Non-constant Hazard Function and Inflation Dynamics

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Abstract

This paper explores implications of nominal rigidity characterized by a non-constant hazard function for aggregate dynamics. I derive the NKPC under an arbitrary hazard function and parameterize it with the Weibull duration model. The resulting Phillips curve involves lagged inflation and lagged expectations. It nests the Calvo NKPC as a limiting case in the sense that the effects of both terms are canceled out under the constant-hazard assumption. Furthermore, I find lagged inflation always has negative coefficients, thereby making it impossible to interpret inflation persistence as intrinsic. The numerical evaluation shows that the increasing hazard function leads to hump-shaped impulse responses of inflation to monetary shocks, and output leads inflation.

JEL classification: E12; E31
Key words: Hazard function, Weibull distribution, New Keynesian Phillips Curve

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1 Introduction

The Calvo pricing assumption (Calvo, 1983) has become predominant in the world of applied monetary analysis under nominal rigidity. Nevertheless, its ubiquity is neither based on theoretical soundness nor on empirical appeal. Instead, the main argument for using this approach is that it gives great tractability, making it useful to analyze various monetary policy issues. Recently, one of the main criticisms of the Calvo approach is that it implies a constant hazard function for the stochastic pricing behavior. Unfortunately, constant hazard functions are largely rejected by empirical evidence from the micro level data (See: e.g. Dhyne et al., 2006, and references cited therein)\(^1\). Given this conflict between theory and empirical evidence, it is important to understand to which extent the constant hazard function is innocuous for aggregate dynamics implied by the model.

To tackle this question, I construct a generalized time-dependent model of nominal rigidity à la Wolman (1999) and derive the New Keynesian Phillips curve (NKPC) conditional on an arbitrary hazard function. This model is useful to study the implications of nominal rigidity for the aggregate dynamics because it models price rigidity in the most general form, so that, except for the time-dependent pricing structure, its implications do not depend on any one specific price setting assumption.

In the analytical results, I show that the NKPC derived from the model involves components including lagged inflation, forward-looking and lagged expectations of inflation and real marginal costs. This version of the Phillips curve nests the Calvo case in the sense that, under a constant hazard function, effects of lagged inflation exactly cancel those of lagged expectations, so that only current real terms and future expectations of inflation remain in the expression, as in the Calvo NKPC. In the general case, however, both lagged inflation and inflation expectations should be present. The reason why lagged inflation and lagged expectations exert opposite effects on the current inflation is due to the following facts. On the one hand, sticky prices ensure that lagged expectations have a long lasting influence on inflation, in that higher expectations of marginal costs leads to higher inflation. On the other hand, the "front-loading" effect deters the current inflation to react to a current economic condition. That means that a high level of past inflation hinders the ability of current inflation to continue on a high course. In the more general setting, both effects work against each other, but in the Calvo model, these two effects just cancel each other out.

Furthermore, I find a general result relating to the debate on the nature of inflation persistence. Starting with Fuhrer and Moore (1995), the older literature believed that the NKPC needs a significant backward-looking component in order to generate the key feature of reduced-form Phillips curve regression: the positive dependence of inflation on its lags. More recently, however, a new consensus has been emerging in the literature, showing that inflation persistence is mainly due to its time-varying persistent trend. Detrended inflation has less significant or even negative autocorrelations. (Cogley and Sbordone, 2006, Bils et al., 2009). For example, Cogley and Sbordone (2006) find that when correctly accounting for the time-varying trend inflation, the purely forward-looking model explains the persistence of the inflation deviation from its trend

\(^1\)The results of those work on the empirical hazard function is not conclusive. Some find strong support for increasing hazard functions (e.g. :Fougere et al., 2005, Goette et al., 2005), while others find evidence in favour of decreasing hazards (e.g.:Alvarez, 2007, Campbell and Eden, 2005).
quite well. From my generalized NKPC, I find that the coefficients on lagged inflation should always be negative, even after controlling for the effects of lagged expectations of inflation. In particular, I show that the lagged expectations of inflation can be transformed into lagged inflation and expectational errors, which reduce to white noises under rational expectations. After transforming all the lagged expectations into lagged inflation, the coefficient on the lagged inflation is still negative, thereby we can conclude that the forward-looking Phillips curve is able to adequately account for the persistence of inflation deviations from the steady state, and that lagged inflation is not important for this persistence.

In the numerical assessment, I parameterize the hazard function with a functional form motivated from the Weibull duration model. By definition, it is a function with two parameters. One parameter is the scale parameter, which controls the average duration of the price adjustment. The other is the shape parameter that determines the monotonic property of the hazard function. By changing the value of the shape parameter, this hazard function enables the incorporation of a wide range of hazard profiles. When simulating the full-scale general equilibrium model, I combine the generalized NKPC with a simple aggregate demand curve and an exogenous nominal money growth process. The simulation results show that the increasing-hazard model generates hump-shaped impulse responses of inflation and real output to the nominal money growth shock. Moreover, impulse responses of output lead those of inflation, which reflects a robust feature of the data.

This paper relates to a number of existing studies. While Mankiw and Reis (2002) also emphasize the role of lagged expectations in propagating shocks through the sticky information assumption, this model finds that a similar mechanism can be motivated within the time-dependent pricing paradigm. Mash (2004) constructs a mixture of the Calvo and Taylor pricing models and shows that the NKPC under increasing hazard functions replicates a large part of persistence in inflation and in the output gap. Parallel to my methodology, Costain and Nákov (2008) parameterize a hazard function in a state-dependent pricing context. The most closely related paper in the literature is Sheedy (2007), who parameterizes the hazard function in such a way that the resulting NKPC has a positive coefficient on lagged inflation given that the hazard function is upward sloping. This result, however, is only valid under his hazard function specification.

The remainder of the paper is organized as follows: in section 1, I introduce the model with generalized time-dependent pricing at the firm’s level and derive the New Keynesian Phillips curve; section 2 shows some analytical results to give the structural interpretation of the coefficients of the generalized NKPC; in section 3, I introduce the calibration strategy of the model’s parameters and present the simulation results; section 4 contains some concluding remarks.

2 The model

In this section, I introduce the generalized time-dependent model of nominal rigidity à la Wolman (1999). The most important components of the model are 1) monopolistic competitive firms who set their prices according to the demand condition and the probabilities for re-optimizing their prices, and 2) firms cannot adjust their price whenever they want, instead, the opportunities for re-adjusting their prices depend on exogenous hazard rates, which are based on the length of
time since the last adjustment.

2.1 Monopolistic competition firms

I consider an economy with a continuum of monopolistic competitive firms which are differentiated with respect to the type of worker they use, indexed by \( i \in \{0, 1\} \). The final goods sector is perfectly competitive and produces a single final good, \( Y_t \), with all intermediate goods using a CES aggregate production function (Dixit and Stiglitz, 1977)

\[
Y_t = \left( \int_0^1 Y_{t,i}^{\frac{\eta-1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}}, \tag{1}
\]

Given this aggregate production function and the market structure, the profit maximization problem of the final-good firm solves the demand function for intermediate goods,

\[
Y_{d,t}^{i} = \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} Y_t, \tag{2}
\]

Where \( P_{i,t} \) denotes the nominal price of good \( i \), and \( P_t \) is the aggregate price for one unit of the final good \( Y_t \). It follows that the welfare-based aggregate price index is obtained by the following expression:

\[
P_t = \left( \int_0^1 P_{t,i}^{1-\eta} \, di \right)^{\frac{1}{1-\eta}}, \tag{3}
\]

2.2 The generalized time-dependent pricing

The principal assumption of the model is that firms cannot adjust their price whenever they want. Instead, opportunities for re-optimizing their prices depend on exogenous probabilities which are related to the length of time since the last adjustment. I summarize this limited price adjustment scheme using an arbitrary hazard function \( h_j \), where \( j \) denotes the period of time elapsed since the last price adjustment \( j \in \{0, J\} \). Because it is difficult to justify that some firms keep their prices constant forever, I assume there is a maximum number of periods in which a firm can fix its price (\( J \)). Firms in the same vintage (\( j \)) have the same probability (\( h_j \)) of adjusting their prices. Note that, for the basic model, I do not parameterize the hazard function, so that the relative magnitudes of the hazard rates are totally free. By doing that, the analytical results derived from the model should be robust to any shape of the hazard function.

2.2.1 Dynamics of the vintage distribution

To aggregate the economy, we need to track the distribution of firms’ vintages. At the end of each period, those firms that reoptimize their prices in the current period are labelled by the ‘vintage 0’, while the other firms move to the next vintage \( j+1 \) because their prices age by one-period. Assume that the ex ante distribution of price vintages is \( \Theta_t = \{\theta_t(0), \theta_t(2) \cdots \theta_t(J-1)\} \), then, after firms re-optimize their prices, the ex post distribution \( \Theta'_t = \{\theta'_t(0), \theta'_t(2) \cdots \theta'_t(J-1)\} \) is obtained by
\[
\theta'_t(j) = \begin{cases} 
\sum_{i=1}^{J} b_j \theta_t(i), & \text{when } j = 0 \\
\alpha_j \theta_t(j), & \text{when } j = 1 \cdots J - 1
\end{cases}
\]

(4)

When period \( t \) is over, this ex post distribution \( \Theta'_t \) becomes an ex ante distribution for the new period \( \Theta_{t+1} \). Table (??) summarizes key notations concerning the dynamics of vintage distribution.

Table ?? is about here

2.2.2 The stationary distribution

As long as the hazard rates are well defined, dynamics of the vintage distribution can be viewed as a Markov process with an invariant distribution \( \Theta \), obtained by solving \( \theta_t(j) = \theta'_t(j - 1) = \theta_{t+1}(j) \). It yields the stationary vintage distribution \( \theta(j) \) as follows:

\[
\theta(j) = \frac{\prod_{i=0}^{j} \alpha_i}{\sum_{n=0}^{J-1} \prod_{i=0}^{n} \alpha_i} = \frac{S_j}{\sum_{n=0}^{J-1} S_n}, \text{ for } j = 0, 2 \cdots J - 1
\]

(5)

Let’s assume the economy converges to this invariant distribution fairly quickly, so that regardless of the initial vintage distribution, I only consider the economy with the above invariant distribution of vintages. For any stationary distribution \( \theta(j) \), the aggregate price index (3) can be rewritten as a distributed sum of all vintage prices, reflecting the fact that all firms setting prices in the same period should choose the same price, assuming no other heterogeneity affects the firms’ price decisions.

The optimal price is defined as \( P^*_{t-j} \), set \( j \) periods ago. It allows for the aggregate price index to be obtained by the weighted sum of the past optimal prices as follows:

\[
P_t = \left( \sum_{j=0}^{J-1} \theta(j) P^*_{t-j} \right)^{\frac{1}{1-\eta}}
\]

(6)

2.2.3 Optimal pricing

In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be adjusted in the near future. Consequently, adjusting firms choose an optimal price that maximizes the discounted sum of real profits over the time horizon during which the new price is expected to hold. The probability that the new price is fixed is given by the survival function \( S(j) \) defined in Table (??). The maximization problem is obtained by

\[
\max_{P_t} \sum_{j=0}^{J-1} S_j E_t \{ Q_{t,t+j} [ Y^d_{t+j} | t ] - T C_{t+j} / P_{t+j} \}
\]

Where \( E_t \) denotes the conditional expectation operator based on the information set at period \( t \), and \( Q_{t,t+j} \) is the stochastic discount factor which is appropriate for discounting real
profits from time \( t + j \) to time \( t \). \( Y^d_{t+j|t} \) denotes real output demand in period \( t + j \) for a firm that resets its price in period \( t \). I implicitly assume here that firms have no monopolistic power in individual labor markets, so that firms do not consider the possibility of their price decisions affecting future real wages and hence future marginal costs. As a result, the optimal price has no direct effect on the future cost.

Firms maximize profits subject to two constraints. The first is the production function

\[ Y_t = Z_t N_t \]  

where \( Z_t \) denotes productivity which is identical across sectors. Log deviation of productivity \( \hat{z}_t \) follows an exogenous stochastic process \( \hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \), where \( \varepsilon_{z,t} \) is white noise and \( \rho_z \in [0,1) \). \( Y^d_{t+j|t} \) denotes demand of real output in period \( t + j \) for a firm that resets its price in period \( t \), which follows:

\[ Y^d_{t+j|t} = \left( \frac{P^*_t}{P^*_{t+j}} \right)^{-\eta} Y_{t+j} \]

The parameter \( \eta \) can be interpreted as the elasticity of substitution among intermediate goods.

This optimization yields the following first order necessary condition:

\[ P^*_t = \left( \frac{\eta}{\eta - 1} \right) \frac{\sum_{j=0}^{J-1} S_j E_t[Q_{t+j} Y_{t+j} P^*_t + 1 MC_{t+j}]}{\sum_{j=0}^{J-1} S_j E_t[Q_{t+j} Y_{t+j} P^*_t + 1]} } \]  

(8)

Where \( MC_{t+j} \) denotes nominal marginal cost. One can see that the optimal price is equal to the markup multiplied by the weighted sum of future nominal marginal costs. The weight depends on the survival rate. In addition, the maximum time horizon \( J \) depends on the speed at which the survival function goes to zero. In the Calvo case, where \( S(j) = (1 - \alpha) \alpha^j \), survival rates approach zero as \( j \) increases, but never reach, thereby making the decision horizon infinite in this case.

2.3 Steady State

Before starting to derive the NKPC in terms of log deviations from the steady state, I define it as follows: in the steady state, all real variables are constant, while, all nominal variables and the price level grows at a constant rate of trend inflation, which is equal to the growth rate of nominal money stock set by the central bank \( g = \bar{\pi} \). If we define \( \bar{X} \) as the steady state value of variable \( X \), then the optimality condition (8) can be rewritten as:

\[ P^*_t = \left( \frac{\eta}{\eta - 1} \right) \frac{\sum_{j=0}^{J-1} S_j E_t[Q_{t+j} Y_{t+j} P^*_t + 1 MC_{t+j}]}{\sum_{j=0}^{J-1} S_j E_t[Q_{t+j} Y_{t+j} P^*_t + 1]} } \]  

(8)

Where \( MC_{t+j} \) denotes nominal marginal cost. One can see that the optimal price is equal to the markup multiplied by the weighted sum of future nominal marginal costs. The weight depends on the survival rate. In addition, the maximum time horizon \( J \) depends on the speed at which the survival function goes to zero. In the Calvo case, where \( S(j) = (1 - \alpha) \alpha^j \), survival rates approach zero as \( j \) increases, but never reach, thereby making the decision horizon infinite in this case.

\[ 2 \]Here I assume the firm type is not the same as the labor type, thereby, in each labor market, all intermediate firms demand some labor from it. As a result, labor markets are competitive, and there is no difference between real wages among individual firms. See Woodford (2003).
\[ \hat{p}_t^* = \frac{\eta \sum_{j=0}^{J} \beta^j S(j) \hat{p}_{t+j}^* \bar{mc}}{\eta - 1} = \frac{\eta \sum_{j=0}^{J} \beta^j S(j) \hat{p}_{t+j}^* \bar{mc}}{\eta - 1} \]

\[ r = \frac{\hat{p}_t^*}{\bar{p}_t} = \frac{\eta - 1}{\eta} \left[ \sum_{j=0}^{J} \beta^j S(j) g^{\eta j} \right] \]

From Equation (9), we see that, at the steady state, the relative price \( r = \frac{\hat{p}_t^*}{\bar{p}_t} \) is a constant, equal to the product of a constant markup, real marginal cost and an extra term, which summarizes the effect of trend inflation on the relative price. When trend inflation is zero \( (g = 1) \), then this term is equal to one, allowing this equation to reduce to the static pricing equation, which expresses the optimal price as the nominal marginal cost multiplied by a constant markup \( \left( \frac{\eta}{\eta - 1} \right) \). When the trend inflation is greater than zero \( (g > 1) \), however, the bracketed term is greater than one, so is the relative price, meaning that the optimal price is adjusted at a higher rate than the trend inflation. The economic intuition is that forward-looking price setters take into account that trend inflation erodes their relative prices over time, so that they need to ‘front-load’ the price when they reoptimize it. Consequently, this higher relative price ratio leads to lower steady state output and hence induces an additional welfare loss caused by the positive trend inflation.

2.4 New Keynesian Phillips curve

To derive the NKPC, I first log-linearize equations (6) and (8) around the zero-inflation steady state. Defining \( \tilde{x}_t = log X_t - log \bar{X}_t \), I obtain following log-linearized equations:

\[ \tilde{p}_t^* = E_t \left[ \sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi} (\bar{mc}_{t+j} + \hat{p}_{t+j}) \right], \text{ where } \Psi = \sum_{j=0}^{J-1} \beta^j S_j \] \( (10) \)

\[ \hat{p}_t = \sum_{k=0}^{J-1} \theta(k) \hat{p}_{t-k}^* \] \( (11) \)

After some tedious algebra, I obtain the generalized NKPC from Equation (10) and (11)\(^3\). To reveal the essential implications of the NKPC for the inflation dynamics, I derive it without trend inflation, i.e. \( g = 1 \).

\(^3\)Log-linearization of price equations and the detailed derivation of NKPC can be found in a technical Appendix, available upon request from the author.
\[
\hat{\pi}_t = \sum_{k=0}^{J-1} \frac{\theta(k)}{1 - \theta(0)} E_{t-k} \left[ \sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi} \tilde{m}_{c_{t+j-k}} + \sum_{i=1}^{J-1} \sum_{j=1}^{J-1-i} \frac{\beta^i S_j}{\Psi} \hat{\pi}_{t+i-k} \right] - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1},
\]

where \( \Phi(k) = \frac{\sum_{j=k}^{J-1} S_j}{\sum_{j=1}^{J-1} S_j} \)

At first sight, this New Keynesian Phillips curve has a much more complex structure than the Calvo NKPC. It involves not only lagged inflation but also lagged expectations that were built into pricing decisions in the past. All coefficients in the NKPC are derived from deep parameters which represent either stationary distributional parameters or the preference parameter. In particular, coefficients before lagged inflation and lagged expectations \( (\frac{\theta(0)}{1-\theta(0)}, \Phi(k)) \), representing the compositional proportion of each dated price in aggregate inflation.

To be more unobstructed, I give an example with \( J = 3 \):

\[
\hat{\pi}_t = \frac{1}{(\alpha_1 + \alpha_1 \alpha_2)} \Psi \tilde{m}_{c_1} + \frac{\alpha_1}{(\alpha_1 + \alpha_1 \alpha_2)} \Psi \tilde{m}_{c_{t-1}} + \frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_1 \alpha_2)} \Psi \tilde{m}_{c_{t-2}} + \frac{1}{\alpha_1 + \alpha_1 \alpha_2} E_{t-1} \left( \frac{\beta \alpha_1}{\Psi} \tilde{m}_{c_{t+1}} + \frac{\beta^2 \alpha_1 \alpha_2}{\Psi} \tilde{m}_{c_{t+2}} + \frac{\beta \alpha_1 + \beta^2 \alpha_1 \alpha_2}{\Psi} \hat{\pi}_{t+1} + \frac{\beta^2 \alpha_1 \alpha_2}{\Psi} \hat{\pi}_{t+2} \right) + \frac{\alpha_1}{\alpha_1 + \alpha_1 \alpha_2} E_{t-2} \left( \frac{\beta \alpha_1}{\Psi} \tilde{m}_{c_{t-1}} + \frac{\beta^2 \alpha_1 \alpha_2}{\Psi} \tilde{m}_{c_{t-2}} + \frac{\beta \alpha_1 + \beta^2 \alpha_1 \alpha_2}{\Psi} \hat{\pi}_{t-1} + \frac{\beta^2 \alpha_1 \alpha_2}{\Psi} \hat{\pi}_{t} \right) - \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_1 \alpha_2} \hat{\pi}_{t-1}
\]

In this example, current inflation depends on marginal costs, lagged inflation and a complex weighted sum of expectations. All coefficients are expressed in terms of non-adjustment rates \((\alpha_j = 1 - h_j)\) and the subjective discount factor \( \beta \).

It is natural to ask why these lagged terms are absent in the Calvo NKPC. Are there new insights of the NKPC that can be gained by relaxing the constant hazard assumption? The answer is yes. In the next section, I use a proposition to elaborate this point formally.

3 Analytical results

3.1 New insights of the generalized NKPC:

**Proposition 1** When assuming the hazard rates are constant w.r.t. time-since-last-adjustment, the generalized NKPC (12) reduces to the standard Calvo New Keynesian Phillips curve.

\[
\hat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tilde{m}_{c_t} + \beta E_t (\hat{\pi}_{t+1})
\]

**Proof.** : see Appendix A.
From the derivation, we see that the Calvo NKPC is equivalent to the following equation:

$$\tilde{\pi}_t = E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i \tilde{m_{ct+i}} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \tilde{\pi}_{t+i} \right)$$

(13)

Lag Equation (13), the following expressions must hold too.

$$\tilde{\pi}_{t-1} = E_{t-1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{ct+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \tilde{\pi}_{t+i-1} \right)$$

$$\tilde{\pi}_{t-2} = E_{t-2} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{ct+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \tilde{\pi}_{t+i-1} \right)$$

These equations reveal why the lagged inflation and lagged expectations are missing in the Calvo NKPC. Given the constant hazard function, the effects of lagged inflation exactly cancel out the effects of lagged expectations, leaving only current variables and forward-looking expectations in the NKPC. In the more general case, however, both lagged inflation and inflation expectations should be present in the Phillips curve.

In addition, given any values between zero and one for non-adjustment rates ($\alpha_j$), coefficients of the lagged inflations are always negative, while coefficients of lagged expectations are always positive. The reason why lagged inflation and lagged expectations exert opposite effects on the current inflation is the following: on the one hand, because the price is sticky, expectations have a long lasting influence on the economy. The higher the expectations of marginal costs, the higher is the inflation; on the other hand, past inflation has a negative impact on current inflation due to the "front-loading" effect. Again, because prices are sticky, firms adjust more than necessary to hedge against the risk that they might not be allowed to re-optimize again in the near future and would be unwilling to react to a current economic condition. The ‘front-loading’ pricing therefore deters the price adjustment needed in the future. Due to this effect, a high level of past inflation hinders the ability of current inflation to continue to be high. In the more general setting, both effects work against each other, but in the Calvo model, these two effects just cancel each other out.

3.2 The role of lagged inflation:

The next question that I can address by using the generalized NKPC is whether the inflation persistence is ‘intrinsic ’ in this model, defined as inflation driven by its own lags with positive coefficients.

To answer this question, we need to identify the sign of the coefficient on lagged inflation, while taking the effects of lagged expectations into account. To do it, I substitute lagged expectations of inflation from the Phillips curve with expectational errors by using the following identities:
\[
E_{t-1} [\hat{\pi}_{t+1}] - E_t [\hat{\pi}_{t+1}] + E_t [\hat{\pi}_{t+1}] = \epsilon_t + E_t [\hat{\pi}_{t+1}]
\]
\[
E_{t-2} [\hat{\pi}_t] - E_{t-1} [\hat{\pi}_t] + E_{t-1} [\hat{\pi}_t] = \epsilon_{t-1} + E_{t-1} [\hat{\pi}_t]
\]
\[
E_{t-1} [\hat{\pi}_t] - \hat{\pi}_t + \hat{\pi}_t = \eta_t + \hat{\pi}_t
\]
\[
E_{t-2} [\hat{\pi}_{t-1}] - \hat{\pi}_{t-1} + \hat{\pi}_{t-1} = \eta_{t-1} + \hat{\pi}_{t-1}
\]

Where \( \epsilon_t \) and \( \eta_t \) are white noises under rational expectations.

Again, when \( J = 3 \), I obtain the following compact form of the NKPC:

\[
\hat{\pi}_t = -\frac{\alpha_2}{1 + \alpha_2 + \beta \alpha_1 \alpha_2} \hat{\pi}_{t-1} + \Omega_1 E_t [\hat{\pi}_{t+1}] + \Omega_2 E_t [\hat{\pi}_{t+2}] + \Omega_3 F_t [\hat{m} \hat{c}] + \Omega_3 \omega_t \tag{14}
\]

\[
\Omega_1 = \frac{\beta + \beta^2 \alpha_2 + \beta^2 \alpha_1 \alpha_2}{1 + \alpha_2 + \beta \alpha_1 \alpha_2}, \quad \Omega_2 = \frac{\beta^2 \alpha_2}{1 + \alpha_2 + \beta \alpha_1 \alpha_2}, \quad \Omega_3 = \frac{(1 + \beta \alpha_1 + \beta^2 \alpha_1 \alpha_2)(1 + \alpha_2)}{1 + \alpha_2 + \beta \alpha_1 \alpha_2}
\]

Where \( F_t [\hat{m} \hat{c}] \) summarizes all expectational terms of marginal cost and \( \omega_t \) is a linear combination of white noise expectational errors under rational expectations. Given any well-defined hazard rate, the alphas lie between zero and one, and the coefficient on \( \hat{\pi}_{t-1} \) is always negative. Since a negative coefficient works against inflation persistence, this theory does not provide support for intrinsic persistence. Instead, persistence should come from the additional moving-average terms of real shocks through the presence of the lagged expectations\(^4\). This result is consistent with empirical evidence shown by Cogley and Sbordone (2006) and Bils et al. (2009), that the purely forward-looking model explains the persistence of the inflation deviation from its trend quite well when applied to the correctly detrended inflation data. Based on these results, we can conclude that the forward-looking Phillips curve is able to adequately account for the persistence of inflation deviations from the steady state, and that lagged inflation is not important for this persistence.

### 4 Numerical experiment

To study the effects of varying shapes of the hazard function on the dynamics of inflation and output gap, I close the model by adding a simple aggregate demand condition and a nominal money growth rule to the NKPC.

\[
\hat{m}_t = \gamma \hat{y}_t \tag{15}
\]
\[
\hat{m} \hat{c}_t = \kappa_1 \hat{y}_t - \kappa_2 \hat{z}_t \tag{16}
\]
\[
\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + g_t \tag{17}
\]
\[
\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_t \tag{18}
\]
\[
g_t = \rho_g g_{t-1} + u_t \tag{19}
\]

Equation (15) represents a simple money-market equilibrium condition, where \( \hat{m}_t \) is the log deviation of the real money balance from the steady state. This equation can be motivated\(^4\)In principle, this result can be extended to the NKPC with an arbitrary length of \( J \).
through the quantity theory of money. with a constant velocity. \( \gamma \) is the income elasticity of money demand. Equation (16) defines the real marginal cost as a linear function of real output gap and the technology shock. \( \kappa_1 \) and \( \kappa_2 \) are positive coefficients, which measure the degree of real rigidity (Ball and Romer, 1990). Equation (17) is a nominal money growth rule, where \( g_t \) denotes the exogenous growth rate of the nominal money stock. For both shocks \((\tilde{z}_t, g_t)\), I assume that they are following AR(1) processes with i.i.d. innovations.

### 4.1 Calibration

In calibration, I use a novel strategy for parameterizing the hazard function. Since the hazard function in this model is defined in terms of the time-since-last-adjustment, it is reasonable to base its calibration on the well-established statistical theory of duration analysis. In particular, the functional form I apply is based on the Weibull distribution with two parameters\(^5\).

\[
h(j) = \frac{\tau}{\lambda} \left( \frac{j}{\lambda} \right)^{\tau-1}
\]

(20)

\( \lambda \) is the scale parameter, which controls the average duration of the price adjustment, while \( \tau \) is the shape parameter to determine the monotonic property of the hazard function. It enables the incorporation of a wide range of hazard functions by using various values of the shape parameter. In fact, any value of the shape parameter that is greater than one corresponds to an increasing hazard function, while values ranging between zero and one lead to a decreasing hazard function. By setting the shape parameter to one, we can retrieve the Poisson process from the Weibull distribution. In the figure (1), I give some examples of hazard functions and the corresponding Weibull distributions with varying values of the shape parameter.

To calibrate the Weibull parameters, I choose the shape parameter \( \tau \) in the range between 1 and 2. Even though this range only covers increasing hazard functions, it is more theoretically justified, because it makes the maximum number of price duration \( J \) finite. Furthermore, I set \( \lambda = 4.5 \), so that it implies an average non-adjustment rate equal to 0.75, a value commonly used for the Calvo model. In the calibration of the preference parameters, I assume \( \beta = 0.9902 \), which implies a steady state real return on financial assets of about four percent per annum. I also choose \( \eta = 10 \), which implies the markup is around 11%. Following Mankiw and Reis (2002), I choose income elasticity of money demand \( \gamma = 0.5 \). Finally, I set \( \kappa_1 = \kappa_2 = 0.15 \), implying a mild degree of 'real rigidity'.

### 4.2 Impulse responses

To evaluate the quantitative performance of the model, I apply the standard algorithm to solve for the log-linearized rational expectation model\(^6\) and report the impulse responses of inflation and output gap.

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\(^5\)In Appendix(B), I give an introduction to the Weibull distribution.

\(^6\)I am grateful to Alexander Meyer-Gohde for helping me to calculate the equilibrium with some extreme parameter values, where the computation involves large numbers of lags and leads of expectations. For the details of the algorithm, you can refer to Meyer-Gohde (2007).
Figure 2 illustrates impulse responses of inflation and output to a transitory 1% increase in the annual nominal money growth rate ($\rho_m = 0$). While, in the Calvo setup, the response of inflation jump on impact and decays monotonically afterwards, it is hump-shaped in the increasing-hazard setting and peaks at around third quarters.

Figure 2 about here

Figure 3 depicts responses to a persistent money growth shock ($\rho_m = 0.95$). When shocks are persistent, both inflation and output responses are hump-shaped. It also exhibits that output leads inflation, a rather robust feature of the data. Under Calvo, however, either inflation and output have their largest correlation contemporaneously (non-persistent shock) or inflation leads output (persistent shock).

Figure 3 about here

The economic intuition behind these results is that, on the one hand, only a few firms adjust their prices immediately after a shock, and more and more adjust later on, thereby postponing the timing of the adjustment. On the other hand, the size of the adjustment is increasing in the time since the shock occurred. The later a firm changes its price, the larger the adjustment it needs to make. In another words, the increasing-hazard pricing affects not only the timing of the price adjustment, but also the average magnitude of firms’ adjustments, in that they tend to increase some periods after the shock, leading to a hump-shaped response.

Conclusion

The central theme of this study is to show non-constant hazard functions induced by the pricing assumption implies different aggregate dynamics. I derive a general New Keynesian Phillips curve, reflecting an arbitrary hazard function. The generalized NKPC involves components including lagged inflation, forward-looking and lagged expectations of inflations and real marginal cost, which nests the standard Calvo Phillips curve as a limiting case.

While the standing theory of the Phillips curve has argued that, in order to generate inflation persistence in the data, the NKPC needs to incorporate the lagged inflation with a significant positive coefficient, which is interpreted as ‘intrinsic inflation persistence’, I however, show that this is not the case in the general time-dependent pricing model. It accounts for inflation persistence not because of the ‘intrinsic’ force, but due to the moving-average precess of real shocks through the presence of lagged expectations. Therefore including lagged expectations is important for the inflation dynamics.

In the numerical exercise, I contribute to the literature with a new approach to parameterize the hazard function by using the Weibull distribution. The main advantage of this approach is that it allows for flexible characteristics in the hazard function, which in turn provides an alternative discipline to calibrate the parameters of the distribution of price vintages. More importantly, this parsimonious approach makes the underlying mechanism transparent. The numerical results show that the increasing hazard function leads to hump-shaped impulse responses of inflation to monetary shocks, and output leads inflation.
Given that handling a more sophisticated pricing model—such as the state-dependent pricing model—is rather challenging, the model presented here is a promising candidate for monetary policy research, in that it compromises of a better modeling of the pricing mechanism and tractability of the model’s solution required for policy analysis.
A Proof for Proposition 1

In the Calvo pricing case, all hazards are equal to a constant between zero and one. Let’s denote the constant hazard as $h = 1 - \alpha$. We can rearrange the NKPC 12 in the following way:

\[
\hat{\pi}_t + \sum_{k=1}^{\infty} \alpha^k \hat{\pi}_{t-k} = (1 - \alpha) \sum_{k=0}^{\infty} \alpha^k E_{t-k} \left( (1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i} + \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-k} \right)
\]

\[
\hat{\pi}_t + \alpha \hat{\pi}_{t-1} + \alpha^2 \hat{\pi}_{t-2} + \cdots = E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)
\]

\[
+ \alpha E_{t-1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right)
\]

\[
+ \alpha^2 E_{t-2} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i-2} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right)
\]

\[
\vdots
\]

Then iterating this equation one period forwards,

\[
\hat{\pi}_{t+1} + \alpha \hat{\pi}_t + \alpha^2 \hat{\pi}_{t-1} + \alpha^3 \hat{\pi}_{t-2} \cdots = E_{t+1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i+1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)
\]

\[
+ \alpha E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)
\]

\[
+ \alpha^2 E_{t-1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right)
\]

\[
\vdots
\]

\[
\hat{\pi}_{t+1} + \alpha(\hat{\pi}_t + \alpha \hat{\pi}_{t-1} + \alpha^2 \hat{\pi}_{t-2} \cdots) = E_{t+1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i+1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)
\]

\[
+ \alpha E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)
\]

\[
+ \alpha^2 E_{t-1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m_{t+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right)
\]

\[
\vdots
\]

Then substitute Equation 21 for the term in the brackets on the left hand side of this equation,
\[ \hat{\pi}_{t+1} + \alpha E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
+ \alpha^2 E_{t-1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
+ \alpha^3 E_{t-2} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i-2} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
\vdots \\
= E_{t+1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i+1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
+ \alpha E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
+ \alpha^2 E_{t-1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i-1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
\vdots \\
\]

After canceling out equaling terms from both sides of the equation, we obtain the following equation:

\[ \hat{\pi}_{t+1} = E_{t+1} \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i+1} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \]

Lagging this equation and rearranging it yields the familiar NKPC of the Calvo model.

\[ \hat{\pi}_t = E_t \left( (1 - \alpha)(1 - \alpha \beta) \sum_{i=0}^{\infty} \alpha^i \beta^i m c_{t+i} + (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \]

\[ \hat{\pi}_t = (1 - \alpha)(1 - \alpha \beta) m c_t + (1 - \alpha) \hat{\pi}_t + \alpha \beta E_t (\hat{\pi}_{t+1}) \]

\[ \hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} m c_t + \beta E_t (\hat{\pi}_{t+1}) \]
B The Weibull distribution

The PDF of Weibull distribution is given by the following expression:

$$Pr(j) = \frac{\tau}{\lambda} \left( \frac{j}{\lambda} \right)^{\tau-1} \exp \left( - \left( \frac{j}{\lambda} \right)^{\tau} \right)$$

and the cumulative probability function is:

$$F(j) = 1 - \exp \left( - \left( \frac{j}{\lambda} \right)^{\tau} \right)$$

The parameters that characterize the Weibull distribution are the scale parameter $\lambda$ and the shape parameter $\tau$. The shape parameter determines the shape of the Weibull’s pdf function, e.g. when $\tau = 1$, it reduces to an exponential case; while with $\tau = 3.4$, the Weibull amounts to the normal distribution. The scale parameter defines the characteristic life of the random process that amounts to the time at which 63.2% of the firms adjust their labor. This can be seen with the evaluation of the cdf function of the Weibull distribution at $j$ equaling the scale parameter $\lambda$. Then we have, $F(\lambda) = 1 - e^{(-1)} = 0.632$.

Note that it relates to the mean duration $\bar{j}$ according to the following equation:

$$\bar{j} = \frac{1}{\alpha} = \lambda \Gamma\left(\frac{1}{\tau} + 1\right),$$

where $\Gamma()$ is the Gamma function.

It follows that the hazard function of Weibull distribution is:

$$h(j) = \frac{\tau}{\lambda} \left( \frac{j}{\lambda} \right)^{\tau-1}$$

Note that this hazard is constant when the shape parameter $\tau$ equals one, and increasing when $\tau$ is greater than one.
## C Tables

<table>
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<tr>
<th>Vintage</th>
<th>Hazard Rate</th>
<th>Non-adj. Rate</th>
<th>Survival Rate</th>
<th>Distribution</th>
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<td>$S_j$</td>
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<td>$S_1 = \alpha_1$</td>
<td>$\theta(1)$</td>
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</tr>
<tr>
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<td>$h_j$</td>
<td>$\alpha_j = 1 - h_j$</td>
<td>$S_j = \prod_{i=0}^{j} \alpha_j$</td>
<td>$\theta(j)$</td>
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Table 1: Notations of the dynamics of price-vintage-distribution.
Figure 1: Hazard function and Weibull distribution with various shape parameters
Figure 2: Impulse responses to a transitory money growth shock ($\rho_m = 0$)
Figure 3: Impulse responses to a persistent money growth shock ($\rho_m = 0.95$)
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