forecasts in Affine Models', *Journal of Finance*, **57**, 405–443 (2002).

Duffie, D. and K. Singleton, 'An Econometric Model of the Term Structure of Interest Rate Swap Yields', *Journal of Finance*, **52(4)**, 1287–1321 (1997).

European Central Bank, 'Euro Money Market Study 2006', *Press Release, download from http://www.ecb.eu/press/pr/date/2007/html/pr070213.en.html*, (2007).

Härdle, W., H. Herwartz and V. G. Spokoiny, 'Time inhomogeneous multiple volatility modeling', *Journal of Financial Econometrics*, **1**, 55–95 (2003).

Hatzmark, M. L., 'Luck Versus Forecast Ability: Determinants of Trader Performance in Futures Markets', *Journal of Business*, **64**, 49–74 (1991).

Johansen, S., *Likelihood–Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, Oxford (1995).

Johnson, R.A. and D. W. Wichern, *Applied Multivariate Statistical Analysis*, Prentice Hall, 5th Ed (2002).

Knez, P. J., R. Litterman and J. Scheinkman, 'Explorations Into Factors Explaining Money Market Returns', *Journal of Finance*, **49(5)**, 1861–1882 (1994).

Lai K. S., 'An Evaluation of Survey Exchange Rate Forecasts', *Economics Letters*, **32**, 61–65 (1990).

Leitch G. and J. E. Tanner, 'Economic Forecast Evaluation: Profits Versus the Conventional Error Measures', *American Economic Review*, **81**, 580–590 (1991).

Litterman, R. and J. Scheinkman, 'Common Factors Affecting Bond Returns', *Journal of Fixed Income*, **June**, 54–61 (1991).

Liu, J., F. A. Longstaff and R. E. Mandell, 'The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks', *The Journal of Business, 79*, 2337-2359 (2006).

Miron, P. and P. Swannell, *Pricing and Hedging Swaps*, Euromoney Publications PLC, London, (1991).

Mönch, E., 'Forecasting the Yield Curve in a Data-Rich Environment: A No–Arbitrage Factor–Augmented VAR Approach', *Discussion Paper, Humboldt–Universität zu Berlin* (2007).

Nelson, C. R. and A. F. Siegel, 'Parsimonious Modeling of the Yield Curve', *Journal of Business*, **60(4)**, 473–489 (1987).

Öller L. E. and B. Barot, 'The Accuracy of European Growth and Inflation Forecasts', *International Journal of Forecasting*, **16**, 293–315 (2000).

Remonola, E. M. and P. D. Wooldridge, 'The Euro Interest Rate Swap Market', *Bank for International Settlements, Quarterly Review, March*, 53–64 (2003).

Steeley, J. M., 'Modeling the Dynamics of the Term Structure of Interest Rates', *Economic and Social Review*, **21(4)**, 337–361 (1990).

Svensson, L. E. O., 'Estimating and Interpreting Forward Interest Rates: Sweden 1992–1994', *NBER Working Paper No. 4871, National Bureau of Economic Research* (1994).

Swanson, N. R. and H. White, 'A Model Selection Approach to Assessing the Information in the Term Structure Using Linear Models and Artificial Neural Networks', *Journal of Business & Economic Statistics*, **13**, 265–275, (1995).

Swanson, N. R. and H. White, 'A Model Selection Approach to Real–Time Macroeconomic Forecasting Using Linear Models and Artificial Neural Networks', *Review of Economics and Business Statistics*, **79**, 540–550, (1997a).

Swanson, N. R. and H. White, 'Forecasting Economic Time Series Using Flexible Versus Fixed Specification and Linear Versus Non Linear Econometric Models', *International Journal of Forecasting*, **13**, 439–461, (1997b).

West, K. D., 'Forecast Evaluation', *In Elliot, G., W. J. Granger and A. Timmermann. (eds.), Handbook of Economic Forecasting*, Elsevier B.V. (2006).

Figures

Evolution of Swap Term Structure
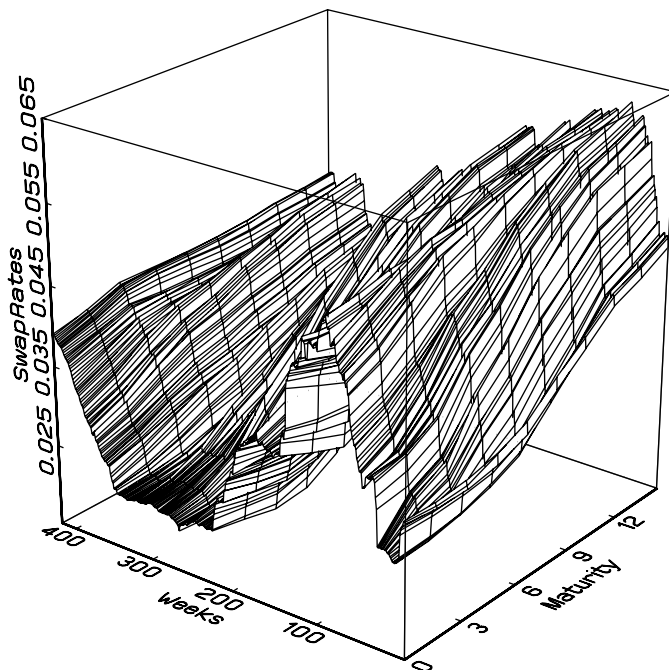
Figure 1. Evolution of the actual swap term structure for the period from February 15, 1999 to March 2, 2007.

Tables

	3m	6m	1yr	2yr	3yr	5yr	7yr	10yr	12yr	15yr	level	slope	curve
Mean	3.138	3.191	3.307	3.544	3.756	4.093	4.354	4.621	4.741	4.878	4.086	0.538	-0.00543
Median	3.013	3.109	3.228	3.490	3.680	3.906	4.117	4.420	4.575	4.759	3.915	0.555	-0.00325
Min	1.984	1.950	1.956	2.010	2.240	2.615	2.850	3.120	3.250	3.395	2.652	0.010	-0.08250
Max	5.211	5.274	5.415	5.583	5.698	5.805	5.900	6.031	6.150	6.295	5.774	0.900	0.08400
StD	0.915	0.919	0.935	0.910	0.876	0.825	0.803	0.776	0.768	0.762	0.816	0.234	0.03122

Table 1. Descriptive statistics of location and dispersion for actual swap rates and shape parameters for the period from February 15, 1999 ($T^* = 1$) to March 2, 2007 ($T^* = 2100$). Level, slope and curvature are measured by $\frac{2yr + 5yr + 10yr}{3}, \frac{10yr}{2} - \frac{2yr}{2}$ and $\frac{2yr}{4} - \frac{5yr}{2} + \frac{10yr}{4}$, respectively. Swap rates are multiplied by 100 for this Table only. In the remaining analysis swap rates are measured as 0.0312 instead of 3.12.

20

	2yr				5yr				10yr			
	$h=1$	$h=5$	$h=10$	$h=15$	$h=1$	$h=5$	$h=10$	$h=15$	$h=1$	$h=5$	$h=10$	$h=15$
	MSFE$*10^6$											
1st	0.200 (126/5/0)	0.962 (252/5/0)	1.954 (252/3/1)	3.025 (252/3/1)	0.196 (252/5/0)	0.938 (252/5/0)	1.852 (252/5/0)	2.861 (252/5/3)	0.153 (126/5/0)	0.697 (252/4/3)	1.384 (252/5/3)	2.095 (252/2/3)
10th	0.203 (42/4/0)	0.971 (252/3/3)	1.969 (252/5/2)	3.040 (252/4/2)	0.201 (63/4/0)	0.953 (189/5/2)	1.882 (189/5/2)	2.903 (252/4/2)	0.155 (63/3/0)	0.699 (252/5/2)	1.390 (252/4/1)	2.142 (252/5/1)
50th	0.213 (126/3/2)	1.043 (42/3/0)	2.154 (126/2/2)	3.357 (63/3/1)	0.212 (42/2/0)	1.014 (63/3/1)	2.070 (63/5/0)	3.287 (63/3/0)	0.160 (63/4/2)	0.740 (63/4/0)	1.543 (63/5/0)	2.466 (63/2/0)
90th	0.654 (126/1/2)	1.556 (126/1/2)	2.774 (126/1/0)	4.128 (126/1/2)	0.655 (126/1/2)	1.550 (126/1/0)	2.740 (126/1/0)	4.064 (126/1/2)	0.578 (126/1/2)	1.170 (126/1/0)	1.976 (126/1/0)	3.019 (42/1/1)
100th	1.609 (252/1/0)	2.608 (252/1/0)	3.897 (252/1/0)	5.266 (252/1/0)	1.806 (252/1/0)	2.729 (252/1/0)	3.895 (252/1/0)	5.128 (252/1/0)	1.696 (252/1/0)	2.302 (252/1/3)	3.074 (252/1/0)	3.892 (252/1/0)
	MDA$*10$											
1st	1.130 (189/2/0)	1.828 (63/3/0)	2.222 (63/3/1)	2.469 (63/3/0)	0.793 (42/5/1)	1.243 (63/3/0)	1.710 (63/4/0)	1.789 (63/4/1)	0.973 (252/3/3)	0.827 (252/3/3)	1.035 (63/1/1)	1.389 (63/1/2)
10th	0.996 (189/2/2)	1.614 (63/2/2)	1.963 (63/3/2)	2.188 (63/2/0)	0.579 (63/2/1)	1.007 (63/3/2)	1.406 (63/2/1)	1.665 (63/5/0)	0.771 (252/4/0)	0.658 (189/3/3)	0.832 (189/2/1)	1.063 (42/1/2)
50th	0.726 (42/2/2)	0.973 (126/3/3)	1.187 (126/2/2)	1.254 (252/5/0)	0.343 (42/2/0)	0.771 (126/3/2)	1.024 (126/2/3)	0.967 (126/5/3)	0.523 (126/4/1)	0.377 (126/2/1)	0.551 (189/5/2)	0.771 (63/3/0)
90th	0.253 (252/1/1)	0.287 (189/1/2)	0.546 (189/4/3)	0.501 (189/3/2)	0.017 (252/3/3)	0.264 (189/3/3)	0.214 (189/4/2)	0.304 (252/2/1)	0.118 (189/2/2)	0.017 (63/2/2)	−0.157 (126/3/2)	−0.377 (126/5/0)
100th	−0.219 (63/1/1)	−0.039 (126/1/0)	0.231 (189/1/3)	0.197 (189/1/3)	−0.433 (189/1/2)	−0.399 (189/1/1)	−0.461 (189/1/3)	−0.067 (189/1/3)	−0.051 (42/2/3)	−0.579 (189/1/3)	−0.934 (189/1/2)	−1.063 (189/1/1)
	MBH$*10^4$											
1st	0.428 (252/5/1)	1.482 (63/2/0)	2.921 (63/3/1)	4.210 (63/3/0)	0.335 (252/5/3)	0.907 (63/3/0)	1.796 (63/4/0)	2.432 (63/3/2)	0.320 (252/3/3)	0.597 (63/1/0)	1.072 (63/1/1)	1.801 (63/1/0)
10th	0.401 (126/2/3)	1.306 (63/5/1)	2.622 (63/2/2)	3.923 (63/2/2)	0.272 (189/4/3)	0.721 (126/2/0)	1.420 (63/5/1)	2.124 (63/2/0)	0.249 (252/4/2)	0.409 (42/3/2)	0.771 (252/3/1)	0.989 (63/5/2)
50th	0.302 (126/3/3)	0.678 (189/5/2)	1.322 (126/3/3)	1.873 (126/2/3)	0.183 (126/3/2)	0.438 (63/1/0)	0.828 (42/4/0)	1.043 (42/1/1)	0.183 (63/5/3)	0.227 (126/1/0)	0.343 (189/4/0)	0.625 (42/5/1)
90th	0.049 (42/1/2)	0.239 (63/1/0)	0.405 (252/4/2)	0.420 (189/5/2)	0.026 (252/1/2)	0.008 (126/1/3)	0.102 (252/4/0)	−0.072 (189/4/0)	0.061 (42/2/3)	−0.066 (42/2/1)	−0.129 (252/1/3)	−0.160 (126/5/2)
100th	−0.108 (63/1/1)	−0.202 (126/1/0)	0.148 (189/1/3)	0.233 (189/1/3)	−0.166 (189/1/2)	−0.562 (189/1/1)	−0.611 (189/1/3)	−0.674 (189/1/2)	−0.048 (189/1/3)	−0.637 (189/1/2)	−1.161 (189/1/2)	−1.590 (189/1/1)

Table 2. Quantiles for MSFE$*10^6$, MDA$*10$ and MBH$*10^4$ out-of-sample forecast performance of $h = 1, 5, 10, 15$ day–ahead forecasts of 2yr, 5yr, 10yr swap rates from $T_1^* = 308$ (April 4, 2000) to $T_2^* = 2085$ (February 9, 2007) for the 100 models $\{M_s\}_{s=1}^{100} = \{\tau^s, K^s, p^s\}_{s=1}^{100}$. Specifications are shown in parentheses.

	1	2	3	4
1	0.600	0.400	0.000	0.000
2	0.400	0.520	0.080	0.000
3	0.000	0.080	0.800	0.120
4	0.000	0.000	0.120	0.880

MSFE

	1	2	3	4
1	0.308	0.231	0.231	0.231
2	0.083	0.333	0.417	0.167
3	0.464	0.179	0.143	0.214
4	0.091	0.273	0.318	0.318

MDA

	1	2	3	4
1	0.600	0.080	0.160	0.160
2	0.280	0.080	0.280	0.360
3	0.040	0.360	0.280	0.320
4	0.080	0.480	0.280	0.160

MBH

Table 3. Transition probability matrices for one day-ahead forecasts of the 2yr swap rate. The first sample period of 889 forecasts ranges from $T_1^* = 308$ (April 4, 2000) to $T^* = 1196$ (August 31, 2004), the second sample period of 889 forecast ranges from $T^* = 1197$ (September 1, 2004) to $T_2^* = 2085$ (February 9, 2007). The first row contains the relative transition frequencies from the models, from the 1st quartile in the first sample half to the 1st, 2nd, 3rd and 4th quartile in the second sample half, etc.

normalized MSFE

	$h = 1$		$h = 5$		$h = 10$		$h = 15$		all horizons	
1	MedStrat	2.992	MedStrat	2.972	MedStrat	2.942	MedStrat	2.904	MedStrat	11.810
2	Av10MSFE	2.990	AvStrat	2.935	AvStrat	2.886	AvStrat	2.832	AvStrat	11.613
3	BunnMSFE	2.990	Av10MSFE	2.875	Av10MSFE	2.640	Av10MSFE	2.407	Av10MSFE	10.913
4	AnoMSFE	2.981	BunnMSFE	2.872	BunnMSFE	2.637	BunnMSFE	2.394	BunnMSFE	10.892
5	MinMSFE	2.981	AnoMSFE	2.857	BunnMBH	2.626	Av10MDA	2.367	AnoMSFE	10.763
6	BunnMDA	2.967	BunnMDA	2.835	Av10MDA	2.620	Av10MBH	2.345	Av10MDA	10.759

normalized MDA

	$h = 1$		$h = 5$		$h = 10$		$h = 15$		all horizons	
1	AnoMSFE	2.378	AnoMSFE	2.662	MedStrat	2.478	AvStrat	2.379	MedStrat	9.498
2	MedStrat	2.321	BunnMSFE	2.538	AvStrat	2.267	MedStrat	2.370	Av10MDA	8.404
3	Av10MSFE	2.052	Av10MDA	2.487	BunnMBH	2.124	Av10MDA	2.047	BunnMDA	8.347
4	MinMSFE	2.019	BunnMDA	2.484	Av10MBH	2.101	Av10MBH	1.973	AnoMSFE	8.095
5	BunnMDA	2.002	Av10MSFE	2.471	AnoMBH	2.044	AnoMDA	1.855	BunnMSFE	7.851
6	BunnMSFE	1.872	BunnMBH	2.401	AnoMDA	2.038	BunnMDA	1.846	Av10MSFE	7.826

normalized MBH

	$h = 1$		$h = 5$		$h = 10$		$h = 15$		all horizons	
1	AnoMSFE	2.595	AnoMSFE	2.543	MedStrat	2.448	AvStrat	2.330	MedStrat	9.563
2	MedStrat	2.491	Av10MDA	2.421	Av10MBH	2.274	MedStrat	2.250	Av10MDA	8.570
3	MaxMDA	2.362	BunnMDA	2.417	BunnMBH	2.255	Av10MBH	2.064	BunnMDA	8.540
4	Av10MSFE	2.338	BunnMSFE	2.386	AvStrat	2.221	Av10MDA	2.063	AnoMSFE	8.531
5	BunnMSFE	2.210	MedStrat	2.373	AnoMBH	2.209	AnoMDA	1.941	BunnMBH	8.396
6	MinMSFE	2.160	BunnMBH	2.328	BunnMDA	2.197	BunnMDA	1.902	BunnMSFE	8.383

Table 4. MSFE, MDA and MBH comparison of adaptive strategies. For a given forecast horizon the sum of normalized losses for forecasts of the 2yr, 5yr and 10yr rates for 1778 rolling forecasts for the period from $T_1^* = 308$ (April 4, 2000) to $T_2^* = 2085$ (February 9, 2007) are provided. Normalization is accomplished with respect to the best and worst unconditional models in terms of MSFE, MDA and MBH. Results for the six best adaptive strategies are shown.

normalized MSFE

	2yr								5yr								10yr							
	h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15	
	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻	nMSFE	≻
MedStrat	**0.997**	86	**0.991**	83	**0.988**	87	**0.987**	88	0.995	79	**0.985**	82	**0.968**	80	**0.957**	79	**0.999**	94	**0.996**	85	**0.986**	72	**0.960**	68
AvStrat	0.982	41	0.973	62	0.967	76	0.969	81	0.987	41	0.973	64	0.947	64	0.929	74	0.991	42	0.988	75	0.971	68	0.934	68
Av10MSFE	0.996	83	0.951	51	0.894	48	0.816	43	**0.996**	85	0.958	50	0.873	45	0.808	49	0.998	63	0.967	45	0.874	36	0.784	44

normalized MDA

	2yr								5yr								10yr							
	h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15	
	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻
MedStrat	**0.842**	76	0.789	77	**0.627**	66	0.688	72	0.798	87	0.747	56	**0.834**	88	**0.806**	80	**0.681**	68	0.792	74	**1.017**	100	**0.876**	91
Av10MDA	0.650	37	0.801	77	0.565	60	**0.733**	77	**0.826**	90	0.822	77	0.731	64	0.764	74	0.374	29	**0.864**	88	0.726	41	0.550	27
BumMDA	0.683	45	**0.843**	84	0.582	61	0.713	74	0.780	82	**0.856**	90	0.674	49	0.624	61	0.538	47	0.784	72	0.760	51	0.509	25

normalized MBH

	2yr								5yr								10yr							
	h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15	
	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻
MedStrat	**0.845**	67	0.745	76	0.611	67	0.568	68	0.906	92	**0.831**	77	0.826	87	0.798	81	**0.740**	77	0.797	83	**1.010**	100	**0.884**	96
Av10MDA	0.587	26	0.847	87	0.623	69	0.773	83	**1.022**	100	0.756	65	**0.830**	88	**0.807**	81	0.289	10	**0.818**	85	0.735	70	0.483	24
BumMDA	0.689	36	**0.900**	92	**0.655**	75	**0.793**	83	0.950	95	0.762	67	0.786	83	0.665	66	0.387	17	0.754	69	0.756	74	0.444	18

Table 5. MSFE, MDA and MBH comparison of adaptive and unconditional strategies. For each forecast exercise (2yr, 5yr, 10yr swap rates, horizons $h = 1, 5, 10, 15$) normalized average losses for 1778 rolling forecasts for the period from $T_1^* = 308$ (April 4, 2000) to $T_2^* = 2085$ (February 9, 2007) is provided in the column labelled nMSFE, nMDA resp. nMBH. Normalization is accomplished with respect to the best and worst unconditional models in terms of MSFE, MDA resp. MBH. The number of unconditional models that perform (strictly) worse than the adaptive strategy given in the first column is shown in the column labelled ≻.

24

normalized MDA

	2yr								5yr								10yr								Sum	Rank
	h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15			
	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻	nMDA	≻		
Naive	0.325	9	0.163	10	0.102	7	0.119	8	**0.798**	87	0.637	39	0.627	46	0.661	64	0.516	44	**0.984**	98	**0.971**	99	**1.046**	100	6.949	(5)
AR42	0.425	19	**0.675**	63	**0.650**	71	**0.703**	73	0.514	27	**0.801**	74	**0.772**	79	**0.788**	79	0.165	10	0.872	89	0.800	66	0.844	89	8.008	(4)
AR252	**0.633**	35	0.500	43	0.401	38	0.366	36	0.697	67	0.466	14	0.399	23	0.297	21	**0.879**	98	0.808	79	0.766	52	0.638	35	6.850	(6)
DL42	0.400	15	0.404	21	0.542	59	0.614	68	0.037	3	0.267	4	0.523	44	0.533	48	0.319	26	0.536	18	0.800	66	0.867	90	5.842	(7)
DL252	0.458	22	0.337	17	0.390	38	0.446	48	0.147	4	-0.007	0	0.041	3	-0.218	0	0.253	20	0.776	70	0.674	30	0.573	28	3.871	(8)
MedStrat	**0.842**	76	0.789	77	**0.627**	66	0.688	72	0.798	87	0.747	56	**0.834**	88	**0.806**	80	**0.681**	68	0.792	74	**1.017**	100	**0.876**	91	9.498	(1)
Av10MDA	0.650	37	0.801	77	0.565	60	**0.733**	77	**0.826**	90	0.822	77	0.731	64	0.764	74	0.374	29	**0.864**	88	0.726	41	0.550	27	8.404	(2)
BumMDA	0.683	45	**0.843**	84	0.582	61	0.713	74	0.780	82	**0.856**	90	0.674	49	0.624	61	0.538	47	0.784	72	0.760	51	0.509	25	8.347	(3)

normalized MBH

	2yr								5yr								10yr								Sum	Rank
	h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15		h = 1		h = 5		h = 10		h = 15			
	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻	nMBH	≻		
Naive	0.265	10	0.227	8	0.087	8	0.113	18	0.453	17	0.602	33	0.528	42	0.567	53	0.347	14	0.851	92	**0.898**	96	**0.866**	96	5.803	(6)
AR42	**0.458**	23	**0.611**	57	**0.596**	66	**0.686**	75	**0.543**	28	**0.773**	67	**0.659**	63	**0.719**	73	0.132	4	0.819	86	0.679	56	0.642	44	7.319	(4)
AR252	0.399	21	0.468	41	0.303	33	0.300	35	0.364	8	0.376	10	0.295	10	0.300	31	0.719	71	0.769	76	0.700	62	0.655	52	5.648	(7)
DL42	0.348	16	0.307	13	0.422	50	0.604	68	0.231	4	0.497	21	0.568	44	0.610	59	0.751	80	0.799	83	0.773	77	0.704	75	6.614	(5)
DL252	0.383	20	0.491	47	0.395	49	0.412	50	0.286	6	0.176	4	0.176	5	0.064	8	**0.802**	89	**0.924**	96	0.811	83	0.709	77	5.628	(8)
MedStrat	**0.845**	67	0.745	76	0.611	67	0.568	68	0.906	92	**0.831**	77	0.826	87	0.798	81	**0.740**	77	0.797	83	**1.010**	100	**0.884**	96	9.563	(1)
Av10MDA	0.587	26	0.847	87	0.623	69	0.773	83	**1.022**	100	0.756	65	**0.830**	88	**0.807**	81	0.289	10	**0.818**	85	0.735	70	0.483	24	8.570	(2)
BumMDA	0.689	36	**0.900**	92	**0.655**	75	**0.793**	83	0.950	95	0.762	67	0.786	83	0.665	66	0.387	17	0.754	69	0.756	74	0.444	18	8.540	(3)

Table 6. MDA and MBH of benchmark strategies and comparison to adaptive strategies. For each forecast exercise the normalized losses for the period from T_1^* to T_2^* are provided in the column labelled nMDA resp. nMBH. Normalization is accomplished with respect to the best and worst unconditional models in terms of MDA resp. MBH. The number of unconditional models that perform (strictly) worse than the strategy given in the first column is shown in the column labelled ≻. The last column shows for a given strategy the sum of the normalized MDA's resp. MBH's over the 12 forecast exercises.

	MDA			
	Naive	AR42	AR252	Sum
MedStrat	9 (4)	8 (2)	10 (5)	27 (11)
Av10MDA	8 (4)	6 (1)	9 (5)	23 (10)
BunnMDA	7 (4)	6 (2)	8 (3)	21 (9)

Adaptive strategy
(significantly) better than
benchmark strategy

	MDA			
	MedStrat	Av10MDA	BunnMDA	Sum
Naive	3 (0)	4 (1)	5 (1)	12 (2)
AR42	4 (0)	6 (1)	6 (1)	16 (2)
AR252	2 (0)	3 (1)	4 (0)	9 (1)

Benchmark strategy
(significantly) better than
adaptive strategy

	MBH			
	Naive	AR42	AR252	Sum
MedStrat	11 (5)	10 (3)	12 (9)	33 (17)
Av10MDA	8 (5)	9 (1)	10 (6)	27 (12)
BunnMDA	9 (5)	8 (2)	9 (5)	26 (12)

Adaptive strategy
(significantly) better than
benchmark strategy

	MBH			
	MedStrat	Av10MDA	BunnMDA	Sum
Naive	1 (0)	4 (1)	3 (1)	8 (2)
AR42	2 (0)	3 (0)	4 (0)	9 (0)
AR252	0 (0)	2 (1)	3 (0)	5 (1)

Benchmark strategy
(significantly) better than
adaptive strategy

Table 7. MDA and MBH comparison of benchmark and adaptive strategies of 1778 rolling forecasts for the period from $T_1^* = 308$ (April 4, 2000) to $T_2^* = 2085$ (February 9, 2007) for 2yr, 5yr, 10yr swap rates and horizons $h = 1, 5, 10, 15$. Each panel provides the number of forecast exercises, out of 12, in which the strategy given in the first column (significantly) outperforms the strategy given in the first row in terms of MDA resp. MBH. Significance is tested using a one–sided null hypothesis, the Diebold–Mariano test statistic, Diebold and Mariano (1995), and the 10% critical value from the standard normal distribution. Note that for the forecast exercise $h = 1$ and the 2yr rate the MedStrat and Naive strategy have a MDA of 0.546.

Appendix

	1	2	3	4
1	0.707	0.287	0.007	0.000
2	0.227	0.530	0.243	0.000
3	0.027	0.170	0.663	0.140
4	0.040	0.013	0.087	0.860

MSFE

	1	2	3	4
1	0.488	0.235	0.129	0.147
2	0.267	0.246	0.236	0.251
3	0.205	0.278	0.269	0.248
4	0.067	0.258	0.348	0.328

MDA

	1	2	3	4
1	0.460	0.163	0.207	0.170
2	0.247	0.207	0.260	0.287
3	0.177	0.283	0.280	0.260
4	0.117	0.347	0.257	0.280

MBH

Table 8. Average transition probability matrices. Transition matrices are averaged for the forecasts for the horizons $h = 1, 5, 10, 15$ and swap rates 2yr, 5yr, 10yr. The first sample period of 889 forecasts ranges from $T_1^* = 308$ (April 4, 2000) to $T^* = 1196$ (August 31, 2004), the second sample period of 889 forecast ranges from $T^* = 1197$ (September 1, 2004) to $T_2^* = 2085$ (February 9, 2007). The first row contains the average relative transition frequencies from the models, from the 1st quartile in the first sample half to the 1st, 2nd, 3rd and 4th quartile in the second sample half, etc.

SFB 649 Discussion Paper Series 2008

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

001   "Testing Monotonicity of Pricing Kernels" by Yuri Golubev, Wolfgang Härdle and Roman Timonfeev, January 2008.

002   "Adaptive pointwise estimation in time-inhomogeneous time-series models" by Pavel Cizek, Wolfgang Härdle and Vladimir Spokoiny, January 2008.

003   "The Bayesian Additive Classification Tree Applied to Credit Risk Modelling" by Junni L. Zhang and Wolfgang Härdle, January 2008.

004   "Independent Component Analysis Via Copula Techniques" by Ray-Bing Chen, Meihui Guo, Wolfgang Härdle and Shih-Feng   Huang, January 2008.

005   "The Default Risk of Firms Examined with Smooth Support Vector Machines" by Wolfgang Härdle, Yuh-Jye Lee, Dorothea Schäfer and Yi-Ren Yeh, January 2008.

006   "Value-at-Risk and Expected Shortfall when there is long range dependence" by Wolfgang Härdle and Julius Mungo, Januray 2008.

007   "A Consistent Nonparametric Test for Causality in Quantile" by Kiho Jeong and Wolfgang Härdle, January 2008.

008   "Do Legal Standards Affect Ethical Concerns of Consumers?" by Dirk Engelmann and Dorothea Kübler, January 2008.

009   "Recursive Portfolio Selection with Decision Trees" by Anton Andriyashin, Wolfgang Härdle and Roman Timofeev, January 2008.

010   "Do Public Banks have a Competitive Advantage?" by Astrid Matthey, January 2008.

011   "Don't aim too high: the potential costs of high aspirations" by Astrid Matthey and Nadja Dwenger, January 2008.

012   "Visualizing exploratory factor analysis models" by Sigbert Klinke and Cornelia Wagner, January 2008.

013   "House Prices and Replacement Cost: A Micro-Level Analysis" by Rainer Schulz and Axel Werwatz, January 2008.

014   "Support Vector Regression Based GARCH Model with Application to Forecasting Volatility of Financial Returns" by Shiyi Chen, Kiho Jeong and Wolfgang Härdle, January 2008.

015   "Structural Constant Conditional Correlation" by Enzo Weber, January 2008.

016   "Estimating Investment Equations in Imperfect Capital Markets" by Silke Hüttel, Oliver Mußhoff, Martin Odening and Nataliya Zinych, January 2008.

017   "Adaptive Forecasting of the EURIBOR Swap Term Structure" by Oliver Blaskowitz and Helmut Herwatz, January 2008.