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Monetary Policy, Determinacy, and the Natural Rate Hypothesis

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Abstract

Imposing the natural rate hypothesis (NRH) can dramatically alter the determinacy bounds on monetary policy by closing the output gap in the long run. I show that the hypothesis eliminates any role for the output gap in determinacy and renders the conditions for determinacy identical for all conforming supply equations. Specializing further to IS demand, determinacy depends only on the parameters in the interest rate rule and a pure forward or backward-looking inflation target is inconsistent with determinacy. Monetary policy that embodies the Taylor principle with respect to contemporaneous inflation delivers a determinate equilibrium in all models that satisfy the NRH.

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1 Introduction

What are the consequences of the natural rate hypothesis (NRH) for monetary policy? I address this question in the context of the determinacy bounds on the policy rule for the nominal interest rate. The NRH, which posits that the output gap is closed on average regardless of nominal quantities, is not fulfilled by the standard model for monetary policy analysis, the sticky-price New Keynesian model.[1] As I show that the bounds under the NRH can differ quite substantially, my analysis demonstrates that New Keynesian determinacy analyses are not robust to the NRH and, as such, requiring fulfilment of this hypothesis is of direct consequence for policy recommendations.

The central contribution of this paper is an equivalence result: for a given demand schedule, the bounds on monetary policy to ensure a determinate equilibrium are identical for all supply equations that satisfy the NRH. This result rests intuitively on the observation that all such supply equations behave identically in the long run, positing a vertical long-run Phillips curve at the natural rate. Thus, the novelty of my analysis is the positive result that identifies the limits to monetary policy for there to be determinacy when the NRH holds.

Several specific results are derived from this equivalence and the long-run restriction of the hypothesis. First, regardless of the specification for demand, output gap targeting is wholly irrelevant for determinacy under the NRH. With the long-run Phillips curve vertical in all NRH specifications, the output gap cannot be leveraged to substitute for vigourous inflation targeting. Under standard dynamic IS demand, the only model parameters relevant for determinacy are those in the interest rate rule itself. The long-run closure of the output gap reduces the IS equation to a Fisher-like equation with no reference to parameters in the demand equation, which combined with the equivalence on the supply side removes reference to all model specific parameters. Additionally, the combination of IS demand and the NRH strips all policies that only target either past or future inflation of any horizon of determinacy for all parameter constellations. In the absence of a dynamic tradeoff in the Phillips curve, forward-looking or backward-looking inflation targets cannot be translated into a nominal anchor for contemporaneous nominal developments. The flip-side of this negative result is that the necessary condition of a more than one-for-one response to inflation (the Taylor principle) can be combined with the additional sufficient condition that more than half of this response be directed towards current inflation to ensure a determinate equilibrium for any supply specification that satisfies the NRH.

[1] See, e.g., Woodford (2003, p. 254), McCallum (2004), Galí (2008, p. 78), and McCallum and Nelson (2011).

McCallum (2004, p. 21) noted that the widespread agreement within the profession regarding the validity of the natural rate hypothesis (NRH) "has seemingly been implicitly overturned, not by argument but merely by example, via the widespread adoption of the famous Calvo (1983) model of nominal price stickiness," a development that Wolman (2007, p. 1366) called "an awkward situation in monetary economics." McCallum (1998) and subsequently has called for monetary policy analysis to be conducted within the confines of NRH. Efforts have been made to reconcile the NRH and the New Keynesian framework, notably Andrés, López-Salido, and Nelson's (2005) examination of the NRH and New Keynesian models with an emphasis on the role of the NRH for models' dynamics or Levin and Yun's (2007) model that brings the standard sticky-price model closer to the NRH by endogenizing the contract length. I extend these efforts to the analysis of determinacy.

Policy recommendations from determinacy analyses come in the form of bounds consistent with a unique saddle-path stable equilibrium on parameter values in a monetary policy rule.[2] These bounds present a "First, do no harm" recommendation for monetary policy to ensure uniqueness of an equilibrium while being silent as to what that equilibrium might optimally be.[3] The notable precursor to my analysis is Carlstrom and Fuerst (2002), who examine determinacy under a narrower scope of interest rate rules in their specific NRH model. They explicitly make the connection between determinacy of nominal variables in a fully flexible model and determinacy in their model with rigidities. Unfortunately, their analysis is limited to their specific model-policy setup; this paper generalizes and extends their insights to a range of policy rules and a class of models that satisfy the NRH.

Methodologically, I foreshadow the formal derivation of the results with an informal thought-experiment approach that derives the entire set of determinacy results under IS demand by exploring borderline stability cases. I compliment the textbook[4] thought experiment of finding the nominal interest rate response to a permanent increase in inflation that leaves the long-run real rate positive with a second thought experiment on the opposite side of the unit circle implying a permanent oscillation in inflation that identifies the remaining determinacy bound.[5] Besides offering access to the main results

[2]While determinacy is neither the only criterium for evaluating the uniqueness of an equilibrium—see, e.g., McCallum (2003) for an overview of some competitors: his MSV as well as E-stability and LS-learning—nor is its validity uncontested—see Cochrane (2011), it is the current standard criterium in the literature. As McCallum (2009, p. 26) notes, the subject appears on "75 different pages in Michael Woodford's hugely influential treatise Interest and Prices (Woodford 2003). In addition, the number of new writings (books, articles, and working papers) with both of the phrases 'indeterminacy' and 'monetary policy' appearing in their text was 166 over the time span January 1995 through June 2008."

[3]See Woodford (2011) for an overview of recent advances in this direction.

[4]See, e.g., Woodford (2003, p. 254) and Galí (2008, pp. 78–79).

[5]If the model is a local approximation, it is important to note that use of permanent shifts in the thought experiments in no way contradicts the applicable locally-bounded-equilibrium approach of Woodford (2003). As made explicit by

through a second and far less formal channel, this approach serves the dual purpose of highlighting that it is indeed the violation of the NRH in the sticky-price supply equations that is at fault, as well as providing an intuitive insight into the nature of the multiplicity that determinacy seeks to avoid. For comparison with the sticky-price literature,[6] an inventory of rules comprising inflation targeting (contemporaneous as well as forward and backward looking), output gap targeting, interest-rate smoothing, as well as exogenous interest rate rules is examined for determinacy.

The formal derivation contributes a lemma to the literature for assessing determinacy in a broad class of linear[7] models with arbitrary leads, lags, and lagged expectations on the basis of two conditions, a standard[8] saddle-point consideration from Anderson (2010) and a resolvability of any remaining undetermined expectations structure from Meyer-Gohde (2010). The generality of the model class examined does not, on its own, rule out isolated singular cases;[9] but excepting for such cases, a general equivalence of the determinacy bounds among all admissible models that satisfy the NRH is proven. Finally, in applying a simplification of the Schur-Cohn criterion, I provide a completed classification of the conditions that sort the zeros to a second order polynomial to either side of the unit circle.

The remainder of the paper is organized as follows. Section 2 briefly examines the standard sticky-price New Keynesian model and its relation to the NRH. I use the thought experiment approach in section 3 to derive the determinacy bounds for the standard sticky-price and models that satisfy the NRH in parallel. In section 4, I prove the main result of the paper, the equivalence proposition for the determinacy bounds within a class of models that satisfy the NRH. Section 5 applies the result to a particular interest rate rule that encompasses several widely studied rules, confirms the results from the thought experiments, and explores the robustness of the one-period horizon for backward and forward-looking targets. Section 6 concludes.

Woodford (2003, pp. 78 & 633), the question of local determinacy reduces to examining whether the linearized system of equations have a unique bounded solution. Thus, the hypothetical unbounded deviations from a rest point serve to coax the boundaries of determinacy and do not undermine the local nature of the approximation.

[6] E.g., Woodford (2003, Ch. 4) provides the literature-standard inventory of determinacy results, or Lubik and Marzo (2007) presents a more recent compendium of determinacy results in a standard sticky-price model. See Bernanke and Woodford (1997), Clarida, Galí, and Gertler (1999), Bullard and Mitra (2002) and Woodford (2003), among many others, for the corresponding and contrasting results from sticky-price models.

[7] In case the system has been linearized, determinacy here is local and not global. For the latter type of determinacy analysis, see Benhabib, Schmitt-Grohé, and Uribe (2001), whose analysis shows that the Taylor principle for determinacy from local analysis does not carry over to the global analysis that takes the zero lower bound of nominal interest rates into account. The analysis here abstracts from such complications and restricts itself entirely to local determinacy.

[8] Anderson's (2010) condition extends Blanchard and Kahn's (1980) to higher leads and lags.

[9] For example, that the demand and supply equations are linearly dependent at some expectational horizon.

2 The New Keynesian Sticky-Price Model and the NRH

The most prominent model for monetary policy analysis currently is the standard New Keynesian sticky-price model with Calvo (1983)-style overlapping contracts in general equilibrium.[10] In its simplest form, the model comprises three equations—demand, supply, and a monetary policy rule—in three variables—inflation, the output gap, and the nominal interest rate. Demand is given by the dynamic IS equation

$$y_t = E_t\left[y_{t+1}\right] - aR_t + aE_t\left[\pi_{t+1}\right] \tag{1}$$

R_t is the nominal interest rate, y_t is the output gap, π_t inflation, and a a positive parameter equal to the intertemporal elasticity of substitution in the simplest configuration. The demand side relates expected changes in the output gap positively to movements in the real interest rate $r_t = R_t - E_t\left[\pi_{t+1}\right]$. The supply side is given by the sticky-price Phillips curve

$$y_t = \frac{1}{\kappa}\left(\pi_t - \beta E_t\left[\pi_{t+1}\right]\right) \tag{2}$$

where β is the discount factor, and κ a positive composite parameter that depends, among others, on the probability of a price change. Of note is the non-standard presentation of the Phillips curve, with the output gap on the left-hand side, which will ease the exposition when examining the fully flexible limiting case.[11] Several different specifications for monetary policy in the form of a Taylor (1993)-type rule for the nominal interest rate will be explored to close the model.

The central focus of my analysis rests on the fulfillment of the NRH by the supply side. As encapsulated by Friedman (1968, p. 11), the NRH captures a central tenet of monetary theory that though "there is always a temporary trade-off between inflation and employment; there is no permanent trade-off." Lucas (1972) provides the formalization of the NRH that I will use,[12] namely that for the NRH to be fulfilled, $E\left[y_t\right] = 0$ must hold for any monetary policy and, hence, regardless of monetary policy. The sticky-price Phillips curve (2) does not in general satisfy the NRH, as has been noted by many including Woodford (2003, p. 254), McCallum (2004), Galí (2008, p. 78), and McCallum and

[10] See, e.g., Woodford (2003, p. 246) or Galí (2008, p. 49) for textbook-length expositions.

[11] Additionally, this presentation is consistent with Modigliani's (1977, p. 5) assessment that the NRH "turns the standard explanation on its head: instead of (excess) employment causing inflation, it is (the unexpected component of) the rate of inflation that causes excess employment."

[12] McCallum (2004, pp. 21–22) draws a distinction between "Friedman's weaker version" and the "stronger Lucas version" of the NRH. The former states that a higher, but constant, rate of inflation cannot permanently affect output and the latter that no path for prices, inflation, inflation growth, etc., can permanently keep output above its natural level. Lucas's (1972) is the version that McCallum (1994) argues should be upheld by monetary models—repeated more directly in McCallum (1998, p. 359)—and will be the version imposed in my analysis.

Nelson (2011), as it does posit a permanent tradeoff. To see this, note that from the perspective of time $t-k$,

(3)
$$E_{t-k}[y_t] = \frac{1}{\kappa} E_{t-k}[\pi_t - \beta\pi_{t+1}]$$

the tradeoff between the output gap and inflation remains unchanged. That is, the sticky-price model posits the same dynamic tradeoff at all expectational horizons. Setting β to one or using indexation[13] may remove a static tradeoff, but cannot remove the dynamic tradeoff in (2).[14] Yet, as $\kappa \to \infty$, the sticky-price Phillips curve in (2) in fact does satisfy the NRH.

(4)
$$\lim_{\kappa\to\infty} y_t = \lim_{\kappa\to\infty} \frac{1}{\kappa}(\pi_t - \beta E_t[\pi_{t+1}]) = 0$$

Of course, in this case, the probability of a price change goes to one and the Phillips curve posits no tradeoff, simply stating that the output gap is always closed. As I proceed to derive the determinacy bounds on monetary policy, note that only the limiting case ($\kappa \to \infty$) of the bounds themselves from the sticky-price model will coincide with the bounds obtained directly under the NRH. This underlines the crucial role that the sticky-price model's permanent tradeoff has played in existing determinacy studies.

3 Determinacy via Thought Experiments

In this section, I derive the determinacy bounds in the basic New Keynesian model, both when the NRH is fulfilled and for the standard sticky-price case, by examining the threshold response of the nominal interest rate to ensure a positive response of the (long-run) real rate to a permanent pattern of change in inflation. Using a thought experiment approach is useful as it dispenses with much of the technical exposition of later sections necessary to formally establish the bounds on determinacy in a broader class of models that satisfy the NRH, enabling more intuitive access to the results. The constant pattern for inflation is the standard[15] thought experiment to evaluate whether the Taylor principle holds under a given monetary policy rule and the constant oscillating pattern captures another Taylor principle-type unit-root threshold case. With this approach, I will demonstrate the dramatic effect that imposing the NRH can have on the traditional determinacy bounds postulated by the New Keynesian model. Under the NRH, output gap targeting of any degree is ineffective in bringing about a determinate equilibrium

[13]See Yun (1996) for a constant indexation, Christiano, Eichenbaum, and Evans (2005) for full dynamic indexation and Smets and Wouters (2003) for a partial dynamic indexation.

[14]For more on this point, see McCallum (2004) and McCallum and Nelson (2011).

[15]See Woodford (2003, p. 254) and Galí (2008, pp. 78–79) for textbook applications.

and forward-looking or backward-looking inflation targeting of any degree fails to secure determinacy unless the central bank smoothes interest rates.

3.1 The Thought Experiments

The Taylor principle, e.g., Taylor (2001, p. 331), captures the idea that for monetary policy to be stabilizing, the real interest rate should increase in the face of a sustained increase in inflation so as to exert a countervailing downward pressure on inflation by contracting demand. This principle can be recast as a thought experiment of the effect on the real rate of a permanent permanent pattern of change in inflation ($d\pi_t$), where the threshold response to ensure a countervailing movement in the long-run real interest rate is calculated via the demand equation. I will identify the threshold response using the same demand equation, the New Keynesian standard dynamic IS equation (1), but will distinguish two cases for the supply side: the sticky-price Phillips curve (2) and the NRH case. That is, I will not specify a specific formulation of the supply curve under the NRH, but will derive the determinacy bounds by simply appealing to the neutrality properties imposed by the NRH.

I will consider two different permanent patterns in inflation to elicit the threshold responses of the nominal interest rate consistent with the Taylor principle. The first pattern is a constant increase and the second an oscillating pattern with a constant amplitude and (trigonometric) period equal to two (model) periods. For the first, textbook standard pattern of a permanent, constant increase in inflation (call it $d\pi_t = d\pi_{t+1}$), the threshold value to bring about the necessary countervailing movement in the real interest rate is given by $\frac{dR_t}{d\pi_t} = 1$: any cumulative reaction of the nominal interest rate greater than the increase in inflation will bring about the necessary increase in long-run real interest rate to contract demand. This can be confirmed in both models (sticky price and NRH) by noting that a constant pattern in inflation translates into a constant pattern in the output gap. For the sticky-price model, through the Phillips curve (2),

$$dy_t = \frac{1}{\kappa}(d\pi_t - d\pi_{t+1}) = \frac{1-\beta}{\kappa}\pi_t = dy_{t+1} \tag{5}$$

and for the NRH model, by virtue of the NRH, the output gap is necessary closed in the long run, so the associated pattern relevant for the long-rung real interest rate is $dy_t = dy_{t+1} = 0$. With a constant output gap in both models, the neutral movement in the long-run real interest rate (call it dr_t) is zero, as can be confirmed by plugging the long-run pattern of the output gap into the demand equation (1)

$$dy_t = dy_{t+1} - a dr_t \Rightarrow 0 = dr_t \tag{6}$$

Thus any positive long-run movement in the real interest rate would present a countervailing force to stabilize the postulated increase in inflation. From the definition of the real interest rate it follows then

$$0 < dr_t \Rightarrow dR_t > \begin{cases} d\pi_{t+1} \\ d\pi_t \\ d\pi_{t-1} \end{cases} \tag{7}$$

The nominal interest rate must move more than one for one with inflation to achieve this increase in the real rate—the celebrated Taylor principle.

The second pattern I will use is non-standard and is designed to adapt Lubik and Marzo's (2007) insight that non-monotonic sunspot dynamics are not only possible in these models, but are associated with some policy and parameter regions of interest. Consider an oscillating pattern with a constant amplitude and (trigonometric) period equal to two (model) periods (call it $-d\pi_t = d\pi_{t+1}$). This is a "unit-root" sunspot in inflation just like in the standard experiment, but with the root located differently on the unit circle (-1 here instead of the 1 above). Unlike the first experiment, the neutral movements in the long-run real rate in the two models is different by virtue of the dynamic tradeoff in the sticky-price Phillips curve discussed in the previous section. Examine the sticky-price model first: the Phillips curve (2) delivers (as $-d\pi_t = d\pi_{t+1}$)

$$dy_t = \frac{1}{\kappa}(d\pi_t - d\pi_{t+1}) = \frac{1+\beta}{\kappa}\pi_t \tag{8}$$

thus (again, as $-d\pi_t = d\pi_{t+1}$), $dy_{t+1} = -dy_t$. Inserting this into the IS demand (1) equation

$$dy_t = dy_{t+1} - a dr_t \Rightarrow -\frac{2}{a}dy_t = dr_t \Rightarrow -2\frac{1+\beta}{a\kappa}d\pi_t = dr_t \tag{9}$$

Monetary policy's rule for the nominal interest rate must now ensure that the real rate is *less* than this threshold value (recall the location on the unit circle is -1)

$$-2\frac{1+\beta}{a\kappa}\pi_t > dr_t \Rightarrow dR_t < \begin{cases} \left(1+2\frac{1+\beta}{a\kappa}\right) d\pi_{t+1} \\ -\left(1+2\frac{1+\beta}{a\kappa}\right) d\pi_t \\ \left(1+2\frac{1+\beta}{a\kappa}\right) d\pi_{t-1} \end{cases} \tag{10}$$

Of note is that this upper bound on the elasticity of the nominal interest rate with respect to inflation is negative for current inflation,[16] but positive for both future and past inflation. Turning to the NRH model for the oscillating inflation pattern, by virtue of the NRH, the output gap is necessary closed in the long run regardless of the pattern of inflation, so the associated pattern relevant for the long-rung real interest rate is $y_t = y_{t+1} = 0$. Thus, from demand (1)

$$dy_t = dy_{t+1} - a dr_t \Rightarrow 0 = dr_t \tag{11}$$

[16] This illustrates that a region of determinacy exists for a *negative* elasticity here, see King (2000, p. 79).

for the real rate to be less than this threshold (again, recall the location on the unit circle),

$$0 > dr_t \Rightarrow dR_t < \begin{cases} d\pi_{t+1} \\ -d\pi_t \\ d\pi_{t-1} \end{cases} \tag{12}$$

As in the sticky-price model, this upper bound is negative for current inflation, but positive for both future and past inflation. King's (2000, p. 79) conclusion that the zone of indeterminacy under current inflation targeting is greater with sticky than with flexible prices is illustrated by this bound with the maximal elasticity precisely -1 for the NRH model but generally less than -1 for the sticky price model. Yet, the relevant range of elasticities in the monetary policy rule is arguably limited to positive elasticities,[17] and the positive upper bounds associated with future and past inflation imply exactly the opposite conclusion. In what follows, I will flesh out the consequences of these two thought-experiment bounds for a broad range of interest rate rules, maintaining the positive elasticity assumption.[18]

3.2 Exogenous Interest Rates

The simplest example of an exogenous interest rate is an interest rate peg that keeps the nominal interest rate equal to some exogenously given constant. Appealing to the thought experiment, it follows immediately that the model cannot be determinate: the interest rate is constant (or exogenous to inflation) and, hence, cannot increase more than one-for-one in the face of a permanent rise in inflation.

The same logic applies to any bounded, exogenous process for the interest rate. Determinacy rests on the homogenous part of the system of difference equations, which captures the response of monetary policy to deviations from some stochastically varying neutral path in the words of King (2000). The interest rate, properly normalized for this neutral path, is again constant and, as such, determinacy can be assessed for any bounded exogenous interest rate under the same conditions as for a constant interest rate. Thus, any constant or bounded exogenous interest rate rule is necessarily associated with indeterminacy—the Sargent and Wallace (1975) result. This corresponds to Woodford (2003, p. 253) and confirms McCallum's (1981) result that a nominal interest rate rule must involve feedback from endogenous variables if Sargent and Wallace's (1975) indeterminacy is to be overcome.

[17] See, e.g., Bullard and Mitra (2002), Woodford (2003), and Lubik and Marzo (2007).

[18] Readers unsatisfied with this assumption are directed to theorem 5.1 in section 5 for the general case.

3.3 Output Gap Targeting

Consider, now, an output gap targeting interest-rate rule

$$R_t = \phi_y y_t \tag{13}$$

I will go right to the determinacy bounds, derived using the thought experiments laid out above. For the sticky-price model, from (7), the response of the nominal interest rate must be greater than that of inflation. In the sticky-price model, movements in the output gap correspond to movements in inflation even in the long run, with the relation, as derived in (5), given by $dy_t = \frac{1-\beta}{\kappa} d\pi_t$. Thus, as $\frac{dR_t}{d\pi_t} = \frac{dR_t}{dy_t}\frac{dy_t}{d\pi_t} = \phi_y \frac{1-\beta}{\kappa}$, if $\phi_y \frac{1-\beta}{\kappa} > 1$ the model is determinate.[19] For the NRH model, the permanent change in inflation has—by virtue of the NRH—no permanent impact on the output gap ($dy_t = 0$). Hence, $\frac{dR_t}{d\pi_t} = \frac{dR_t}{dy_t}\frac{dy_t}{d\pi_t} = 0$ and the response of the nominal interest rate cannot exceed the necessary threshold value with an interest rate rule that targets only the output gap. The second experiment places an upper bound on the threshold response of the nominal interest rate. But it is negative and with a positive tradeoff in the sticky-price model, no positive degree of output gap targeting can fulfill this condition. As in the first experiment, the nominal interest rate will not respond at all in the long run under the NRH and no degree of output gap targeting can bring about the necessary response in the long-run real rate.

Contrary to Woodford's (2003, pp. 254–255) claim that "a large enough [response to] *either* [the output gap or inflation] suffices to guarantee determinacy," (emphasis in the original) if the feedback from endogenous variables is limited to the output gap, no degree of output gap targeting can induce determinacy. The culprit: the non-vertical long-run Phillips curve in sticky-price models allows monetary policy to substitute output gap targeting for inflation targeting so as to satisfy the Taylor Principle, a possibility not available if the NRH is upheld. Without a permanent output-inflation tradeoff as in the sticky-price model, the Taylor principle cannot be fulfilled under a pure output gap target.

3.4 Contemporaneous Inflation Targeting

Consider now an extended (by interest rate smoothing and output gap targeting) contemporaneous inflation targeting rule

$$R_t = \phi_R R_{t-1} + \phi_\pi \pi_t + \phi_y y_t \tag{14}$$

[19] See Woodford (2003, p. 254) and Galí (2008, p. 77): set $\phi_\pi = 0$ and note that Woodford's (2003, p. 254) $\frac{\phi_x}{4}$ is equivalent to my ϕ_y (see Woodford (2003, p. 246)).

Turning to the first thought experiment, the effect on the real rate of a permanent increase in inflation ($d\pi_t = d\pi_{t+1}$). For the sticky-price model, the threshold response of the nominal interest rate is just equal to that of inflation, see (7). Inserting this threshold response along with the response of the output gap to inflation from the previous section into (14) yields $\frac{dR_t}{d\pi_t} = \phi_R + \phi_\pi + \phi_y \frac{1-\beta}{\kappa}$. This must be greater than the borderline case of one, which delivers $\phi_\pi > 1 - \phi_R - \phi_y \frac{1-\beta}{\kappa}$.[20] Under the NRH, as before, the permanent change in inflation has no permanent impact on the output gap ($dy_t = 0$). With the threshold response of one, it now follows from (14) that $\frac{dR_t}{d\pi_t} = \phi_R + \phi_\pi > 1$. The Taylor principle can be satisfied under the NRH only by a sufficiently vigorous cumulative reaction ($\frac{1}{1-\phi_R} d\pi_t$) of the nominal interest rate to inflation.

The interpretation under the NRH is the one generally given to the Taylor principle, despite the pervasiveness of the sticky-price model in the literature which allows monetary policy to substitute a reaction to the output gap for a reaction to inflation at the rate $\frac{1-\beta}{\kappa}$ (the long-run slope of the Phillips curve). Under the NRH, the monetary authority is unable to make such a trade with a long-run vertical Phillips curve and must, then, satisfy the Taylor principle directly. The second experiment of oscillating inflation again places a negative upper bound, see the case of π_t in (10), on the threshold response of the nominal interest rate: restricting policy rules to positive elasticities, no parameter combination can satisfy this condition.

The results under the NRH reiterate the conclusion that output gap targeting is irrelevant for determinacy. The absence of parameters outside of monetary policy has the convenient attribute that determinacy can be evaluated on the merits of the interest rate rule alone. Woodford's (2003, p. 255) indirect interpretation of Taylor (2001), requiring parameter estimations of the sticky-price Phillips curve is not necessary under the NRH model. Indeed, taking Woodford's (2003, p. 255) analysis literally, one could just as easily conclude that indeterminacy in the pre-Volker era was due to too small a concern of monetary policy for the real economy and not necessarily to too weak of a reaction to inflation; a conclusion which could not be reached if the NRH is upheld.

[20]See Woodford (2003, p. 255), but note that his $\frac{\phi_x}{4}$ is equivalent to my ϕ_y.

3.5 Forward-Looking Inflation Targeting

Consider an extended (again, by interest rate smoothing and output gap targeting) inflation-forecast target

$$R_t = \phi_R R_{t-1} + \phi_\pi E_t\left[\pi_{t+1}\right] + \phi_y y_t \tag{15}$$

I again turn to the thought experiments, though now the second experiment will be of consequence. First, in the case of a permanent increase in inflation, the same lower bound is derived as in the case of contemporaneous inflation targeting. In the special case $\phi_y = \phi_R = 0$, this lower bound is given by $\phi_\pi > 1$ in both the standard sticky-price model and NRH models. Allowing for interest rate smoothing enables the monetary authority to spread the needed more than one-for-one increase in the nominal rate over many periods in accordance with the cumulative Taylor principle. The presence of output gap targeting then differentiates the results of sticky-price and NRH models, with the non-vertical Phillips curve of the former allowing the monetary authority to substitute output gap targeting for the necessary inflation targeting at the rate $\frac{1-\beta}{\kappa}$, the long-run slope of the Phillips curve.

Turning to the second experiment with oscillating inflation ($-d\pi_t = d\pi_{t+1}$), the threshold response of the nominal interest rate for the sticky-price model, see (10), is now given by $dR_t = \left(1 + 2\frac{1+\beta}{a\kappa}\right) d\pi_{t+1}$. From (15), noting the oscillatory paths and the tradeoff between inflation and the output gap, it follows that $dR_t = \phi_R dR_{t-1} + \phi_\pi d\pi_{t+1} + \phi_y dy_t$ becomes $dR_t = -\phi_R\left(1 + 2\frac{1+\beta}{a\kappa}\right) d\pi_{t+1} + \phi_\pi d\pi_{t+1} - \phi_y \frac{1+\beta}{\kappa} d\pi_{t+1}$ or $dR_t = \left(-\phi_R\left(1 + 2\frac{1+\beta}{a\kappa}\right) + \phi_\pi - \phi_y\frac{1+\beta}{\kappa}\right) d\pi_{t+1}$. The response of the nominal interest rate must be below the threshold value, see (10), and this requires $-\phi_R\left(1 + 2\frac{1+\beta}{a\kappa}\right) + \phi_\pi - \phi_y\frac{1+\beta}{\kappa} < 1 + 2\frac{1+\beta}{a\kappa}$, or $\phi_\pi < (1+\phi_R)\left(1 + 2\frac{1+\beta}{a\kappa}\right) + \phi_y\frac{1+\beta}{\kappa}$.[21] For the NRH model, the threshold response of the nominal interest rate, see (12), is given by $dR_t = d\pi_{t+1}$. From (15), $dR_t = \phi_R dR_{t-1} + \phi_\pi d\pi_{t+1} + \phi_y dy_t$ which, noting the oscillatory paths and that the permanent change in inflation has no permanent impact on the output gap ($dy_t = 0$), becomes $dR_t = -\phi_R d\pi_{t+1} + \phi_\pi d\pi_{t+1}$ or $dR_t = (-\phi_R + \phi_\pi)\, d\pi_{t+1}$. For the response of the nominal interest rate to be below the threshold value, this requires $-\phi_R + \phi_\pi < 1$ or $\phi_\pi < 1 + \phi_R$.

The lower bound requires the interest rate to follow the Taylor Principle, necessitating an active interest rate. The upper bound, however, requires that the interest rate not be overly aggressive, lest "the output gap and inflation [be] projected to converge back to the steady state *regardless* of their values in the current period."[22] The difference is that in the absence of interest-rate smoothing, the two

[21] See Woodford (2003, p. 258), noting that his ρ corresponds to my ϕ_R and his $\frac{\phi_x}{4}$ to my ϕ_y.
[22] Levin, Wieland, and Williams (2003, p. 628). Emphasis in the original.

bounds collapse, meaning every interest rate rule of this type is either too aggressive or not aggressive enough.[23] The history dependence induced by feedback on the lagged interest rate is enough to open a window of determinacy for the forward-looking rule. Yet again, determinacy is independent of the degree of output gap targeting and of parameters outside the interest rate rule, contrary to sticky-price analyses. The lower bound conforms to Woodford's (2003, p. 96) inertial modification of the Taylor Principle: the cumulative response of the nominal interest rate must react more than one-to-one to a sustained deviation in inflation. Woodford (2003, p. 259) remarks that with "coefficients in the range that is likely to be of practical interest, [the upper bound does] not seem likely to be a problem;" likewise Galí (2008, p. 79). These assurances are less convincing if one requires the NRH to be fulfilled.

In the special case $\phi_y = \phi_R = 0$, note that the determinacy region disappears under the NRH: a pure inflation-forecast targeting rule is necessarily indeterminate.[24] Contrary to sticky-price models, there is no region of determinacy for pure inflation-forecast targeting rules. The pervasive indeterminacy under the NRH would certainly seem to be more consistent with Woodford's (2000) discussion of the non-optimality of purely forward-looking monetary policy rules than the analogous analysis in sticky-price models.

3.6 Backward-Looking Inflation Targeting

Consider an extended (again, by interest rate smoothing and output gap targeting) backward-looking inflation target

$$R_t = \phi_R R_{t-1} + \phi_\pi \pi_{t-1} + \phi_y y_t \tag{16}$$

The determinacy bounds for the backward-looking target are identical to those for the forward looking target, noted also by Lubik and Marzo (2007). To see this using the thought experiments, observe that the first experiment delivers the same lower bound is derived as in the case of contemporaneous inflation targeting. This follows as timing is irrelevant in this experiment ($\pi_t = \pi_{t+1}$). Turning to the second experiment with oscillating inflation ($-\pi_t = \pi_{t+1}$), the threshold response of the nominal interest rate for the sticky-price model, see (10), is now given by $dR_t = \left(1 + 2\frac{1+\beta}{a\kappa}\right) d\pi_{t-1}$. From

[23]This indeterminacy can also be seen by considering the specific NRH supply side: $y_t = 0$. This reduces (1) to $R_t = E_t[\pi_{t+1}]$, the Fisher-like equation from the frictionless case. The pure inflation forecast targeting rule is $R_t = \phi_\pi E_t[\pi_{t+1}]$. The system of these two equations can at most pin down R_t and $E_t[\pi_{t+1}]$, leaving π_t undetermined. See footnote 49 of King (2000, p. 80).

[24]The result of Carlstrom and Fuerst (2002) and Michael Dotsey as communicated to King (2000, p. 80).

(16), noting the oscillatory paths and the tradeoff between inflation and the output gap, it follows that $dR_t = \left(-\phi_R\left(1+2\frac{1+\beta}{a\kappa}\right)+\phi_\pi-\phi_y\frac{1+\beta}{\kappa}\right)d\pi_{t-1}$. The response of the nominal interest rate must be below the threshold value, see (10), and this requires $\phi_\pi < (1+\phi_R)\left(1+2\frac{1+\beta}{\kappa 1}\right)+\phi_y\frac{1+\beta}{\kappa}$ as with forward-looking moentary policy. For the NRH model, the threshold response of the nominal interest rate, see (12), is given by $dR_t = d\pi_{t-1}$. From (16), $dR_t = \phi_R dR_{t-1}+\phi_\pi d\pi_{t-1}+\phi_y dy_t$ which becomes $dR_t = (-\phi_R+\phi_\pi)\,d\pi_{t-1}$. For the response of the nominal interest rate to be below the threshold value, this requires $\phi_\pi < 1+\phi_R$, as in the case of a forward looking target.

I now turn to the formal analysis of the paper to generalizes these results to a class of models that satisfy the NRH and a wider range of interest rate rules. The results derived informally here for a number of specific monetary policy rules will be confirmed formally in section 5.1 and an interpretation for the differences between the NRH case and the sticky model will be given in section 5.2, building on the intuition developed with the formal results in the coming sections.

4 Determinacy and the Natural Rate Hypothesis

This section derives the main result of the paper; after having introduced the class of models and derived the conditions for determinacy in the class, I prove an equivalence property among these conditions for models in the class that satisfy the NRH. The main result is that all models, saving a few isolated singularities, that satisfy the NRH are determinate under the same conditions on monetary policy for a given demand equation; that is, the nature of the short-run tradeoff (if any) in the supply equation is inconsequential for determinacy bounds on monetary policy. Additionally, if the NRH is fulfilled, then the degree of output gap targeting (if any) is irrelevant for determinacy—the output gap is closed in the long run by virtue of the NRH and provides no leverage for monetary policy to fulfill the Taylor principle. Finally, if demand is given by standard IS demand, the only parameters relevant for determinacy are those in the interest rate rule of monetary policy. The class of models encompasses all three-equation models in the output gap, inflation, and the nominal interest rate with finite leads, lags, and expectational lags. The number of leads, lags, and expectational lags is arbitrary leading to a rather general class of rational expectations models. The standard New Keynesian model is a special case and I show how it fits into the broader class I define. Lucas's (1972) NRH as applied to the class I study reduces to the condition introduced by Carlstrom and Fuerst (2002) and I link the NRH to a parameter restriction on sums of coefficient in the supply equation.

4.1 Model Class

Consider the following class of three-equation (i.e., supply, demand, and monetary policy) linear rational-expectations models:

$$0 = \sum_{i=0}^{p} \sum_{j=-m}^{n} Q(i,j) E_{t-i} X_{t+j}, \;\; X_t = \begin{bmatrix} R_t & \pi_t & y_t \end{bmatrix}', \;\; 0 \le p, m, n < \infty \tag{17}$$

where y_t is the output gap, π_t inflation, and R_t the nominal interest rate. With three variables and three equations, the matrices $Q(i,j)$'s are of dimension 3×3. The class encompasses all linear rational-expectations models in the three variables of interest that (i) have a finite number of leads (given by n), (ii) have a finite number of lags (given by m), and (iii) have expectations formed at horizons from t into the finite past $t-p$.[25] The compact presentation of the class in (17) is chosen for technical reasons that will become apparent later but I shall first explore a particular example, the standard New Keynesian model, to make it more transparent.

The standard New Keynesian model has three equations in inflation, the output gap, and the nominal interest rate (say $R_t = \phi_\pi \pi_t + \phi_y y_t$, Taylor's (1993) rule) and has finite leads (namely, one: $n = 1$), lags (none $m = 0$), and expectational lags (none $p = 0$). Thus, it fits within the class introduced above and can be brought into the form of (17)

$$0 = \sum_{j=0}^{1} Q(0,j) E_t X_{t+j} = \begin{bmatrix} 0 & -\frac{1}{\kappa} & 1 \\ -a & 0 & -1 \\ -1 & \phi_\pi & \phi_y \end{bmatrix} \begin{bmatrix} R_t \\ \pi_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 & \frac{\beta}{\kappa} & 0 \\ 0 & a & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t\left[R_{t+1}\right] \\ E_t\left[\pi_{t+1}\right] \\ E_t\left[y_{t+1}\right] \end{bmatrix} \tag{18}$$

with only two 3×3 matrices, $Q(0,0)$ and $Q(0,1)$, being non-zero.

Notice in contrast to the standard New Keynesian model, a more general structure of model is permitted by (17). Besides requiring supply and demand only to fit into the finite lead, lag, and expectational lag structure of the class, monetary policy too only needs fits into the class. If monetary policy is the third equation of (17),[26] given by

$$0 = \sum_{i=0}^{p} \sum_{j=-m}^{n} Q_{3,\cdot}(i,j) E_{t-i}\left[X_{t+j}\right] \tag{19}$$

The class, of course, captures a wide range of interest rate rules found in the literature, including the current and forward-looking inflation targeting, interest rate smoothing, and output gap targeting as examined in Woodford (2003) as well as the rules in Bullard and Mitra (2002).

[25] Note that the absence of exogenous driving forces in (17) and constants is without loss of generality. The conditions for determinacy remain the same if (17) is appended with stationary driving forces (i.e., I am investigating the properties of the homogenous component of the system of difference equations).

[26] Where $Q_{3,\cdot}(i,j)$ is the row vector given by the the third row of $Q(i,j)$.

4.2 The NRH in the Model Class

For the NRH to be fulfilled, $E[y_t] = 0$ must hold for any monetary policy. In the class of finite models defined in (17), this reduces to the following condition from Carlstrom and Fuerst (2002),[27]

Definition 4.1 (The Natural Rate in the Class of Models in (17)).

(20) $$E_{t-k}[y_t] = 0 \; \forall t$$

Within the class there exists some maximal expectational lag after which the slope of the Phillips curve no longer changes. Thus, if the long-run Philips curve is to be vertical as demanded by the NRH, it must do so at some finite horizon, say k.

The list of dynamic models that satisfy (20) includes: Andrés, López-Salido, and Nelson's (2005, p. 1034) "Sticky information, staggered á la Taylor," as found also in Koenig (2004), Collard, Dellas, and Smets (2009), and Woodford (2011); other models of finitely staggered predetermined prices such as Fischer (1977) and Blanchard and Fischer (1989, pp. 390–394); Carlstrom and Fuerst's (2002, p 81-82) model in this spirit; the Mussa-McCallum-Barro-Grossmann "P-bar model"—see McCallum (1994) and McCallum and Nelson (2001); as well as the expectational Phillips curve of Lucas (1973)—see also Sargent and Wallace (1975)—that formalized the rational expectations revolution. With the exception of Calvo sticky-information models with an infinite distribution of expectational lags,[28] every linear intertemporal model that claims to satsify Lucas's (1972) NRH also satisfies the condition of Carlstrom and Fuerst (2002), repeated here as (20).

The NRH can also be seen as a parameter restriction for models in (17), given by

(21) $$\sum_{i=0}^{k} Q_{1,h}(i,j) = 0, \; \forall j = -m, \ldots, n \text{ and } h = 1,2$$

(22) $$Q_{1,h}(i,j) = 0, \; \forall j = -m, \ldots, n, i > k \text{ and } h = 1,2$$

where the subscripts in $Q_{1,h}(i,j)$ refer to the h'th column of the row associated with the supply equation, assumed without loss of generality to be the first row. This condition requires that inflation and the nominal interest rate at each lead and lag drop out of the supply equation upon application of the

[27] An obvious extension of (20) that would be inconsequential for the analysis of determinacy but useful in analyzing stochastic properties would allow the righthand side to differ from zero by a stationary stochastic process orthogonal to the remaining variables in the system.

[28] For technical reasons requiring the analysis of non-constant coefficient difference equations, I examine the determinacy properties of this model separately and cast doubt on the purported adherence of the model, whose origins can be found in Mankiw and Reis (2002) and Bénassy (2003), to the NRH.

conditional expectations operator k periods ago

$$
\begin{aligned}
0 &= E_k\left[\sum_{i=0}^{p}\sum_{j=-m}^{n} Q_1(i,j)E_{t-i}\left[R_{t+j} \quad \pi_{t+j} \quad y_{t+j}\right]'\right] \\
&= \left(\sum_{i=0}^{k}\sum_{j=-m}^{n} Q_1(i,j)\right)E_{t-k}\left[R_{t+j} \quad \pi_{t+j} \quad y_{t+j}\right]' + \sum_{i=k}^{p}\sum_{j=-m}^{n} Q_1(i,j)E_{t-i}\left[R_{t+j} \quad \pi_{t+j} \quad y_{t+j}\right]' \\
&= \left(\sum_{i=0}^{k}\sum_{j=-m}^{n} Q_{1,3}(i,j)\right)E_{t-k}\left[y_{t+j}\right] + \sum_{i=k}^{p}\sum_{j=-m}^{n} Q_{1,3}(i,j)E_{t-i}\left[y_{t+j}\right]
\end{aligned}
$$

which is assumed throughout to be a saddle-point equation with resolvable expectations in the output gap and,[29] hence, the natural rate property follows and the output gap is closed on average irrespective of monetary policy.

4.3 Determinacy: A Lemma

Here, I derive a lemma for the conditions under which determinacy obtains in the general class of models in (17). This lemma splits the determinacy property into a standard saddle-point problem and a resolvability of the lagged expectation structure.

The following lemma establishes necessary and sufficient conditions for models in the class (17) to have a unique stationary solution. There are two conditions that need to be fulfilled. First, the Blanchard and Kahn (1980) saddle-point condition needs to be fulfilled. Due to the variable number of leads and lags in (17), Anderson's (2010) explicit extension to such a case is used, eliminating the burden of expanding the state space to fit Blanchard and Kahn's (1980) canonical form. The second condition collects the set of equations needed to resolve the lagged expectations,[30] extending Meyer-Gohde (2010) to the variable leads and lags of Anderson (2010), and requires that it have a unique solution. The lemma itself contains little novelty, but combines existing results to be flexible enough for the rather general class of models in (17).[31]

Lemma 4.2. *For the system (17) to be determinate, i.e., to have a unique stationary solution,*

[29] If not, there exists the potential for an interaction of the nominal side with an increasing-returns-to-scale type of indeterminacy as explored by Weder (2008). This is, however, beyond the scope of this paper.

[30] An example of how lagged expectations can be unresolvable is explored starting with (25).

[31] Boyd and Dotsey (1996) provide a somewhat similar result, though their uniqueness proof leaves a set of coefficients arbitrary leading them to the apparently paradoxical conclusion that "the general solution is not unique." Additionally, by assuming the underlying homogenous system to be non-singular and using the resulting Jordan decomposition, they are able to characterize a rank condition analytically. The generalized Schur decomposition, here from Anderson (2010) or likewise Klein (2000) among others, I use here allows for singular cases to be analyzed, but comes at the cost of the analytical analysis of this rank condition, which I abstract from following common practice in the literature. See also footnote 33 for more on this point.

1. *The model that results from replacing all lagged expectations (those formed at time $t-p$ for $p>0$) with their time t values must have a unique saddle-point stable solution.*

2. *The sequence of one-step expectation errors induced by the structure of lagged expectations must have a unique solution.*

Proof. See appendix B. ☐

Again, the first condition requires that the model be determinate if all lagged expectations are replaced with time t expectations, Anderson's (2010) saddle point. While the second requires that one can uniquely resolve the lagged expectations, formalizing Whiteman's (1983, pp. 29–36) insight that resolving lagged expectations, solving ”withholding constraints” in his terminology or removing "arbitrary MA coefficients" in Boyd and Dotsey's (1996), is not generally a trivial task.

4.4 Determinacy and Monetary Policy

In this section, I apply the lemma from the foregoing section to establish the link between determinacy in NRH models and determinacy in their fully frictionless counterparts, excepting for the indeterminacy of expectational non-resolvability. This link enables me to decouple the examination of determinacy from the particular supply curve in an analysis so long as it satisfies the NRH, leading to the equivalence result in theorem 4.6. Finally, the ability to abstract from the particular supply curve allows me to establish general results for determinacy given standard dynamic IS demand for all supply sides that satisfy the NRH in the admissible class.

First, I shall describe the frictionless counterpart benchmark. Here, there is no impediment to firms' setting the optimal, full-information price every period. It follows by definition that the output gap is always zero

$$y_t = 0 \; \forall t \tag{23}$$

i.e., the special case of $k=0$ in the NRH definition (20). This is a rather extreme version of the NRH also satisfied by the limiting case of the New Keynesian model examined in section 2. In this case, IS demand (1) reduces to

$$R_t = E_t[\pi_{t+1}] \tag{24}$$

After k periods have passed since some disturbance from equilibrium, a supply side that fulfills (20) behaves identically to that of (23), i.e., applying the conditional expectations operator to the LHS of both supply sides yields zero—$E_{t-k}[(23)] = E_{t-k}[(20)] = 0$. Hence, given a common specification for the remainder of the model, any two models that satisfy (20) for some k are identical in the long-run (or indeed, after k).

Proposition 4.3. *Consider a model in (17) that satisfies the NRH of (20). The model is determinate only if the corresponding frictionless model—i.e., one satisfying (23)—is determinate.*

Proof. If the model is determinate, parts 1 and 2 of lemma 4.2 are fulfilled. But part 1 of lemma 4.2 is the same for the frictionless model and is thus likewise fulfilled. This leaves part 2 of lemma 4.2. But part 2 is necessarily fulfilled by the frictionless model (the matrix (B-15) in the proof of lemma 4.2 is lower triangular with all diagonal entries equal one, hence necessarily nonsingular). Thus, the frictionless model that satisfies (23) is also determinate. □

Thus, a necessary condition for determinacy in any model that satisfies the NRH is that the corresponding frictionless model is determinate. The foregoing proposition proves the necessity of determinacy in the underlying frictionless model, showing essentially that the eigenvalue counting method of Blanchard and Kahn (1980) is the same regardless of actual value of k.

Proposition 4.4. *Consider a determinate frictionless model—i.e., one that satisfies (23) in (17). There exist corresponding NRH models—i.e., that satisfy (20) for $k > 0$—that are indeterminate.*

Proof. As the frictionless model is determinate, part 1 of lemma 4.2 is fulfilled. This system is the same for the NRH model and hence part 1 of lemma 4.2 is fulfilled by the NRH model. Part 2 is necessarily fulfilled by the frictionless model (the matrix (B-15) in the proof of lemma 4.2 is lower triangular with all diagonal entries equal one, hence necessarily nonsingular), but the lagged expectation structure is unrestricted in the NRH model. Thus, part 2 of lemma 4.2 need not be fulfilled by the NRH model (the matrix (B-15) in the proof of lemma 4.2 can in general be singular) and there exist indeterminate NRH models whose the corresponding frictionless models are determinate. □

Lemma 4.2 establishes that the saddle-point property of the underlying matrix polynomial is insufficient to conclusively establish determinacy: it does not necessarily follow that a model that satisfies the NRH is determinate when its frictionless counterpart is. This shows, for example, that the equivalence between nominal and real determinacy in the NRH model of Carlstrom and Fuerst (2002) does not carry over without exception to all models that satisfy the NRH. As Whiteman (1983, p. 33) points out, “the conditions for existence and uniqueness of solutions to withholding equations are quite different from those for the general expectational difference equation.”

A simple, univariate example will illustrate. Consider the following system

$$aE_t\left[\theta_{t+1}\right] = b\theta_t + cE_{t-1}\left[\theta_t\right] \tag{25}$$

In the absence of expectations (that is, in the form of part 1 of lemma 4.2), this reduces to

$$a\theta_{t+1} = (b+c)\,\theta_t \tag{26}$$

which is saddle-point stable if $|\frac{b+c}{a}| > 1$. But the original equation does have expectations and, indeed, lagged expectations that need to be resolved for part 2 of lemma 4.2. Taking expectations of (25) at the highest expectational lag (here $t-1$) yields

$$aE_{t-1}\left[\theta_{t+1}\right] = (b+c)\,E_{t-1}\left[\theta_t\right] \tag{27}$$

defining $\tilde{\theta}_{t-1} = E_{t-1}\left[\theta_t\right]$, inserting into the above and lagging forward yields

$$aE_t\left[\tilde{\theta}_{t+1}\right] = (b+c)\,\tilde{\theta}_t \tag{28}$$

an equation whose saddle-point properties are the same as (26). Thus, if $|\frac{b+c}{a}| > 1$, there is a unique stable solution. This confirms proposition 4.3 and rests on the underlying “eigenvalue count” being the same. As the system is homogenous, this unique solution is $\tilde{\theta}_t = 0$. Recalling the definition of $\tilde{\theta}_t$ and inserting into (25) yields

$$0 = b\theta_t \tag{29}$$

Consider now the special case $b = 0$: the foregoing does not deliver a unique solution for θ_t, even though the condition for saddle-point stability, now $|\frac{c}{a}| > 1$, can still be fulfilled. This results is embodied in proposition 4.4 and rests on the singularity of the the system needed to “resolve” the expectations structure. Of course, $b = 0$ is a special case and it need not hold generally: hence, an isolated singularity. In the context of a multivariate model, like those in (17), a singularity in the expectations structure would occur if demand and supply are linearly dependent at some expectational horizon and hence the model is ill-formulated, or monetary policy succeeds in recreating such a linear dependency through a fortuitous selection of policy and parameter values. While neither seems likely within the the analysis here, neither can be excluded a priori and such singularities might be of greater importance in other analyses.

Moving past such singular cases for the analysis, the correspondence between determinacy under the NRH and determinacy in the corresponding frictionless model has some strong implications. To aid in the analysis that follows, I shall define a class of models that excludes such isolated singularities, so that the analysis can focus on regular cases

Definition 4.5 (The Class of Regular Models in (17))**.** *The class of models in this definition is the class of models in (17) restricted to rule out the non-resolvability in part 2 of lemma 4.2.*

Restricting attention to models in definition 4.5, an equivalence theorem, the central result of the paper, follows straightforwardly

Theorem 4.6. *Consider models in definition 4.5 and fix the demand equation and monetary policy.*

1. *If the model is determinate under one supply equation that satisfies (20), it is determinate under all supply equations that satisfy (20).*

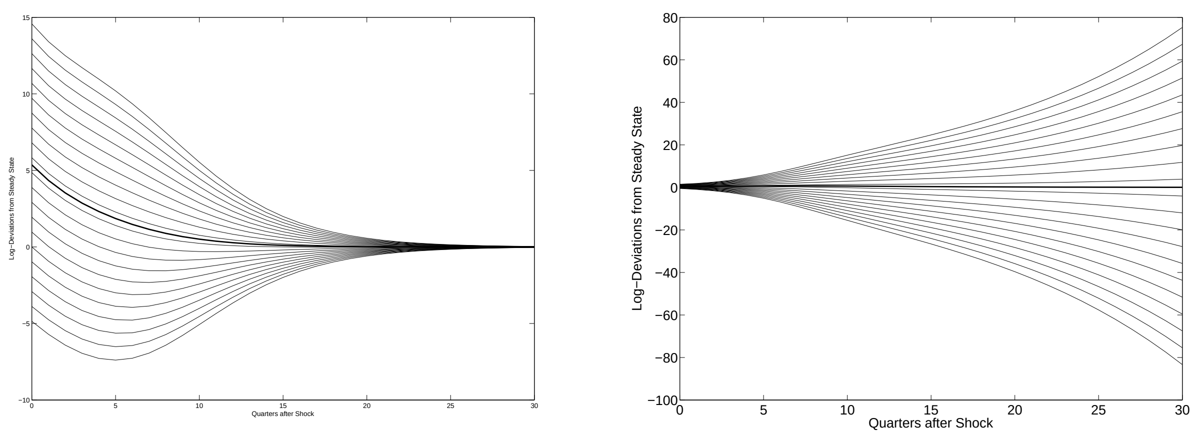
2. *If the model is not determinate under one supply equation that satisfies (20), it is not determinate under all supply equations that satisfy (20).*

Thus, the determinacy bounds are the same for all supply equations that satisfy (20).

Proof. The non-resolvability in part 2 of lemma 4.2, proposition 4.4 has been ruled out by definition 4.5. Thus, a model in this class that satisfies the NRH defined in (20) is determinate if and only if the corresponding model that satisfies (23) is determinate. This must hold for all k and thus holds for all $\tilde{k} < k$. Any supply equation that satisfies the NRH at a horizon $\tilde{k} < k$, necessarily satisfies it at the horizon k as well. Thus, for a given k, all supply equations that satisfy the NRH are determinate if and only if the corresponding frictionless model is determinate. □

The intuition is as follows. If the model satisfies the NRH, then the output gap must on average be equal to zero independent of monetary policy. This guarantees the existence (but not necessarily the uniqueness) of a stationary output gap independent of inflation and the nominal interest rate. Were $k = 0$, there would be complete separation between the real and nominal sides of the economy and monetary policy would serve only to establish nominal determinacy with the output gap necessary determined. For $k > 0$, the lack of a complete separation by assumption links nominal and real determinacy: without a unique solution for the nominal side, the link between the output gap and the nominals at horizons less than k implies a unique solution for the output gap cannot be pinned down. If a unique solution for the nominal side can be determined by long-run demand and monetary policy, this solution selects, through the link at horizons less than k, a single solution for the output gap.

Therefore, as there is a unique stationary solution for the output gap if and only if there is a unique stationary solution for inflation and the nominal interest rate in the frictionless counterpart model, the conditions for determinacy are identical for all supply equations that satisfy (20).[32] The situation is exemplified graphically in figure 1: each path for the output gap is associated with a single path for inflation. All the different paths of the output gap in figure 1a converge even though all but one of the paths for inflation, depicted in figure 1b, diverge. Selecting the unique stationary path among the different paths for inflation selects a particular path for the output gap (the more heavily weighted lines in the figure), thus determining both through consideration solely of inflation. The particular pattern for the output gap associated with each path for inflation—i.e., the particular form of the short-run Phillips tradeoff—is entirely inconsequential.



(a) Output Gap (b) Inflation

Figure 1: Hypothetical Impulse Responses with Different Initial Conditions

The irrelevance of the degree and manner of output gap targeting for determinacy, as well as that of the exact nature of the short-run tradeoff in the Phillips curve relationship follows immediately.

Corollary 4.7. *Consider a model within the class of definition 4.5 with any supply equation satisfying (20). Determinacy is independent of the parameters both in the interest rate rule, however formulated within the class, pertaining to the output gap and of those in the supply equation.*

Proof. From theorem 4.6, I may choose any supply equation to establish determinacy. Choosing (23), the output gap is always closed. But this eliminates the parameters in monetary policy pertaining to the output gap. Furthermore, from theorem 4.6, it follows directly that the parameters in the supply equation are irrelevant. □

[32]Saving, of course, for the caveat of the non-resolvability in part 2 of lemma 4.2.

Again, if the model satisfies the NRH, then the output gap must on average be equal to zero independent of monetary policy. There is simply no role for targeting the real side in determinacy. Furthermore, as the conditions for determinacy are the same for all supply equations that satisfy the NRH, theorem 4.6, parameters of a specific supply equation formulation must be irrelevant.

Specializing to standard dynamic IS demand, a more specific statement can be made: a complete independence of the conditions for determinacy from all parameters outside of monetary policy except those that relate to inflation and the nominal interest rate

Corollary 4.8. *Consider a model within the class of definition 4.5 with demand given by (1) and any supply equation satisfying (20). Determinacy is a function solely of the parameters in the interest rate rule, however formulated within the class, pertaining to inflation and the interest rate.*

Proof. Following theorem 4.6, any supply equation that satisfies (20) can be used to establish determinacy. From corollary 4.7, choosing (23) renders all parameters in the supply equation and those in monetary policy related to the output gap moot. Examining (1), from (23) , (1) becomes (24), eliminating the parameters in the demand equation. Thus, the only parameters in the model remaining that can affect determinacy are those in the interest rate rule pertaining to inflation and the interest rate. □

Dynamic IS demand ultimately derives from a consumption Euler or Lucas asset pricing equation and arbitrage-free pricing of assets leads to a Fisher equation linking the pricing of real and nominal assets, $r_t = R_t + E_t[\pi_{t+1}]$. As the classical dichotomy sets in with the model approaching the long run, the real rate becomes exogenous (and can thus be normalized to zero) to the determinacy question, which operates through the nominal side in the long run under the NRH as argued above. Demand then only contributes this Fisher relation (24), a parameter-less equation, to the long-run system.

5 Interest Rate Rules and the NRH

Of particular interest are the determinacy results regarding specific policies, such as inflation targeting, forward-looking and backward-looking monetary policy, interest rate smoothing, and output gap targeting. To that end, let monetary policy now be described by the following interest rate rule

$$R_t = \phi_R R_{t-1} + \phi_\pi \left(\psi_1 E_t[\pi_{t+1}] + \psi_2 \pi_t + \psi_3 \pi_{t-1}\right) + \phi_y y_t, \text{ where } \psi_1 + \psi_2 + \psi_3 = 1 \tag{30}$$

where ϕ_R describes the degree of interest-rate smoothing, ϕ_π of inflation targeting, and ϕ_y of output gap targeting. The coefficients $\psi_{1,2,3}$ nest contemporaneous inflation targeting ($\psi_2 = 1$), inflation forecast targeting ($\psi_1 = 1$), and a purely backward-looking inflation target ($\psi_3 = 1$).

Theorem 5.1. *For every model in definition 4.5 with demand given by (1), any supply equation satisfying (20), and monetary policy described by (30), the stable equilibrium is determinate if*[33]

$$|\phi_R + \phi_\pi \psi_2| > |1 - \phi_\pi (\psi_1 + \psi_3)| \tag{31}$$

Furthermore, if $|\phi_R + \phi_\pi \psi_2| < |1 - \phi_\pi (\psi_1 + \psi_3)|$, *the stable equilibrium is*

$$\begin{array}{l} \textit{indeterminate if } |\phi_\pi \psi_3| < |1 - \phi_\pi \psi_1| \\ \textit{non-existent if } |\phi_\pi \psi_3| > |1 - \phi_\pi \psi_1| \end{array} \tag{32}$$

Proof. Following theorem 4.6, I may choose any supply equation that satisfies the NRH to establish determinacy. Choosing (23) reduces the demand equation to (24) and the interest rate rule, (30), to $R_t = \phi_R R_{t-1} + \phi_\pi (\psi_1 E_t [\pi_{t+1}] + \psi_2 \pi_t + \psi_3 \pi_{t-1})$. Combining delivers the system

$$\begin{bmatrix} -\phi_\pi \psi_1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} E_t [\pi_{t+1}] \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} \phi_\pi \psi_2 & \phi_\pi \psi_3 & \phi_R \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ R_{t-1} \end{bmatrix} \tag{33}$$

The generalized eigenvalues, z, solve $0 = -z \left(z^2 (1 - \phi_\pi \psi_1) - z (\phi_\pi \psi_2 + \phi_R) - \phi_\pi \psi_3\right)$. With two backward-looking and one forward-looking variable, determinacy requires two roots inside and one outside the unit circle. One eigenvalue is necessarily inside being equal to 0. Therefore, of the remaining two roots, exactly one must be outside and one inside the unit circle for determinacy. If the remaining two are both inside the unit circle, the stable equilibrium is indeterminate, and if they are both outside the unit circle, the stable equilibrium is non-existent. The two remaining roots solve $z^2 (1 - \phi_\pi \psi_1) - z (\phi_\pi \psi_2 + \phi_R) - \phi_\pi \psi_3 = 0$, and, the bounds on the eigenvalues follow directly from corollary A.2 in appendix A. □

Reiterating the results from 4.8, determinacy depends only on parameters in the interest rate rule and is independent of the degree of output gap targeting.

Additionally, a specific case of the foregoing is of particular interest for monetary policy, that of positive reaction coefficients.

[33] It is to be understood that the analysis will be abstracting from cases where the relevant eigenvalues lie on the unit circle. Following Woodford (2003, p. 254), in such a case, the linearized models examined here are insufficient to address the question of local determinacy. There is the additional issue of "translatability" of the stable manifold—i.e., Klein's (2000, p. 1413) Assumption 4.5 or Blanchard and Kahn's (1980, p. 1308) full-rank condition—that ensures the stable manifold can be associated with the initial conditions of the backward-looking variables that I abstract from likewise following Woodford (2003, pp. 672–673).

Corollary 5.2. *For $\phi_R, \phi_\pi \geq 0$,*[34] *the determinacy bounds from theorem 5.1 become*

$$\begin{aligned}
&\textit{Determinacy:} && \phi_\pi > 1-\phi_R \textit{ and } \phi_\pi(1-2\psi_2) < 1+\phi_R \\
&\textit{Indeterminacy:} && \textit{either} \begin{cases} \phi_\pi < 1-\phi_R \\ \phi_\pi(1-2\psi_2) > 1+\phi_R \textit{ and } \phi_\pi\psi_1 > 1+\phi_\pi\psi_3 \end{cases} \\
&\textit{Non-existence:} && \phi_\pi(1-2\psi_2) > 1+\phi_R \textit{ and } \phi_\pi\psi_1 < 1+\phi_\pi\psi_3
\end{aligned} \tag{34}$$

Proof. See appendix C. ☐

In (30), the general rule that allows for any combination of forward-looking, backward-looking, and contemporaneous inflation targeting, a sufficient condition to eliminate the upper bound is that majority of the emphasis is given to contemporaneous inflation targeting ($\psi_2 > 1/2$)—see figure 2 and corollary 5.2. The indeterminacy of aggressive purely forward-looking policy (high $\phi_\pi\psi_1$) can be mitigated or eliminated entirely if the monetary authority places sufficient weight on current and past indicators , reinforcing Woodford's (2000) warning that monetary policy needs to be history dependent. If the emphasis is shifted towards past inflation rather (high ψ_3) than a (cumulative) contemporaneous response (high ψ_2 and ϕ_R), indeterminacy may be replaced by non-existence, placing the history dependence motivation under a caveat.

With corollary 5.2, I can proceed to analyze specific cases of the interest-rate rule (30) as derived informally with two thought experiments in section 3.

5.1 Confirming the Special Cases of Section 3

The determinacy bounds for the standard sticky-price New Keynesian model and when the NRH is fulfilled were derived in section 3 using two thought experiments. Using theorem 5.1, I will confirm the bounds derived there formally. Additionally, I will identify regions where determinacy does not obtain as being associated with indeterminacy or nonexistence.

5.1.1 Exogenous Interest Rates

A constant (or constant with respect to some exogenously evolving target) interest rate rule reduces (30) to $R_t = 0$, which requires $\phi_\pi = \phi_y = \phi_R = 0$. Inserting these values into theorem 5.1, it follows immediately that $|\phi_R + \phi_\pi\psi_2| = |0| > |1-\phi_\pi(\psi_1+\psi_3)| = |1|$ cannot hold and thus the model is not

[34] Restricting the parameters to be positive also implies that the bounds should be of the form, taking the first bound in the corollary as an example, $\phi_\pi > \max(1-\phi_R, 0)$, see Lubik and Marzo (2007). These non-negativity bounds have been left implicit to reduce clutter.

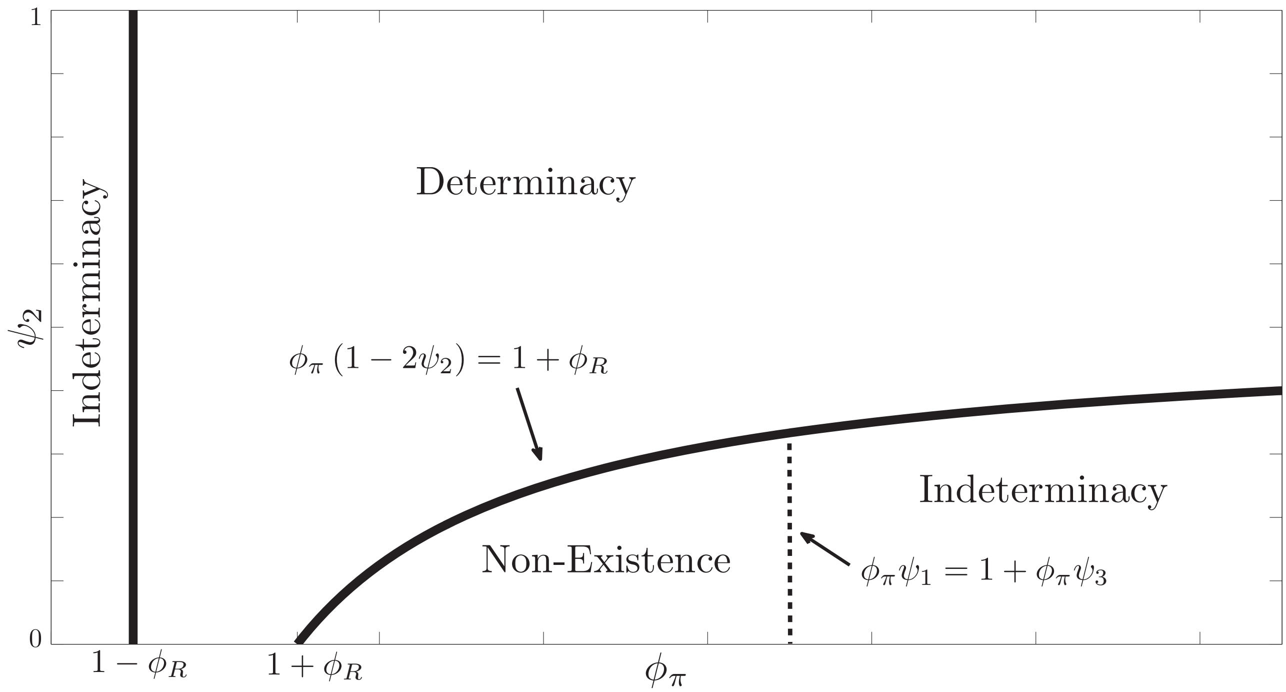


Figure 2: Determinacy Regions from Theorem 5.1

determinate, confirming the result in section 3.2. Furthermore, $|\phi_\pi\psi_3| = |0| < |1-\phi_\pi\psi_1| = |1|$ does hold and thus the model is indeterminate.

5.1.2 Output Gap Targeting

An interest rate rule with only output gap feedback reduces (30) to $R_t = \phi_y y_t$, which requires $\phi_\pi = \phi_R = 0$. Inserting these values into theorem 5.1, it follows immediately that $|\phi_R + \phi_\pi\psi_2| = |0| > |1-\phi_\pi(\psi_1+\psi_3)| = |1|$ cannot hold and thus the model is not determinate, just as in section 3.3. Furthermore, $|\phi_\pi\psi_3| = |0| < |1-\phi_\pi\psi_1| = |1|$ does hold and thus the model is indeterminate.

5.1.3 Contemporaneous Inflation Targeting

An interest rate rule with contemporaneous inflation targeting reduces (30) to $R_t = \phi_R R_{t-1} + \phi_\pi\pi_t + \phi_y y_t$, which requires $\psi_1 = \psi_3 = 0$ and, therefore, $\psi_2 = 1$. Recalling that the exercise focused only on positive coefficient values and inserting these values for the ψ's into corollary 5.2, it follows immediately that $\phi_\pi > 1-\phi_R$ and $\phi_\pi(1-2\psi_2) < 1+\phi_R$ translate into $\phi_\pi > 1-\phi_R$ and $0 < 1+\phi_R$. Thus, the model is determinate only when $\phi_\pi > 1-\phi_R$, e.g. when the cumulative reaction of the nominal interest rate to inflation is more than one-for-one, confirming the result of section 3.4. Going further, it is apparent from corollary 5.2 that when the model is not determinate under a contemporaneous inflation target with positive coefficients, $\phi_\pi < 1-\phi_R$, it is necessarily indeterminate.

5.1.4 Forward-Looking Inflation Targeting

An interest rate rule with feedback from future inflation reduces (30) to $R_t = \phi_R R_{t-1} + \phi_\pi E_t[\pi_{t+1}] + \phi_y y_t$, which requires $\psi_2 = \psi_3 = 0$ and, therefore, $\psi_1 = 1$. Recalling that the exercise focused only on positive coefficient values and inserting these values for the ψ's into corollary 5.2, it follows immediately that $\phi_\pi > 1-\phi_R$ and $\phi_\pi(1-2\psi_2) < 1+\phi_R$ translate into $\phi_\pi > 1-\phi_R$ and $\phi_\pi < 1+\phi_R$. Thus, the model is determinate only when $1-\phi_R < \phi_\pi < 1+\phi_R$. That is, the monetary authority should respect the Taylor principle, but not overly aggressive, confirming the result of section 3.5. Going further, in the absence of determinacy, $\phi_\pi(1-2\psi_2) > 1+\phi_R$ and $\phi_\pi\psi_1 < 1+\phi_\pi\psi_3$ from corollary 5.2 translates into $\phi_\pi > 1+\phi_R$ and $\phi_\pi\psi_1 < 1$, which cannot hold for positive ϕ_R. Thus when the model is not determinate under a forward looking inflation target with positive coefficients, $1-\phi_R < \phi_\pi < 1+\phi_R$, it is necessarily indeterminate.

5.1.5 Backward-Looking Inflation Targeting

An interest rate rule with feedback from past inflation reduces (30) to $R_t = \phi_R R_{t-1} + \phi_\pi \pi_{t-1} + \phi_y y_t$, which requires $\psi_1 = \psi_2 = 0$ and, therefore, $\psi_3 = 1$. Recalling that the exercise focused only on positive coefficient values and inserting these values for the ψ's into corollary 5.2, it follows immediately that $\phi_\pi > 1-\phi_R$ and $\phi_\pi(1-2\psi_2) < 1+\phi_R$ translate into $\phi_\pi > 1-\phi_R$ and $\phi_\pi < 1+\phi_R$. Thus, the model is determinate only when $1-\phi_R < \phi_\pi < 1+\phi_R$. That is, the monetary authority should respect the Taylor principle, but not overly aggressive, confirming the result of section 3.6 and repeating the result from above from the forward-looking rule. Going further, in the absence of determinacy, $\phi_\pi(1-2\psi_2) > 1+\phi_R$ and $\phi_\pi\psi_1 < 1+\phi_\pi\psi_3$ from corollary 5.2 translates into $\phi_\pi > 1+\phi_R$ and $0 < 1+\phi_\pi$, admitting non-existence for the overly aggressive inflation response. Thus with a backward looking inflation target with positive coefficients, the model is determinate when $1-\phi_R < \phi_\pi < 1+\phi_R$, indeterminate when $\phi_\pi < 1-\phi_R$, and displays non-existence when $1+\phi_R < \phi_\pi$.

5.2 Comparison with Sticky Prices

The results for the special cases of (30), examined above, are gathered in table 1. The table juxtaposes the determinacy regions in an NRH model with those in the standard sticky-price model, derived with the thought experiments in section 3.[35] Under standard parameter assumptions—i.e., $a, \kappa > 0$ and $0 < \beta < 1$—where the determinacy regions differ, the sticky-price models implies a larger region of determinacy for the plausible space of positive interest rate rule elasticities. Appealing to the "near-unanimous acceptance of the Friedman-Phelps-Lucas view that there is no exploitable long-run tradeoff between inflation and output or employment[,]" (McCallum 1999, p. 182) I conclude that the sticky-price model can mislead monetary authorities into adapting policies that fail to yield a determinate equilibrium. It can be seen that even the extreme parameterization of $\beta = 1$ does not suffice to bring the determinacy results of the sticky-price model inline with those of a NRH model. Indeed, there only parameterization that brings the determinacy regions of the sticky-price model in the line with those of a NRH model is the parameterization that brings the sticky-price model inline with the NRH, $\kappa \to \infty$, as can be easily observed in the table.[36] For the long-run Phillips curve to be vertical in the case of the standard sticky-price model, it must always be vertical.

Table 1: Determinacy Regions: Comparison of NRH and Sticky Prices

Interest Rate Rule	Sticky Prices	NRH
	Non-Price-Related Feedback	
$R_t = 0$	$\emptyset$	$\emptyset$
$R_t = \phi_y y_t$	$\phi_y > \frac{\kappa}{1-\beta}$	$\emptyset$
	Forward-Looking Inflation Targeting	
$R_t = \phi_\pi E_t[\pi_{t+1}]$	$1 < \phi_\pi < 1 + 2\frac{1+\beta}{a\kappa}$	$\emptyset$
$R_t = \phi_R R_{t-1} + \phi_\pi E_t[\pi_{t+1}] + \phi_y y_t$	$\phi_\pi > 1 - \phi_R - \frac{1-\beta}{\kappa}\phi_y$ $\phi_\pi < (1+\phi_R)\left(1 + 2\frac{1+\beta}{a\kappa}\right) + \phi_y \frac{1+\beta}{\kappa}$	$\phi_\pi > 1 - \phi_R$ $\phi_\pi < 1 + \phi_R$
	Backward-Looking Inflation Targeting	
$R_t = \phi_\pi \pi_{t-1}$	$1 < \phi_\pi < 1 + 2\frac{1+\beta}{a\kappa}$	$\emptyset$
$R_t = \phi_R R_{t-1} + \phi_\pi \pi_{t-1} + \phi_y y_t$	$\phi_\pi > 1 - \phi_R - \frac{1-\beta}{\kappa}\phi_y$ $\phi_\pi < (1+\phi_R)\left(1 + 2\frac{1+\beta}{a\kappa}\right) + \phi_y \frac{1+\beta}{\kappa}$	$\phi_\pi > 1 - \phi_R$ $\phi_\pi < 1 + \phi_R$
	Contemporaneous Inflation Targeting	
$R_t = \phi_\pi \pi_t$	$\phi_\pi > 1$	$\phi_\pi > 1$
$R_t = \phi_R R_{t-1} + \phi_\pi \pi_t + \phi_y y_t$	$\phi_\pi > 1 - \phi_R - \frac{1-\beta}{\kappa}\phi_y$	$\phi_\pi > 1 - \phi_R$

Recall that, in general, the sticky-price New Keynesian model does not satisfy the NRH. Consequently, the sticky-price model is not even asymptotically isomorphic to its frictionless equivalent:

[35] Detailed formal derivations of the determinacy results for the sticky-price model can be found in Bullard and Mitra (2002), Woodford (2003), Lubik and Marzo (2007), and Galí (2008) among others.

[36] This implies a rule of thumb to judge whether a study's particular determinacy results stem from its violation of the NRH: namely examine the limit of the determinacy condition as the Phillips curve becomes vertical.

with this permanent link between the nominal and real side of the economy, nominal and real determinacy must be simultaneously ascertained.[37] In assessing determinacy in the sticky-price model, one is forced to look at paths along which the violation of the NRH may be consequential. In terms of the thought experiment of section 3, potential equilibria associated with a permanent non-diminishing pattern in inflation are excluded from the permissible set of equilibria, such that only bounded equilibria remain in the set. Hence, the assessment of determinacy necessitates the examination of paths that become unbounded in order to exclude them from the set of admissible equilibria. It is here that the NRH becomes decisive: due to the permanency of the output-inflation tradeoff in sticky price models, a path can become unbounded (and hence excludable) solely due to an unabated mutual interaction with inflation and the output gap are feeding boundlessly into each other. In contrast, such a permanent link is impossible in a model that satisfies the NRH and the output gap is necessarily bounded by virtue of the NRH. Therefore, a model's violation of the NRH can force an otherwise bounded equilibria to become unbounded, distorting policy recommendations in terms of bounds on coefficients in the monetary authority's policy rule so as to ensure the uniqueness of a bounded equilibrium.

That imposing the NRH leads to such general results underlines the long-run nature of determinacy analyses. It is perhaps all to easy to forget that determinacy ultimately rests on ruling out explosive, or divergent, paths.[38] It is the behavior in the long run that establishes whether a particular path is diverging. Determinacy is a question of the long run and it is in the long run by its restriction on the average output gap that the NRH is of consequence.

5.3 Higher Leads and Lags?

The determinacy bounds in theorem 5.1 imply that neither a pure forward-looking ($R_t = \phi_\pi [\pi_{t+1}]$) nor pure backward-looking ($R_t = \phi_\pi \pi_{t-1}$) inflation target is associated with determinacy. In the following, I will prove that this is not restricted to the one period horizon used in the theorem, but extends straightforwardly to any (finite) horizon.

[37] As McCallum (2003, p. 1157) put it, "the [Calvo] form of sticky prices [...] is such that the model continues to include nominal variables even when monetary policy supplies no nominal anchor, because private behavior involves a type of dynamic money illusion."

[38] According to Cochrane (2011), the problem is even worse: macroeconomics appears to have forgotten why and whether it can rule out such paths all. The analysis here is agnostic on that issue, taking as given the standard practice of analyzing determinacy to ascertain whether a unique equilibrium is present. It should, however, be noted that the analysis here suggests that determinacy under the NRH and with standard IS demand reduces to the simple model used by Cochrane (2011) to fix ideas, implying that determinacy for the entire class of models that satisfies Lucas's (1972) NRH rest directly upon the resolution of simple model in the Cochrane (2011) critique.

Theorem 5.3. *A model within the class of definition 4.5 with demand given by (1), any supply equation satisfying (20), and monetary policy described by* $R_t = \phi_\pi E_t\left[\pi_{t+j}\right]$ *has a unique stable equilibrium (determinacy) only if* $j = 0$.

Proof. Following theorem 4.6, I may choose any supply equation that satisfies the NRH to establish determinacy. Choosing (23) reduces the demand equation to (24), combining with the interest rate rule, $R_t = \phi_\pi E_t\left[\pi_{t+j}\right]$, yields

$$E_t\left[\pi_{t+1}\right] = \phi_\pi E_t\left[\pi_{t+j}\right] \tag{35}$$

For $j \geq 1$, the characteristic equation of the system is $F(z) = \phi_\pi z^j - z$. As the system is entirely forward looking, all zeros need to lie outside the unit circle. But the zeros solve $\left(\phi_\pi z^{j-1} - 1\right) z = 0$, one of which is obviously equal to zero.

For $j \leq -1$, the characteristic equation of the system is $F(z) = \phi_\pi z^{-j} - z$. Stability now requires one zero outside and j inside the unit circle. But the zeros solve $\left(\phi_\pi - z^{j+1}\right) z = 0$, all of which are on the same side of the unit circle depending on ϕ_π.

For $j = 0$, the characteristic equation of the system is $F(z) = \phi_\pi - z$. Stability requires one zero outside the unit circle, which holds if $|\phi_\pi| > 1$. □

The intuition here is simple. In the case of backward-looking rules, the necessary vigor in monetary policy's reaction to satisfy the Taylor principle is immediately too vigorous, implying that inflation is always explosive. In the case of forward-looking rules, by the fact that agents with IS demand are also forward looking, the model contains no reference to current inflation whatsoever; satisfying the Taylor principle does not preclude transient self-fulfilling inflation sunspots. In the case of a pure target, monetary policy *must* focus on the current state of inflation.

6 Conclusion

I have presented results on determinacy and monetary policy that are applicable in an entire class of intertemporal models restricted only to satisfy the NRH. As argued by McCallum (1998), this is precisely the class upon which analyses of monetary policy should concentrate and my results indicate that the determinacy analyses in much of the New Keynesian literature are not robust to the NRH. The general results can be be summarized as reinforcing the importance of the Taylor principle, dismissing the relevance of output gap targeting, and questioning the safety of forward-looking and backward-looking monetary policy in terms of determinacy.

A Appendix: Eigenvalue Bounds Using Jury's Criterion

This appendix derives bounds on eigenvalues of a two-dimensional system using Jury's (1961) simplification of the Schur-Cohn criterion.[39]

Lemma A.1. *The polynomial with real coefficients*

$$F(z) = a_2 z^2 + a_1 z + a_0 \tag{A-1}$$

has both zeros inside the unit circle if and only if

$$|a_0| < |a_2|$$
$$|a_0 + a_2| > |a_1| \tag{A-2}$$

Proof. See Jury (1961, p. 342). □

This statement of the two-dimensional stability problem does not require the normalization of the leading coefficient of the polynomial, which greatly simplifies the following.

Corollary A.2. *The zeros of the polynomial with real coefficients*

$$F(z) = a_2 z^2 + a_1 z + a_0 \tag{A-3}$$

are bounded with respect to the unit circle as follows

$$\begin{aligned} &\textit{Both inside} && |a_0 + a_2| > |a_1| \textit{ and } |a_0| < |a_2| \\ &\textit{Both outside} && |a_0 + a_2| > |a_1| \textit{ and } |a_0| > |a_2| \\ &\textit{One inside and one outside} && |a_0 + a_2| < |a_1| \end{aligned} \tag{A-4}$$

Proof. The condition for both zeros inside the unit circle is a repetition of lemma A.1. The condition for both zeros to be outside the unit circle is a simple application of lemma A.1 to the polynomial $z^2 F(1/z) = a_0 z^2 + a_1 z + a_2$, where the zeros of the new polynomial are reciprocals of the zeros of the original polynomial. Excluding both of the foregoing cases, only three possibilities remain: (1) one zero lies inside and one lies outside the unit circle, (2) both zeros lie on the unit circle, (3) one zero lies on the unit circle and the other lies either inside or outside the unit circle. As noted by Jury (1961, p. 342) (or as can be simply confirmed by evaluating $F(1)$ and $F(-1)$), the condition $|a_0 + a_2| < |a_1|$

[39] This also corrects Lubik's (2007, p. 409) claim that a necessary and sufficient condition for both roots of the polynomial $p(\lambda) = \lambda^2 - tr\lambda + det$ to lie outside the unit circle is $|det| > 1$ and $|tr| < 1 + det$. This can be shown incorrect by way of a simple counterexample. Set $tr = 0$ and let $det = -9$, it follows that the zeros of $p(\lambda)$ are given by ± 3, both of which are obviously outside the unit circle. Lubik's (2007, p. 409) second condition, however, $|tr| < 1 + det$, is not fulfilled ($1 + det = -8$ is necessarily negative, contradicting the condition that $|tr| = 0 < 1 + det$). Corollary A.2 in the text here corrects this result. None of the subsequent results in Lubik (2007) were affected by this error.

rules out a real root on the unit circle. This immediately rules out possibility (3), as the only unit roots that remain must be complex roots, which necessarily come in conjugate pairs. Additionally, possibility (2) is then only possible with complex conjugates on the unit circle. Thus, if complex conjugates on the unit circle can be ruled out using $|a_0+a_2|<|a_1|$, then only possibility (1) remains.

I will rule out possibility (2) with complex conjugates by contradiction. Accordingly, assume $|a_0+a_2|<|a_1|$ and that the zeros of $F(z)$ are given as $z_{1,2}=h\pm iv$, where $h=-\frac{a_1}{2a_2}$ and $v=\frac{(4a_2a_0-a_1^2)^{1/2}}{2a_2}$. As the pair must lie on the unit circle, the modulus must be equal to one: $R^2=1=h^2+v^2$. Substituting for h and v in the modulus, $a_2=a_0$ must then hold. From the assumption of complex roots follows additionally that the discriminant of the polynomial is negative: $a_1^2-4a_2a_0<0$, or rearranging $|a_1|<2\sqrt{a_2a_0}$. Using the fact that $a_2=a_0$, $2\sqrt{a_2a_0}=2|a_0|=|2a_0|=|a_0+a_2|$, and combining with the rearranged discriminant condition yields $|a_1|<|a_0+a_2|$, a contradiction, as $|a_0+a_2|<|a_1|$. □

B Appendix: Proof of Lemma 4.2

By the Wold theorem,[40] any stationary process can be represented as

(B-5) $$X_t=\sum_{l=0}^{\infty}\theta_l\varepsilon_{t-l}+\Xi_t,\text{ where }E\varepsilon_t=0\text{ and }E\varepsilon_t\varepsilon_{t+j}'=0,\ \forall j\neq 0$$

and Ξ_t is an orthogonal linearly deterministic process, forecastable perfectly from its own history. Starting with the indeterministic part, and inserting into (17)

(B-6) $$0=\sum_{j=0}^{n}\left[\sum_{l=0}^{\infty}\left(\sum_{i=0}^{min(p,l)}Q(i,j)\right)\theta_{l+j}\varepsilon_{t-l}\right]+\sum_{j=1}^{m}\left[\sum_{l=0}^{\infty}\left(\sum_{i=0}^{min(p,l+j)}Q(i,j)\right)\theta_l\varepsilon_{t-l-j}\right]$$

Defining $\tilde{Q}(l,j)=\sum_{i=0}^{minp,l}Q(i,j)$ and rewriting

(B-7) $$0=\sum_{j=0}^{n}\left[\sum_{l=0}^{\infty}\tilde{Q}(l,j)\theta_{l+j}\varepsilon_{t-l}\right]+\sum_{j=1}^{m}\left[\sum_{l=0}^{\infty}\tilde{Q}(l+j,j)\theta_l\varepsilon_{t-l-j}\right]$$

This must hold for all realizations of ε_t. Comparing coefficients yields

(B-8) $$0=\sum_{j=0}^{n}\tilde{Q}(l,j)\theta_{l+j}+\sum_{j=1}^{m}\tilde{Q}(l,j)\theta_{l-j}$$

a time-varying system of difference equations with initial conditions $\sum_{j=1}^{m}\theta_{-j}=0$. But as $\tilde{Q}(p+i,j)=\tilde{Q}(p,j),\ \forall i\geq 0$, the system of difference equations has constant coefficients after and including p.

This system can be written as

(B-9) $$0=\sum_{j=-m}^{n}\tilde{Q}_jX_{t+j}$$

where $\tilde{Q}_j=\sum_{i=0}^{p}Q(i,j)$ and coincides with Anderson's (2010) canonical form in the case that all

[40] See, e.g., Sargent (1987, pp. 286–290), as well as Priestley (1981, pp. 756–758).

lagged expectations in (17) are replaced with their time t values. If the solution to this system is unique, its stable solution can be written as

$$\theta_l = B\left[\theta'_{l-m} \quad \ldots \quad \theta'_{l-1}\right]', \ \forall l \geq p \tag{B-10}$$

The first p (block) equations—remembering the initial conditions—can be gathered into

$$\mathbf{Q}\left[\theta'_0 \quad \ldots \quad \theta'_{n+p-1}\right]' = 0 \tag{B-11}$$

giving $3p$ equations in $3(p+n)$ variables, with the s^{th} block row of $\mathbf{Q}$ given by

$$\left[0_{max(0,s-1-m)} \quad \tilde{Q}\left(s-1,-min(s-1,m),n\right) \quad 0_{p-s}\right] \tag{B-12}$$

where 0_i is a $3\times 3i$ block vector of zeros and $\tilde{Q}\left(a,b,c\right) = \left[\tilde{Q}(a,b) \quad \tilde{Q}(a,b+1) \quad \ldots\tilde{Q}(a,c)\right]$.

Equation (B-10) yields $3n$ more equations that can be collected as

$$\mathbf{B}\left[\theta'_0 \quad \ldots \quad \theta'_{n+p-1}\right]' = 0 \tag{B-13}$$

with the s^{th} block row of $\mathbf{B}$ given by

$$\left[0_{max(0,s+p-m-1)} \quad -\tilde{B}\left(min(p+s-1,m)\right) \quad I \quad 0_{n-s}\right] \tag{B-14}$$

where I is a 3×3 identity matrix and $\tilde{B}\left(a\right)$ being the last $3\times 3a$ elements of the $3\times 3m$ matrix B that forms Anderson's (2010, p. 7) convergent autoregressive solution to (B-9).

Stacking (B-11) and (B-13) yields the system $\left[\mathbf{Q}' \quad \mathbf{B}'\right]'\left[\theta'_0 \quad \ldots \quad \theta'_{n+p-1}\right]'$, where

$$\begin{bmatrix}\mathbf{Q}\\ \mathbf{B}\end{bmatrix}\begin{bmatrix}\theta_0\\ \vdots\\ \theta_{n+p-1}\end{bmatrix} = 0 \tag{B-15}$$

is a square matrix.

The system (B-8) is homogenous. Thus, one stationary solution is given by $\theta_l = 0$, $\forall i$, the fundamental solution in the absence of exogenous driving forces. If $\left[\mathbf{Q}' \quad \mathbf{B}'\right]'$ is nonsingular and if (B-9) is saddle-point stable, then this is the only stationary solution.

Only Ξ_t remains. Inserting it into (17), it follows that this can also be written as (B-9). If there is a unique solution in past values of Ξ_t, the solution can be written in the same form as (B-10), which must be zero when taken to its remote past from the stability of (B-10).

C Appendix: Proof of Corollary 5.2

C.1 Bounds for Determinacy

For positive coefficients, $|\phi_R + \phi_\pi\psi_2| > |1-\phi_\pi\left(\psi_1+\psi_3\right)|$ from theorem 5.1 becomes (i) $\phi_R + \phi_\pi\psi_2 > |1-\phi_\pi\left(\psi_1+\psi_3\right)|$ and there are three possibilities depending on the sign of $1-\phi_\pi\left(\psi_1+\psi_3\right)$.

(1): $1-\phi_\pi(\psi_1+\psi_3)<0$. Here, (i) becomes $\phi_R+\phi_\pi\psi_2>-1+\phi_\pi(\psi_1+\psi_3)$ which can be rearranged as $\phi_\pi(\psi_1-\psi_2+\psi_3)<1+\phi_R$. Using the relationship $\psi_1+\psi_2+\psi_3=1$ (i.e., the ϕ's are relative weights), the foregoing can be rewritten as $\phi_\pi(1-2\psi_2)<1+\phi_R$.

(2): $1-\phi_\pi(\psi_1+\psi_3)>0$. In this case, (i) becomes $\phi_R+\phi_\pi\psi_2>1-\phi_\pi(\psi_1+\psi_3)$ which can be rearranged as $\phi_\pi(\psi_1+\psi_2+\psi_3)>1-\phi_R$. Using the relationship $\psi_1+\psi_2+\psi_3=1$ (i.e., the ϕ's are relative weights), the foregoing can be rewritten as $\phi_\pi>1-\phi_R$.

(3): $1-\phi_\pi(\psi_1+\psi_3)=0$. Now, (i) becomes $\phi_R+\phi_\pi\psi_2>0$, which is necessarily satisfied.

Combining the three possibilities yields the two constraints in the corollary.

C.2 Bounds for Indeterminacy and Non-Existence

From theorem 5.1, there are two conditions that must be fulfilled: (ii) $|\phi_R+\phi_\pi\psi_2|<|1-\phi_\pi(\psi_1+\psi_3)|$ rules out determinacy, along with (iii a) $|\phi_\pi\psi_3|<|1-\phi_\pi\psi_1|$ for indeterminacy and (iii b) $|\phi_\pi\psi_3|>|1-\phi_\pi\psi_1|$ for non-existence.

C.2.1 Discriminating between Indeterminacy and Non-Existence

The conditions (iii a & b) become (iv) $\phi_\pi\psi_3\gtrless|1-\phi_\pi\psi_1|$ for positive coefficients and there are three possibilities depending on the sign of $1-\phi_\pi\psi_1$.

(1): $1-\phi_\pi\psi_1>0$. Here, (iv) becomes $\phi_\pi\psi_3\gtrless 1-\phi_\pi\psi_1$ which can be rearranged as $\phi_\pi(\psi_3+\psi_1)\gtrless 1$. Thus $\phi_\pi(\psi_3+\psi_1)>1$ is associated with non-existence and $\phi_\pi(\psi_3+\psi_1)<1$ with indeterminacy. As it was assumed that $1-\phi_\pi\psi_1>0$, the range of non-existence emanating from this possibility is bounded above, $1-\phi_\pi\psi_3<\phi_\pi\psi_1<1$ or $1<\phi_\pi(\psi_1+\psi_3)<1+\phi_\pi\psi_3$; the range of indeterminacy is bounded below by the assumption of positive coefficients, $0<\phi_\pi(\psi_3+\psi_1)<1$.

(2): $1-\phi_\pi\psi_1<0$. Now, (iv) becomes $\phi_\pi\psi_3\gtrless-1+\phi_\pi\psi_1$ which can be rearranged as $\phi_\pi\psi_1\gtrless 1+\phi_\pi\psi_3$. Thus $\phi_\pi\psi_1<1+\phi_\pi\psi_3$ is associated with non-existence and $\phi_\pi\psi_1>1+\phi_\pi\psi_3$ with indeterminacy. As it was assumed that $1-\phi_\pi\psi_1<0$, the range of non-existence emanating from this possibility is bounded below, $1<\phi_\pi\psi_1<1+\phi_\pi\psi_3$.

(3): $1-\phi_\pi\psi_1=0$. In this case, $\phi_\pi\psi_3>|1-\phi_\pi\psi_1|$ always holds for any positive value of ψ_3 and $\phi_\pi\psi_3<|1-\phi_\pi\psi_1|$ cannot hold. Thus, $\phi_\pi\psi_1=1$ and $\psi_3>0$ non-existence, but not indeterminacy, is possible.

Combining these three possibilities, the range $0<\phi_\pi(\psi_3+\psi_1)<1$ is associated with indeterminacy, $1-\phi_\pi\psi_3<\phi_\pi\psi_1<1+\phi_\pi\psi_3$ with non-existence, and $1+\phi_\pi\psi_3<\phi_\pi\psi_1$ with indeterminacy again. Note that if $\psi_3 = 0$, the non-existence region disappears and the indeterminacy regions become connected with indeterminacy now possible over the whole positive real line. This concludes the part of the analysis on distinguishing between non-existence or indeterminacy, but taking as given that one of the two will in fact prevail.

C.2.2 Joint Bounds for Indeterminacy and Non-Existence

Turning to the remaining condition of the two that must be fulfilled for there to be either indeterminacy or non-existence, namely whether one of two must prevail at all, condition (ii) must be met. This is the mirror image of the condition for determinacy above and, as above, there are three possibilities depending on the sign of $1-\phi_\pi(\psi_1+\psi_3)$.

(1): $1-\phi_\pi(\psi_1+\psi_3)<0$. Here, (ii) becomes $\phi_R+\phi_\pi\psi_2<-1+\phi_\pi(\psi_1+\psi_3)$ which can be rearranged as $\phi_\pi(\psi_1-\psi_2+\psi_3)>1+\phi_R$. Using the relationship $\psi_1+\psi_2+\psi_3=1$ (i.e., the ϕ's are relative weights), the foregoing can be rewritten as $\phi_\pi(1-2\psi_2)>1+\phi_R$.

(2): $1-\phi_\pi(\psi_1+\psi_3)>0$. In this case, (ii) becomes $\phi_R+\phi_\pi\psi_2<1-\phi_\pi(\psi_1+\psi_3)$ which can be rearranged as $\phi_\pi(\psi_1+\psi_2+\psi_3)<1-\phi_R$. Using the relationship $\psi_1+\psi_2+\psi_3=1$ (i.e., the ϕ's are relative weights), the foregoing can be rewritten as $\phi_\pi<1-\phi_R$.

(3): $1-\phi_\pi(\psi_1+\psi_3)=0$. Now, (ii) becomes $\phi_R+\phi_\pi\psi_2<0$, which is necessarily violated.

C.2.3 Combining: Individual Bounds for Indeterminacy and Non-Existence

Combining the three possibilities, there are two regions of indeterminacy or non-existence: $\phi_\pi<1-\phi_R$ (when $1-\phi_\pi(\psi_1+\psi_3)>0$) and $\phi_\pi(1-2\psi_2)>1+\phi_R$ (when $1-\phi_\pi(\psi_1+\psi_3)<0$). Combining this with the result above dividing the positive real line into three regions (indeterminacy, non-existence, and indeterminacy again), I conclude the following. In the region $1-\phi_\pi(\psi_1+\psi_3)>0$, only indeterminacy is possible. Hence the region $\phi_\pi<1-\phi_R$ is associated with indeterminacy. In the region $1-\phi_\pi(\psi_1+\psi_3)<0$, the portion bounded by $\phi_\pi\psi_1<1+\phi_\pi\psi_3$ is associated with non-existence and the remainder by indeterminacy. Hence, there is indeterminacy if $\phi_\pi(1-2\psi_2)>1+\phi_R$ and $\phi_\pi\psi_1>1+\phi_\pi\psi_3$ and the stable equilibrium is non-existent if $\phi_\pi(1-2\psi_2)>1+\phi_R$ and $\phi_\pi\psi_1<1+\phi_\pi\psi_3$.

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