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Pricing Chinese rain: a multi-site multi-period equilibrium pricing model for rainfall derivatives*

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Abstract

Many industries are exposed to weather risk which they can transfer on financial markets via weather derivatives. Equilibrium models based on partial market clearing became a useful tool for pricing such kind of financial instruments. In a multi-period equilibrium pricing model agents rebalance their portfolio of weather bonds and a risk free asset in each period such that they maximize the expected utility of their incomes constituted by possibly weather dependent profits and payoffs of portfolio positions. We extend the model to a multi-site version and apply it to pricing rainfall derivatives for Chinese provinces. By simulating realistic market conditions with two agent types, farmers with profits highly exposed to weather risk and a financial investor diversifying her financial portfolio, we obtain equilibrium prices for weather derivatives on cumulative monthly rainfall. Dynamic portfolio optimization under market clearing and utility indifference of these representative agents determines equilibrium quantity and price for rainfall derivatives.

Keywords:

rainfall derivatives; equilibrium pricing; space-time Markov model

JEL Classification: C22, C51, G13.

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1 Introduction

Weather derivatives (WDs) are a special kind of contingent claims which have a particular weather index as its underlying, e.g. average temperature, snowfall, rainfall, etc. Since their first introduction in 1996 in OTC market, the trading volume of WDs has an increasing trend. Since 1999 an electronic trading place for standardized WDs was set up on Chicago Mercantile Exchange, where WDs on many cities in North America, Europe, Australia and Asia are currently traded. There are WDs contingent on different temperature indices, hurricanes, frost, snowfall and rainfall. Nevertheless, OTC markets are important due to high dependence of weather indices, especially snowfall and rainfall, on the measurement location.

Investors can diversify their financial portfolio with WDs uncorrelated with financial markets. Especially important are WDs for industries exposed to weather risk, they can use these financial instruments for hedging their weather dependent profits. Energy, tourism and agriculture are the sectors, where WDs have a hedging potential due to weather dependent profits.

One of the most weather sensitive production sectors is agriculture, where weather is an indirect production factor and has a great impact on revenue, see Musshoff et al. (2010). It is known that cultivation of such important agricultural products as wheat, corn, rice, etc., is rather nonsensitive to nonextreme fluctuations of temperature (the opposite is the case for energy demand) but is nevertheless influenced by the amount of rainfall. We devote this paper therefore to pricing derivatives on rainfall important for hedging agricultural yield fluctuations.

Traditionally this kind of hedging in agriculture was taken over by crop insurance, Glauber et al. (2002). However, crop insurance has major disadvantages: adverse selection, moral hazard and high costs arising by the valuation of eventual losses, see Just et al. (1999) and Quiggin et al. (1993) on these issues. Due to their nature WDs help to overcome these problems and the underlying index is normally measured by the weather stations with low cost. Nevertheless, WDs can be used effectively for hedging of agricultural volumetric risks, see Musshoff et al. (2010) on the hedging potential of rainfall derivatives for German farmers.

In China, one of the world's largest agricultural producers, farmers are exposed to pronounced weather risks (The World Bank (2007), Turvey and Kong (2010)), this hedging potential of WDs can play an important role in stabilizing the income of farmers and stimulating therefore agricultural investment flow. According to The World Bank (2007) the existing agricultural insurance schemes are too expensive for Chinese agricultural producers. The existing gap between the willingness to pay for agricultural insurance and the willingness to accept it can be potentially overcome with trading of WDs as a more flexible alternative to agricultural crop insurance.

However, pricing of WDs is not straightforward, since their underlying is non tradeable and the market is incomplete. Classical arguments imposing existence of a unique pricing measure or perfect replication strategy fail in this case. Staum (2008) gives a survey of pricing methodologies in incomplete markets.

Most of the literature on WDs is devoted to pricing temperature derivatives. Many authors use a risk-neutral valuation of WDs, e.g. Alaton et al. (2002) derive their pricing model under assumption of a constant market price of risk, a premium for taking risk in the incomplete market, and temperature following an Ornstein-Uhlenbeck process. Others infer the market price of risk parameter using different assumptions of its form from the market, Härdle and López Cabrera (2011). The drawbacks of these models are the unavoidable assumption about the form of the market price of risk (e.g constant or zero market price of risk) and Gaussian type of the underlying process imputed by the Brownian motion. Their applicability to rainfall as a point process is limited though.
An alternative to finding a proper risk neutral pricing measure is to use utility

based pricing through utility indifference and equilibrium. Indifference pricing applied to temperature and based on the utility indifference of the market players is used by Yamada (2007), Brockett et al. (2006) and Davis (2001). Prices for rainfall derivatives using indifference arguments were derived by Carmona and Diko (2005), who use a Bartlett-Lewis Poisson Cluster Process in continuous time to obtain a closed form of indifference prices for buyer and seller. Leobacher and Ngare (2011) stay in discrete time and assume rainfall follows a Markovian process to derive the indifference prices. However, by indifference pricing alone one obtains a buyers and a sellers price, a unique price for a WD is unknown. One can overcome this major disadvantage of indifference pricing by imposing the equilibrium condition.

Equilibrium pricing models result in one equilibrium price and quantity which clear the market, the price is thereby a result of the interaction between supply and demand for WDs of the market participants. In continuous time equilibrium pricing under the assumption that the underlying weather index follows a mean-reverting Brownian motion were suggested by Horst and Müller (2007) and Chaumont et al. (2006). Consumption based discrete time pricing by equilibrium in financial and goods market can be found in Cao and Wei (2004). Finally, Lee and Oren (2009) proposed an equilibrium pricing model in a multi-commodity and a single planning period setting and later Lee and Oren (2010) developed a multi-period extension. In this model agents rebalance their portfolio of commodities, WDs with a payoff being a predefined function of a particular weather index, and risk free asset in each period such that they maximize the expected utility of their profits composed by their weather dependent incomes and payoffs of portfolio positions.

The models above do pricing of WDs at a single site. However, farmers are usually exposed to basis risk arising from the fact that the average yield is generally not perfectly correlated with rainfall at a particular site, due to the point process nature of the rainfall and possible geographical stretch of the fields, Musshoff et al. (2010). If the weather station, where measurements can be completed, is not exactly at the farmer's location, then she should be able to set up a better hedging portfolio by including WDs on two or more sites, which rainfall her yield is correlated to. On the other hand an investor can diversify her financial risks by holding a basket of WDs for several sites, Brockett et al. (2006).

Taking the considerations above into account we extend the single site model of Lee and Oren (2010) to a multi-site setting and apply the model to pricing rainfall bonds. Thereby we do not consider commodities in the portfolio, since with the exception of the large scale extreme events as flood and drought, agricultural commodity prices are uncorrelated with the rainfall amount in a specific area and can hedge therefore only the price risks.

We also compare the prices arising from the multi-site model to the single-site pricing of Lee and Oren (2010) using the concept of association (Esary et al. (1967), Joag-Dev and Proschan (1983)) of weather indices in different locations.

The structure of the paper is as follows. In section 2 we derive the equilibrium pricing model for rainfall derivatives. Section 3 applies the model to pricing rainfall derivatives for Chinese provinces. In section 4 we conclude.

2 The Pricing Model

Consider a simplified market model where weather bonds on the set of geographical sites $\mathcal{S}$ are priced at times $t = 0, 1, \ldots, T$. The involved agents maximize their expected utility (here we use exponential utility) with and without weather bonds and attain their demand/supply in terms of utility indifference. The market clearing condition determines then the equilibrium quantities and prices.

The set of agents J contains farmers with weather dependent profit and an investor who specializes in issuing weather bonds. Farmers take positions in weather bonds of the nearby sites to hedge weather caused fluctuations in their profits. The investor holds positions in weather bonds on all relevant sites. By weather bonds here and in the following we mean WDs with a pay-off proportional to the prespecified weather index, e.g. cumulative rainfall, the tick value is set to one income currency unit.

The price of the sth weather bond, $s \in \mathcal{S}$, at time $t = 0, \ldots, T$, $W_{t,s}$, is modeled as a positive random variable on a probability space $\{\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=0}^T, \mathrm{P}\}$, where $(\mathcal{F}_t)_{t=0}^T$ is a natural filtration of $\mathcal{F}$ and incorporates the information level of the agents at time t. $W_t = (W_{t,s_1}, W_{t,s_2}, \ldots, W_{t,s_n})^\top$, $s_i \in \mathcal{S}$, $i = 1, \ldots, n$, $n = |\mathcal{S}|$ is a vector of equilibrium prices and $|Z|$ denotes the number of elements of Z. The price process $(W_t)_{t=0,\ldots,T}$ is adapted to the filtration $(\mathcal{F}_t)_{t=0}^T$.

The agents are exposed to the self-financing convex constraints which arise from holding dynamic portfolios such that at each t they contain $\alpha_{j,t} = (\alpha_{j,t,s_1}, \ldots, \alpha_{j,t,s_n})^\top$, $s_i \in \mathcal{S}$, $i \leq n$ weather bonds and $\beta_{j,t}$ risk free assets B_t with constant return r. Every jth agent determines an optimal self-financing trading strategy $(\alpha_{j,t}, \beta_{j,t})_{t=0,1,\ldots,T}^\top$ predictable with respect to the filtration $(\mathcal{F}_t)_{t=0}^T$. To proceed with the model formulation we need to state the following assumptions:

A1 Agents preferences are expressed by an exponential utility function of the form $U_i(x) = -\exp(-a_i x)$, $i = j, m$, $j, m \in J$.
This utility exhibits constant risk aversion (Pratt (1964)) and compared to the isoelastic and logarithmic utility is more convenient for calculations.

A2 $(W_t)_{t=0,\ldots,T}$ is adapted to the filtration $(\mathcal{F}_t)_{t=0}^T$.

A3 $(\alpha_{j,t}, \beta_{j,t})_{t=0,1,\ldots,T}^\top$ predictable with respect to $(\mathcal{F}_t)_{t=0}^T$.

A4 Agents price by utility indifference in each time period.

Assumption **A4** implies that agents maximize their expected utilities in each period $t \leq T$ with and without WD at $s \in \mathcal{S}_x \subset \mathcal{S}$, $x \in J$ and demand or supply this WD such that both expected utilities are equal. Each agent $j \in J$ is faced with the following discrete time stochastic control system:

$$S_{j,t+1} = f_{j,t}\{S_{j,t}, (\alpha_{j,t,s})_{s\in\mathcal{S}_j}, Y_{j,t+1}\}, \quad t = 0, 1, \ldots, T \tag{1}$$

where $S_{j,t} = \{V_{j,t}, (W_{t,s})_{s\in\mathcal{S}_j}\}$ denotes the state of the system at time t for agent $j \in J$, it incorporates to time t available portfolio value $V_{j,t}$ and equilibrium WD prices $(W_{t,s})_{s\in\mathcal{S}_j}$. In the system (1) $(\alpha_{j,t,s})_{s\in\mathcal{S}_j}$ are the controls of the agent. $Y_{j,t+1}$ is the random influence factor which is in our case the next period prices for weather bonds contained in portfolio of the agent j $(W_{t+1,s})_{s\in\mathcal{S}_j}$ and $Y_{j,T}$ is the random value of the chosen weather index at T. Finally $f_{j,t} : \mathbb{R}^{|\mathcal{S}_j|+1} \times \mathbb{R}^{|\mathcal{S}_j|} \times \mathbb{R}^{|\mathcal{S}_j|} \mapsto \mathbb{R}^{|\mathcal{S}_j|+1}$ is some deterministic function which maps to the next state of the stochastic system.

Now we consider the two types of agents (farmers and an investor) separately. We keep the index $j \in J$ for farmers and we index investor relevant variables with $m \in J$.
The profit $\Pi_{j,T}$, $j \in J$ at T of farmer j is:

$$\begin{aligned}\Pi_{j,T} &= I_j(p) + \sum_{s\in S_j} \alpha_{j,T,s} W_{T,s} + \beta_{j,T} B_T \qquad (2)\\ &= I_j(p) + V_{j,T}\end{aligned}$$

with $I_j \overset{\text{def}}{=} I_j(p)$ an income, correlated with the weather indices $(W_{T,s})_{s\in\mathcal{S}_j}$. The production price p is assumed to be constant and $\sum_{s\in S_j} \alpha_{j,T,s} W_{T,s}$, $\beta_{j,T} B_T$ are then the payoffs of the WDs and the risk free asset at traded stations $s \in \mathcal{S}_j \subset \mathcal{S}$, set of sites the income of the farmer j is dependent on.

An example herefor would be a farmer, whose income I_1 is correlated to the cumulative rainfall at T $(R_{Ts})_{s\in\mathcal{S}_1}$ of the set of neighbor stations $\mathcal{S}_1 = \{a, b\} \subset \mathcal{S} = \{a, b, c\}$ with the correlation parameters $\rho_a, \rho_b \neq 0$. This farmer could then hedge rainfall caused fluctuations of her income by holding a portfolio of WDs with payoff at T $\alpha_{1,T,a} R_{T,a} + \alpha_{1,T,b} R_{T,b}$.

Based on (2) and corresponding to the j-farmer's utility maximization value function $J(V_{j,t} - \sum_{s\in\mathcal{S}_j} \alpha_{j,t+1,s} W_{t,s}, \sum_{s\in\mathcal{S}_j} \alpha_{j,t+1,s} W_{t,s})$ is defined as

$$J(V_{j,t} - \sum_{s\in\mathcal{S}_j} \alpha_{j,t+1,s} W_{t,s}, \sum_{s\in\mathcal{S}_j} \alpha_{j,t+1,s} W_{t,s}) = \max_{\{\alpha_{j,t+1,s}\}_{s\in\mathcal{S}_j}} \mathsf{E}_t \left\{ U_j\left(\Pi_{j,T}\right)\right\}$$
$$\text{s.t. } \sum_{s\in\mathcal{S}_j} \alpha_{j,t+1,s} W_{t,s} + \beta_{j,t+1} B_t - V_{j,t} = 0.$$

Consider now the investor $m,\ m \in J$ who issues WD at all sites in $\mathcal{S}$. The profit of the investor at T is:

$$\Pi_{m,T} = -\sum_{s\in\mathcal{S}} \alpha_{m,T,s} W_{T,s} + \beta_{m,T} B_T = V_{m,T}$$

with $\sum_{s\in\mathcal{S}} \alpha_{m,T,s} W_{T,s}$, $\beta_{m,T} B_T$ payoffs of the WDs and the risk free asset.

Then the value function corresponding to the investor's utility maximization $J(V_{m,t} + \sum_{s\in\mathcal{S}} \alpha_{m,t+1,s} W_{t,s}, -\sum_{s\in\mathcal{S}} \alpha_{m,t+1,s} W_{t,s})$ is given by

$$J(V_{m,t} + \sum_{s\in\mathcal{S}} \alpha_{m,t+1,s} W_{t,s}, -\sum_{s\in\mathcal{S}} \alpha_{m,t+1,s} W_{t,s}) = \max_{\{\alpha_{m,t+1,s}\}_{s\in\mathcal{S}}} \mathsf{E}_t \left\{ U_m\left(\Pi_{m,T}\right)\right\}$$
$$\text{s.t. } \sum_{s\in\mathcal{S}} \alpha_{m,t+1,s} W_{t,s} - \beta_{m,t+1} B_t + V_{m,t} = 0.$$

These value functions along with assumption **A4** allow us to derive the supply and demand functions for the WD at s in t time period. By applying the equilibrium condition we can determine for each farmer j and investor m the equilibrium quantities $(\alpha^*_{j,t,s})_{s\in\mathcal{S}_j}$ and $(\alpha^*_{m,t,s})_{s\in\mathcal{S}}$ and the resulting equilibrium prices $(W^*_{t,s})_{s\in\mathcal{S}}$ for each $t = 0, \ldots, T-1$, such that

$$\sum_{j\in J\setminus\{m\}} \alpha^*_{j,t+1,s} = \alpha^*_{m,t+1,s},$$
$$W^*_{t,s} = W_{t,s}(\alpha^*_{j,t+1,s}), \quad \text{for all } s \in \mathcal{S} \text{ and } t = 0, \ldots, T-1.$$

where $W_{t,s}(\alpha_{j,t+1,s})$ denotes the reverse demand or supply for the WD at site

s of the agent j in time t. Its specific form is given in Propositions 2.1 and 2.2.
By substituting the equilibrium prices and quantities into the agents value functions we can find their optimal value functions, which are

- for the farmers:

$$J(V_{j,t} - \sum_{s \in \mathcal{S}_j} \alpha^*_{j,t+1,s} W^*_{t,s}, \sum_{s \in \mathcal{S}_j} \alpha^*_{j,t+1,s} W^*_{t,s}) \stackrel{\text{def}}{=} J^*_{j,t}(V_{j,t}), \ j \in J \setminus \{m\},$$

- for the investor:

$$J(V_{m,t} + \sum_{s \in \mathcal{S}} \alpha^*_{m,t+1,s} W^*_{t,s}, - \sum_{s \in \mathcal{S}} \alpha^*_{m,t+1,s} W^*_{t,s}) \stackrel{\text{def}}{=} J^*_{m,t}(V_{m,t}).$$

PROPOSITION 2.1 *The reverse supply for the WD at site $s' \in \mathcal{S}_j$ and the value function of farmer j at time t, $W_{t,s'}(\alpha_{j,t+1,s'})$ and $J^*_{j,t}(V_{j,t})$, are defined:*

$$\begin{aligned}
W_{T-1,s'}(\alpha_{j,T,s'}) &= \frac{1}{a_j R \alpha_{j,T,s'}} \\
&\quad \log \frac{E_{T-1}\left[\exp\left\{-a_j(I_j + \sum_{s \in \mathcal{S}_j \setminus \{s'\}} \alpha_{j,T,s} W_{T,s})\right\}\right]}{E_{T-1}\left[\exp\left\{-a_j(I_j + \sum_{s \in \mathcal{S}_j} \alpha_{j,T,s} W_{T,s})\right\}\right]} \quad (3)\\
J^*_{j,T-1}(V_{j,T-1}) &= -\exp\{-a_j V_{j,T-1} R\}\,\Theta_{j,T-1} \\
\Theta_{j,T-1} &= \exp\{a_j \sum_{s \in \mathcal{S}_j} \alpha^*_{j,T,s} W^*_{T-1,s} R\} \\
&\quad E_{T-1}\big[\exp\{-a_j(I_j + \sum_{s \in \mathcal{S}_j} \alpha^*_{j,T,s} W_{T,s})\}\big],
\end{aligned}$$

$$
\begin{aligned}
W_{t,s'}(\alpha_{j,t+1,s'}) &= \frac{1}{a_j\alpha_{j,t+1,s'}R^{T-t}} \\
&\quad \log\frac{E_t\{\exp(-a_j\sum_{s\in\mathcal{S}_j\setminus\{s'\}}\alpha_{j,t+1,s}W_{t+1,s}R^{T-(t+1)})\Theta_{j,t+1}\}}{E_t\{\exp(-a_j\sum_{s\in\mathcal{S}_j}\alpha_{j,t+1,s}W_{t+1,s}R^{T-(t+1)})\Theta_{j,t+1}\}}, \\
J^*_{j,t}(V_{j,t}) &= -\exp(-a_jV_{j,t}R^{T-t})\Theta_{j,t} \\
\Theta_{j,t} &= \exp(a_jR^{T-t}\sum_{s\in\mathcal{S}_j}\alpha^*_{j,t+1,s}W^*_{t,s}) \\
&\quad E_t\{\exp(-a_jR^{T-(t+1)}\sum_{s\in\mathcal{S}_j}\alpha^*_{jt+1s}W_{t+1s})\Theta_{jt+1}\}, \\
\textit{with } R &= 1+r,\ 0\le t<T-1.
\end{aligned}
$$

The proof of Proposition 2.1 is given in the appendix.

PROPOSITION 2.2 *The reverse demand for the WD at site $s'\in\mathcal{S}$ and the value function of the investor m at time t, $W_{t,s'}(\alpha_{m,t+1,s'})$ and $J^*_{m,t}(V_{m,t})$, are:*

$$
\begin{aligned}
W_{t,s'}(\alpha_{m,t+1,s'}) &= \frac{1}{a_m\alpha_{m,t+1,s'}R^{T-t}} \\
&\quad \log\frac{E_t\left[\exp\left\{a_m(\sum_{s\in\mathcal{S}}\alpha_{m,t+1,s}W_{t+1,s}R^{T-(t+1)})\right\}\right]}{E_t\left[\exp\left\{a_m\sum_{s\in\mathcal{S}\setminus\{s'\}}\alpha_{m,t+1,s}W_{t+1,s}R^{T-(t+1)}\right\}\right]}, \quad (4) \\
J^*(V_{m,t}) &= \exp(a_mV_{m,t}R^{T-t}) \\
\textit{with } R &= 1+r,\ 0\le t\le T.
\end{aligned}
$$

The proof of Proposition 2.2 is given in the appendix.

With the proposed setup we are in the position to analyze the properties of farmers' supply and investor's demand under different underlying spatial dependence types in a one-period multi-site pricing model. The following propositions specify the shifts of the demand/supply curves assuming positive or negative dependence between sites.

To specify the directional dependence of weather indices across locations we adopt a concept of positive/negative association of random variables introduced in Esary et al. (1967) and Joag-Dev and Proschan (1983). According to their definitions random variables $\mathbf{Z} = (Z_1, \dots, Z_l)$ are positively (negatively) associated if $\mathrm{Cov}\{f(\mathbf{Z}), g(\mathbf{Z})\} \geq 0$ ($\mathrm{Cov}\{f(\mathbf{Z}), g(\mathbf{Z})\} \leq 0$) for all nondecreasing functions f, g for which the corresponding expectations exist, see Definition 1.1 in Esary et al. (1967) and Definition 2.1 in Joag-Dev and Proschan (1983). This concept of association is a very broad one and includes many types of the underlying dependence structures, e.g. Pearson correlation as a special case.

In the following $W_{T-1,s'}(\alpha_{m,T,s'})$ and $W_{T-1,s'}(\alpha_{j,T,s'})$ remains the demand of the investor and the supply the farmer j for the weather bond in site s' respectively, defined in (3) and (4) in a multi-site setting and

$$W^S_{T-1,s'}(\alpha_{m,T,s'}) = (a_m R \alpha_{m,T,s'})^{-1} \log \mathsf{E}_{T-1} \left\{ \exp\left(a_m \alpha_{m,T,s'} W_{T,s'}\right) \right\}$$

the demand of the investor and

$$W^S_{T-1,s'}(\alpha_{j,T,s'}) = (a_j R \alpha_{j,T,s'})^{-1} \log \frac{\mathsf{E}_{T-1} \left\{ \exp(-a_j I_j) \right\}}{\mathsf{E}_{T-1} \left[\exp\left\{ -a_j (I_j + \alpha_{j,T,s'} W_{T,s'}) \right\} \right]}$$

the supply of the farmer j in the same site in a single-site setting (see Lee and Oren (2010) for the derivation). We assume a one-period multi-site setting from now forth.

PROPOSITION 2.3 *If $(W_{T,s})_{s \in \mathcal{S}}$ are positive (negative) associated random variables, then for $(\alpha_{m,T,s})_{s \in \mathcal{S}} \geq 0$ and $a_m > 0$ the investors demand for the WD in s' shifts upwards (downwards) resulting in higher (lower) equilibrium prices in comparison to the single-site case, keeping other conditions equal.*

Proof. From the definition of the positive (negative) association we have:

$$\text{Cov}[f(W_{T,s'}), g\{(W_{T,s})_{s\in\mathcal{S}\setminus\{s'\}}\}] \underset{(\leq)}{\geq} 0, \text{ for all nondecreasing } f, g$$

$$\mathsf{E}[f(W_{T,s'}), g\{(W_{T,s})_{s\in\mathcal{S}\setminus\{s'\}}\}] \underset{(\leq)}{\geq} \mathsf{E}\{f(W_{T,s'})\}\mathsf{E}[g\{(W_{T,s})_{s\in\mathcal{S}\setminus\{s'\}}\}] \quad (5)$$

Note, that for $T = 1$ $\mathcal{F}_0$ is trivial and therefore:

$$\mathsf{E}_{T-1}[f(W_{T,s'}), g\{(W_{T,s})_{s\in\mathcal{S}\setminus\{s'\}}\}] = \mathsf{E}[f(W_{T,s'}), g\{(W_{T,s})_{s\in\mathcal{S}\setminus\{s'\}}\}].$$

Using (5) we have:

$$\begin{aligned}
W_{T-1,s'}(\alpha_{m,T,s'}) = & \frac{1}{a_m R \alpha_{m,T,s'}} \\
& \log \frac{\mathsf{E}_{T-1}\left\{\exp\left(a_m \alpha_{m,T,s'} W_{T,s'}\right) \exp\left(a_m \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T,s} W_{T,s}\right)\right\}}{\mathsf{E}_{T-1}\left\{\exp\left(a_m \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T,s} W_{T,s}\right)\right\}} \\
\underset{(\leq)}{\geq} & \frac{1}{a_m R \alpha_{m,T,s'}} \\
& \log \frac{\mathsf{E}_{T-1}\{\exp\left(a_m \alpha_{m,T,s'} W_{T,s'}\right)\}\mathsf{E}_{T-1}\{\exp(a_m \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T,s} W_{T,s})\}}{\mathsf{E}_{T-1}\{\exp(a_m \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T,s} W_{T,s})\}} \\
= & \frac{1}{a_m R \alpha_{m,T,s'}} \log \mathsf{E}_{T-1}\left\{\exp\left(a_m \alpha_{m,T,s'} W_{T,s'}\right)\right\} \\
= & W^S_{T-1,s'}(\alpha_{m,T,s'}).
\end{aligned}$$

and the assertion follows. ■

The result of Proposition 2.3 shows, that positive (negative) dependencies in underlying weather risks force the investor to shrink (to expand) his demand due to higher (lower) risks she bears by buying weather bonds with dependent payoffs. The diversification effect as known from portfolio theory is not relevant here, since the amount of money invested is not restricted to a limited capital.

We note that in the case of independent $(W_{T,s})_{s\in\mathcal{S}}$ they possess both negative and positive association property, see Esary et al. (1967) Theorem 2.1 and Joag-Dev and Proschan (1983) Property P_5. Therefore in this case an investor would price as in a single site case.

PROPOSITION 2.4 *If $W_{T,s'}$ is independent of $(W_{T,s})_{s\in\mathcal{S}_j\setminus\{s'\}}$ and I_j and $(W_{T,s})_{s\in\mathcal{S}_j}$ are positive associated, then for $(\alpha_{j,T,s})_{s\in\mathcal{S}_j} \leq 0$and $a_j > 0$ the farmers supply for the WD in s' shifts downwards resulting in lower equilibrium prices in comparison to the single-site case, keeping other conditions equal. A similar result can be obtained for the opposite association.*

Proof.

$$W_{T-1,s'}(\alpha_{j,T,s'}) = \frac{1}{a_j R\alpha_{j,T,s'}} \log \frac{\mathsf{E}_{T-1}\left\{\exp(-a_j I_j)\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T,s})\right\}}{\mathsf{E}_{T-1}\left[\exp\{-a_j(I_j + \alpha_{j,T,s'} W_{T,s'})\}\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T,s})\right]}$$

By the definition of association:

$$\begin{aligned}&\mathsf{E}_{T-1}\{-\exp(-a_j I_j)\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T,s})\}\\ &\qquad\geq \mathsf{E}_{T-1}\{-\exp(-a_j I_j)\}\mathsf{E}_{T-1}\{\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T,s})\}\end{aligned}$$

hence

$$\begin{aligned}&\mathsf{E}_{T-1}\{\exp(-a_j I_j)\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T,s})\}\\ &\qquad\leq \mathsf{E}_{T-1}\{\exp(-a_j I_j)\}\mathsf{E}_{T-1}\{\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T,s})\} \qquad (6)\end{aligned}$$

For the denominator we have:

$$
\begin{aligned}
&\mathsf{E}_{T-1}[\exp\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\}\exp(-a_j\sum_{s\in\mathcal{S}_j\setminus\{s'\}}\alpha_{j,T,s}W_{T,s})] \\
&= \mathsf{E}_{T-1}(\mathsf{E}[\exp\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\} \\
&\qquad \exp(-a_j\sum_{s\in\mathcal{S}_j\setminus\{s'\}}\alpha_{j,T,s}W_{T,s})|I_j,(W_{T,s})_{s\in\mathcal{S}_j\setminus\{s'\}}]) \qquad (7)\\
&= \mathsf{E}_{T-1}(\mathsf{E}[\exp\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\}|I_j] \\
&\qquad \exp(-a_j\sum_{s\in\mathcal{S}_j\setminus\{s'\}}\alpha_{j,T,s}W_{T,s})) \qquad (8)\\
&\geq \mathsf{E}_{T-1}(\mathsf{E}[\exp\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\}|I_j]) \\
&\qquad \mathsf{E}_{T-1}(\exp(-a_j\sum_{s\in\mathcal{S}_j\setminus\{s'\}}\alpha_{j,T,s}W_{T,s})) \qquad (9)\\
&= \mathsf{E}_{T-1}\left[\exp\left\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\right\}\right]\mathsf{E}_{T-1}\{\exp(-a_j\sum_{s\in\mathcal{S}_j\setminus\{s'\}}\alpha_{j,T,s}W_{T,s})\}. \\
&\qquad (10)
\end{aligned}
$$

Above equalities (7) and (8) result using properties of conditional expectations and the independence of $W_{T,s'}$ from $(W_{T,s})_{s\in\mathcal{S}_j\setminus\{s'\}}$. The inequality in (9) is obtained using the assumptions of the theorem and the increasing property of $\mathsf{E}[\exp\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\}|I_j]$ in I_j. Using (6) and (10) we get:

$$
\begin{aligned}
W_{T-1,s'}(\alpha_{j,T,s'}) &\leq \frac{1}{a_jR\alpha_{j,T,s'}}\log\frac{\mathsf{E}_{T-1}\left\{\exp(-a_jI_j)\right\}}{\mathsf{E}_{T-1}\left[\exp\left\{-a_j(I_j+\alpha_{j,T,s'}W_{T,s'})\right\}\right]} \\
&= W^S_{T-1,s'}(\alpha_{j,T,s'}),
\end{aligned}
$$

which completes the proof. ■

Proposition 2.4 yields that farmer is willing to supply the same quantity of weather bond for a lower price in comparison to the single-site case, whenever the dependence between her income and weather bond payoffs of other sites is positive, which also implies the assumption of $(\alpha_{j,T,s})_{s\in\mathcal{S}_j}\leq 0$ in the case of independence between payoff of site s' and other sites from $\mathcal{S}_j$, so that she bears less risk.

3 Application to Pricing Chinese Rain

In practice the pricing according to the model includes further aspects. Having the pricing model at hand we still have to concretize some steps, taken abstract in the previous section, as the statistical modeling of the relevant weather variable, in our case rainfall, and the quantification of the relationship between the rainfall and the farmers income.

station	number	latitude	longitude	start date	end date
Changde	57662	29.05	111.68	19510101	20091130
Enshi	57447	30.28	109.47	19510801	20091130
Yichang	57461	30.70	111.30	19520701	20091130

Table 1: Description of the rainfall data and stations.

We illustrate the later on the application to pricing Chinese rain. We base our calculations on the rainfall data of three weather stations in a Chinese provinces Hunan and Hubei, summarized in Table 1 and Figure 1. The data was acquired via Research Data Center of CRC 649 (Collaborative Research Center 649: Economic risk).

We calculate the price of the rainfall bonds for April traded on these three stations. The cumulative rainfall of April will be taken for the illustration purposes because of it's relative importance in the cultivating rice in China.

3.1 Rainfall Dynamics

Because of the multi-site nature of the pricing model, the model for rainfall should also account for its spatial characteristics. Following the traditional way of modeling daily rainfall, see e.g. Richardson (1981), we first model the rainfall occurrences at site s' in day t $X_{s',t}$ as a Markov chain with two states.

$$X_{s',t} = \begin{cases} 1 \text{ (wet,} \geq 0.1\text{mm),} \\ 0 \text{ (dry,} < 0.1\text{mm),} \end{cases}$$

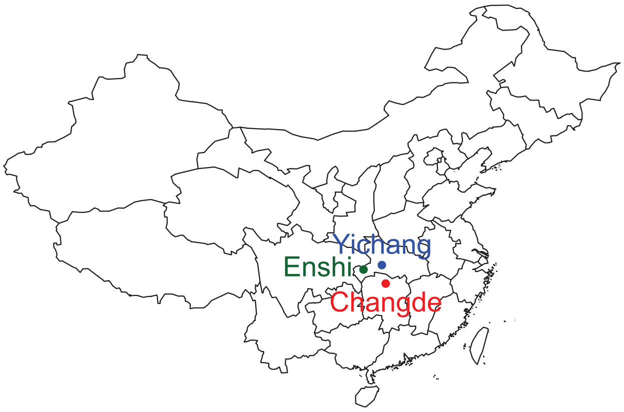

Figure 1: Three stations in China considered in the example.

The threshold of 0.1 mm for the definition of "wet" and "dry" was taken according to the data description. The multi-site feature is added through the contemporal dependencies on the state in neighbor locations, Kim et al. (2008). The stations, where the rainfall occurrences depend only on it's own past, follow a single-site Markov model of second order with transition probabilities:

$$P(X_{s',t}|X_{s',t-1}, X_{s',t-2}),$$

Stations, where the occurrences depend on the state of the neighbors follow a multi-site Markov model of first time and space order with transition probabilities:

$$P(X_{s,t}|X_{s,t-1}, X_{s',t}),$$

here $X_{s,t}$ are rainfall occurrences in station s at time t and s subordinate to s'.

To determine which station follows which model we have to find an optimal path from the "parent" station to other stations, based on some criteria. As in Kim et al. (2008) we find the optimal permutation of stations maximizing the cumulative distance of spatial transition probabilities:

$$\pi^*(\mathcal{S}) = \underset{\pi(\mathcal{S})\in\Pi(\mathcal{S})}{\arg\max} \Big\{ \sum_{s\in\pi(\mathcal{S})} D(s) \Big\},$$

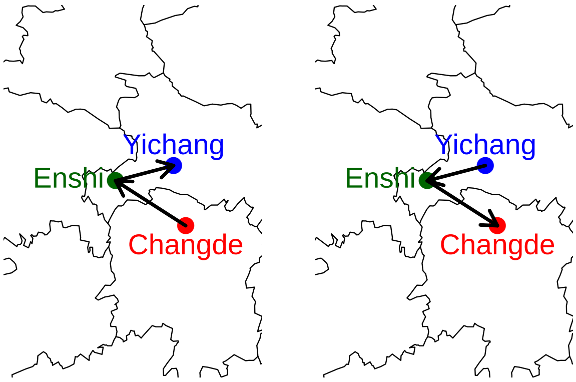

Figure 2: Optimal paths between stations in April: 1.04–28.04 showed in the left plot and 29.04–7.05 in the right plot.

where $\pi(\mathcal{S})$ permutation and $\Pi(\mathcal{S})$ set of all permutations on the set of stations $\mathcal{S}$, $D(s) = |0.5 - P(X_{s,t} = 1|X_{s',t} = 1)| + |0.5 - P(X_{s,t} = 1|X_{s',t} = 0)|$ distance of spatial transition probabilities, s subordinate to s', $s, s' \in \mathcal{S}$.

For the data of the three stations in China, we first test the plausibility of the multi-site modeling using a χ^2-independence test, Hiscott (1981). The test rejects pairwise independence of the stations on $< 1\%$ significance level with p-values near zero. The resulting optimal paths on example of April are illustrated in Figure 2. In the time span of 1.04-28.04 Changde appears to be the parent station, meaning the rainfall occurrence process of Changde follows a single site Markov model of the second order, the rainfall occurrences of Enshi and Yichang depend on those of Changde and Enshi respectively and follow a Markov model of the first order in space and time. For the time span of 29.04-7.05 dependence direction is the other way around.

The positive rainfall amount is given only on wet days. The distribution of the rainfall amount conditioned on a rainy day $R_{s,t}|X_{s,t} = 1$ is assumed to be a mixture of two exponential distributions with a time dependent mixing parameter α_{st} and time changing means β_{1st}, β_{2st}. This is a standard dis-

station	empirical		simulated	
	mean	sd	mean	sd
Changde	168.6	75.2	172.7	65.2
Enshi	72.1	44.2	73.5	34.4
Yichang	93.5	69.6	89.8	50.1

Table 2: sample means and standard deviations of cumulative rainfall of April in each station in comparison to the means and standard deviations resulting from 10'000 simulation steps.

tribution of the rainfall amount in the literature, see Woolhiser and Roldán (1982), Chapman (1997) and Wilks (1998). The parameters were estimated from the rainfall data using Maximum Likelihood and a rolling window of 29 days centered on the day of interest. The average mixing parameter over time and space $\bar{\alpha}$ was 0.35, average $\bar{\beta}_1, \bar{\beta}_2$ were $(0.78, 11.29)^\top$.

Table 2 presents the sample means and standard deviations of cumulative rainfall in each station, as well as means and standard deviations resulting from 10'000 simulation steps using the underlying model described above.

3.2 Quantification of Income-Rainfall relationship and Pricing Example

The income-rainfall relationship for pricing should be estimated on the individual farmer level due to its idiosyncratic character. A simple method to quantify this relationship is to calculate the realized correlation of the income to the cumulative rainfall of the nearby stations and to use the empirical marginal income distribution. Because of the lack of the data, we suggest to specify this on a farm level. For our example we use hypothetical correlation parameters and adopt a normal distribution as the marginal income distribution.

Since homogeneous agents would act identically, we can simplify the general setting of Section 2 to the market of three representative agents: two farmers and an investor. Investor trades rainfall bonds on three stations in China. Farmers are situated such that their income is correlated to the two of the three traded stations. The income of farmer 1 (I_1) is correlated with rainfall of the corresponding station with ρ_{11}, ρ_{12}. The correlations of the income of farmer 2 (I_2) are ρ_{21} and ρ_{23}, see Table 3 for a more detailed specification. Other correlations are zero. In this setting farmers are inclined to buy rainfall bonds on the two stations with nonzero correlation to their income.

	Changde	Enshi	Yichang
I_1	$\rho_{11} = 0.3$	$\rho_{12} = 0.6$	$\rho_{13} = 0$
I_2	$\rho_{21} = 0.6$	$\rho_{22} = 0$	$\rho_{23} = 0.3$

Table 3: ρ-values used for simulation.

We compute the expectations in (3) and (4) using the MCMC algorithm with 10000 simulation steps to generate the distribution of the cumulative rainfall from the Markov model described above. As the initial state we assume no rain in two previous days in all stations. A Gaussian Copula is used to generate the income with the given correlations to the rainfall outcomes, the marginal distribution of the income is taken to be a Gaussian one with hypothetical mean 500 and variance 100.

The resulting prices for the cumulative rainfall bond of April are presented in Figure 3. Thereby the risk aversion parameters $a_1 = a_2 = a_m$ were set to 0.01, the correlations to 0.6, 0.3 as in Table 3, product price p to 1 and the risk free interest rate r to 5.3% p.a. The bottom plot in Figure 3 shows the prices of a one-period model (dashed) compared to a two-period model (solid lines in the left part). After observing the rainfall of the first 15 days in April (upper plot left part) agents can update their expectations and rebalance their portfolios respectively. As the result of this interaction the equilibrium prices of the second period (bottom plot right part) establish.

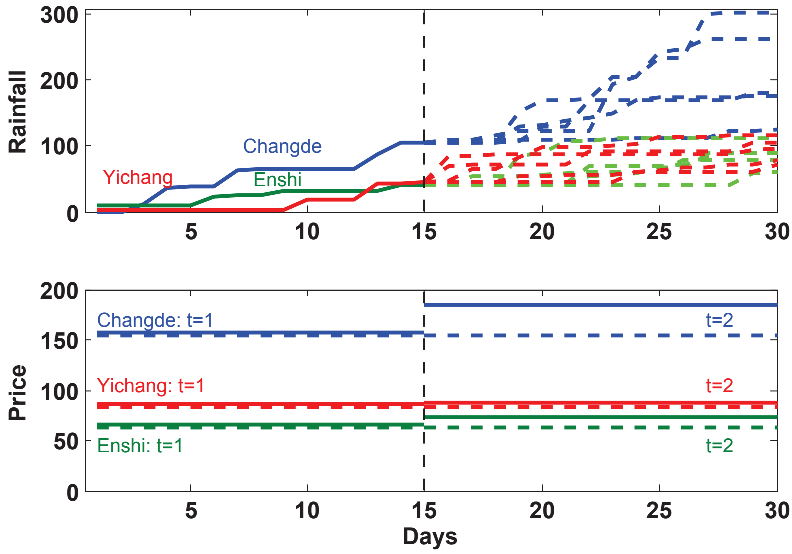

Figure 3: Upper plot: hypothetical paths of the rainfall observed by the agents in the three stations till the rebalancing time (solid lines) and possible cumulative rainfall paths after the rebalancing (dashed lines). Lower plot: simulated equilibrium prices for bonds on the cumulative rainfall in a one period model (dashed lines) and in the two-period model (solid lines) to the left – for the first planning period, to the right – for the second given the rainfall up to the rebalancing.

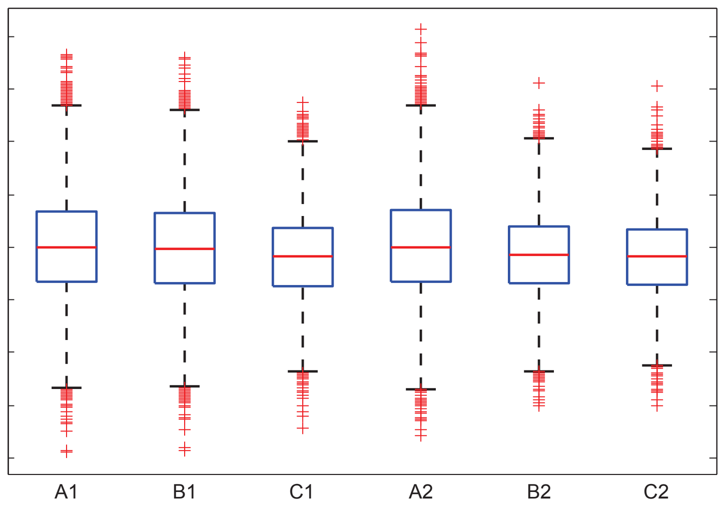

Figure 4: Boxplots of the farmers income distributions with and without WDs. A1, A2 indicate the income distributions of the two farmers before trading WDs, B1, B2 – with trading WDs only at the station Changde and C1,C2 – with trading WDs at several stations.

Figure 4 shows a more desirable distribution of incomes of farmers when they use WDs in comparison to no WDs in portfolio. Although the median of the income with WDs is insignificantly lower than without, the spread and interquartile range are substantially lower with WDs. A multi-site WD trading allows thus to reduce the basis risk of the farmers, such that hedging of their income becomes more efficient as they can trade several sites instead of a single one. This can be seen from Figure 4 by comparing the boxplots of the income of farmer 1: C1 (WD on two stations) to A1 (no WDs) and B1 (WD on a single station) and of farmer 2: C2 to A2 and B2 respectively. The presented income distributions in cases B1 and B2 are calculated for the situation where farmers trade WDs on Changde only, in cases C1 and C2 farmer 1 trades WDs on Changde and Enshi and farmer 2 – on Changde and Yichang. In this case farmers are better off in terms of their income distribution.

4 Conclusion

We proposed a multi-site multi-period model for pricing WDs. Our approach is based on utility indifference of the representative agents and on the partial equilibrium on the market of WDs. We derived supply and demand for WDs in each period and site in this setting and compared the results to a single-site model of Lee and Oren (2010) using the concept of association of Esary et al. (1967).

We applied the proposed model to pricing rainfall bonds on three sites in Chinese provinces. Thereby we used historical rainfall data of these provinces and simulated a two-period three-agent economy. We obtained equilibrium prices and quantities for rainfall bonds of the provinces. The results of the simulation could indeed show an improvement in farmers income distributions in comparison to the case of no WDs and to the single-site weather trading case.

5 Appendix

To prove Proposition 2.1 we use the following

REMARK 5.1

$$V_{j,t} = R(V_{j,t-1} - \sum_{s\in\mathcal{S}_j} \alpha_{,,t,s} W_{t-1,s}) + \sum_{s\in\mathcal{S}_j} \alpha_{j,t,s} W_{t,s}, \tag{11}$$

since

$$\begin{aligned} V_{j,T-k} &= R(V_{j,T-k-1} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k-1,s}) + \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k,s} \\ &= R(\sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k-1,s} + \beta_{j,T-k} \frac{B_{T-k}}{R} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k-1,s}) \\ &+ \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k,s} = \beta_{j,T-k} B_{T-k} + \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k,s} \\ &= V_{j,T-k},\ \ k \leq T-1, j \in J \setminus \{m\}. \end{aligned}$$

Proof. The maximized utility of farmer j in $(T-1)$ with WD at $s' \in \mathcal{S}_j$ is:

$$\begin{aligned} J(V_{j,T-1} &- \sum_{s\in\mathcal{S}_j} \alpha_{j,T,s} W_{T-1,s}, \sum_{s\in\mathcal{S}_j} \alpha_{j,T,s} W_{T-1,s}) \\ &= \mathsf{E}_{T-1}[U_j\{I_j + \sum_{s\in\mathcal{S}_j} \alpha_{j,T,s} W_{T,s} + (V_{j,T-1} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T,s} W_{T-1,s})R\}] \end{aligned} \tag{12}$$

The maximized utility of farmer j in $(T-1)$ without WD at $s' \in \mathcal{S}_j$:

$$\begin{aligned} J(V_{j,T-1} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T-1,s}, \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T-1,s}) \\ = \mathsf{E}_{T-1}[U_j\{I_j + (V_{j,T-1} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s} W_{T-1,s})R\}] \end{aligned} \tag{13}$$

Using **A1** and **A4** we equalize (12) and (13) and get:

$$\mathsf{E}_{T-1}$$
$$\big[-\exp\big\{-a_j(I_j + \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s}W_{T,s}) - a_j(V_{j,T-1} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s}W_{T-1,s})R\big\}\big]$$
$$= \mathsf{E}_{T-1}\big[-\exp\big\{-a_j(I_j + \sum_{s\in\mathcal{S}} \alpha_{j,T,s}W_{T,s}) - a_j(V_{j,T-1} - \sum_{s\in\mathcal{S}} \alpha_{j,T,s}W_{T-1,s})R\big\}\big],$$

With **A2**,**A3**:

$$\exp\{a_j\alpha_{j,T,s'}W_{T-1,s'})R\} = \frac{\mathsf{E}_{T-1}\left[\exp\left\{-a_j(I_j + \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s}W_{T,s})\right\}\right]}{\mathsf{E}_{T-1}\left[\exp\left\{-a_j(I_j + \sum_{s\in\mathcal{S}_j} \alpha_{j,T,s}W_{T,s})\right\}\right]},$$

and thus

$$W_{T-1,s'} = \frac{1}{a_jR\alpha_{j,T,s'}}\log\frac{\mathsf{E}_{T-1}\left[\exp\left\{-a_j(I_j + \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T,s}W_{T,s})\right\}\right]}{\mathsf{E}_{T-1}\left[\exp\left\{-a_j(I_j + \sum_{s\in\mathcal{S}_j} \alpha_{j,T,s}W_{T,s})\right\}\right]} \tag{14}$$

for $s \in \mathcal{S}_j$.
The value function of farmer j at $T-1$ is then:

$$J^*_{j,T-1}(V_{j,T-1}) = -\exp\{-a_jV_{j,T-1}R\}\,\Theta_{j,T-1,s} \tag{15}$$

with

$$\Theta_{j,T-1} = \exp\{a_j\sum_{s\in\mathcal{S}_j}\alpha^*_{j,T,s}W^*_{T-1,s}R\}\mathsf{E}_{T-1}\big[\exp\{-a_j(I_j + \sum_{s\in\mathcal{S}_j}\alpha^*_{j,T,s}W_{T,s})\}\big].$$

Using Bellman's Principal of optimality, Bellman (1952) in $T-2$:

$$
\begin{aligned}
&J(V_{j,T-2} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T-1,s} W_{T-2,s}, \sum_{s\in\mathcal{S}_j} \alpha_{j,T-1,s} W_{T-2,s}) = \mathsf{E}_{T-2}\big\{J^*_{j,T-1}(V_{j,T-1}))\big\} \\
&\quad = \mathsf{E}_{T-2}\left[-\exp\left\{-a_j V_{j,T-1} R\right\}\Theta_{j,T-1,s}\right], \\
&\quad \overset{5.1}{=} \mathsf{E}_{T-2}[-\exp\{-a_j(V_{j,T-2} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T-1,s} W_{T-2,s})R^2 \\
&\quad - a_j \sum_{s\in\mathcal{S}_j} \alpha_{j,T-1,s} W_{T-1,s} R\}\Theta_{j,T-1}],
\end{aligned} \tag{16}
$$

and

$$
\begin{aligned}
&J(V_{j,T-2} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-1,s} W_{T-2,s}, \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-1,s} W_{T-2,s}) \\
&\quad = \mathsf{E}_{T-2}[-\exp\{-a_j(V_{j,T-2} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-1,s} W_{T-2,s})R^2 \\
&\quad - a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-1,s} W_{T-1,s}\}\Theta_{j,T-1}].
\end{aligned} \tag{17}
$$

From (16) and (17) using **A2**–**A4**:

$$
\begin{aligned}
&\exp\left\{a_j \alpha_{j,T-1,s'} W_{T-2,s'} R^2\right\} \\
&= \frac{\mathsf{E}_{T-2}\{\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-1,s} W_{T-1,s} R)\Theta_{j,T-1}\}}{\mathsf{E}_{T-2}\left[\exp\left\{-a_j \sum_{s\in\mathcal{S}_j} \alpha_{j,T-1,s} W_{T-1,s} R\right\}\Theta_{j,T-1}\right]},
\end{aligned}
$$

$$
W_{T-2,s'} = \frac{1}{a_j R^2 \alpha_{j,T-1,s'}} \log \frac{\mathsf{E}_{T-2}\{\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-1,s} W_{T-1,s} R)\Theta_{j,T-1}\}}{\mathsf{E}_{T-2}\left[\exp\left\{-a_j \sum_{s\in\mathcal{S}_j} \alpha_{j,T-1,s} W_{T-1,s} R\right\}\Theta_{j,T-1}\right]}.
$$

The value function of farmer j at $T-2$ is then:

$$
\begin{aligned}
J^*_{j,T-2}(V_{j,T-2}) &= -\exp\{a_j V_{j,T-2} R^2\} \exp\{a_j \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-1,s} W^*_{T-2,s} R^2\} \\
&\quad \mathsf{E}_{T-2}[\exp\{-a_j \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-1,s} W_{T-1,s} R\} \Theta_{j,T-1}] \\
&= -\exp(a_j V_{j,T-2} R^2) \Theta_{j,T-2},
\end{aligned}
$$

with

$$
\begin{aligned}
&\Theta_{j,T-2} = \\
&\exp\{a_j \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-1,s} W^*_{T-2,s} R^2\} \mathsf{E}_{T-2}\big[\exp\{-a_j \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-1,s} W_{T-1,s} R\} \Theta_{j,T-1}\big].
\end{aligned}
$$

For $T-k$, $k \leq T$ we define recursively:

$$
\begin{aligned}
W_{T-k,s'} &= \frac{1}{a_j \alpha_{j,T-k+1,s'} R^k} \qquad (18) \\
&\qquad \log \frac{\mathsf{E}_{T-k}\{\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-k+1,s} W_{T-k+1,s} R^{k-1}) \Theta_{j,T-k+1}\}}{\mathsf{E}_{T-k}\{\exp(-a_j \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k+1,s} W_{T-k+1,s} R^{k-1}) \Theta_{j,T-k+1}\}} \\
J^*_{j,T-k}(V_{j,T-k}) &= -\exp(-a_j V_{j,T-k} R^k) \Theta_{j,T-k} \qquad (19) \\
\Theta_{j,T-(k+1)} &= \exp(a_j R^{k+1} \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-k,s} W^*_{T-(k+1),s}) \\
&\qquad \mathsf{E}_{T-(k+1)}\{\exp(-a_j R^k \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-k,s} W_{T-k,s}) \Theta_{j,T-k}\}. \qquad (20)
\end{aligned}
$$

We prove it by backward induction over $T-k$:

- for $T-1$ above.
- with induction assumption (19):

$$
\begin{aligned}
&J(V_{j,T-(k+1)} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-(k+1),s}, \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-(k+1),s}) \\
&= \mathsf{E}_{T-(k+1)}\{J^*_{j,T-k}(V_{j,T-k})\} \\
&= \mathsf{E}_{T-(k+1)}[-\exp\{-a_j V_{j,T-k} R^k\}\Theta_{j,T-k}] \\
&\overset{5.1}{=} \mathsf{E}_{T-(k+1)}\big(-\exp[-a_j\{(V_{j,T-(k+1)} - \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-(k+1),s}) R^{k+1} \\
&+ \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k,s} R^k\}]\Theta_{j,T-k}\big).
\end{aligned}
$$

$$
\begin{aligned}
&J(V_{j,T-(k+1)} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-k,s} W_{T-(k+1),s}, \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-k,s} W_{T-(k+1),s}) \\
&= \mathsf{E}_{T-(k+1)}\big(-\exp[-a_j\{(V_{j,T-(k+1)} - \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-k,s} W_{T-(k+1),s}) R^{k+1} \\
&+ \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-k,s} W_{T-k,s} R^k\}]\Theta_{j,T-k}\big).
\end{aligned}
$$

$$
\begin{aligned}
W_{T-(k+1),s'} &\overset{\mathbf{A4}}{=} \frac{1}{a_j \alpha_{j,T-k,s'} R^{k+1}} \\
&\log \frac{\mathsf{E}_{T-(k+1)}\{\exp(-a_j \sum_{s\in\mathcal{S}_j\setminus\{s'\}} \alpha_{j,T-k,s} W_{T-k,s} R^k)\Theta_{j,T-k}\}}{\mathsf{E}_{T-(k+1)}\{\exp(-a_j \sum_{s\in\mathcal{S}_j} \alpha_{j,T-k,s} W_{T-k,s} R^k)\Theta_{j,T-k}\}}.
\end{aligned}
$$

For the value function of farmer j at $T-(k+1)$ we have:

$$
\begin{aligned}
&J^*_{j,T-(k+1)}(V_{j,T-(k+1)}) \\
&= -\exp(-a_j V_{j,T-(k+1)} R^{k+1}) \exp(a_j \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-k,s} W^*_{j,T-(k+1)} R^{k+1}) \\
&\mathsf{E}_{T-(k+1)}\{\exp(-a_j \sum_{s\in\mathcal{S}_j} \alpha^*_{j,T-k,s} W_{T-k,s} R^k)\Theta_{j,T-k}\} \\
&\overset{(20)}{=} -\exp(-a_j V_{j,T-(k+1)} R^{k+1})\Theta_{j,T-(k+1)}.
\end{aligned}
$$

Then it holds

$$
\begin{aligned}
&J^*_{j,T-(k+2)}(V_{j,T-(k+2)}) = \; - \exp(-a_j V_{j,T-(k+2)} R^{k+2}) \Theta_{j,T-(k+2)} \\
&= \mathsf{E}_{T-(k+2)}\{J^*_{j,T-(k+1)}(V_{j,T-(k+1)})\} \\
&= \mathsf{E}_{T-(k+2)}\{- \exp(-a_j V_{j,T-(k+1)} R^k) \Theta_{j,T-(k+1)}\} \\
&\overset{\mathbf{A2,A3}}{=} \; - \exp(-a_j V_{j,T-(k+2)} R^{k+2}) \\
&\exp(a_j \sum_{s \in \mathcal{S}_j} \alpha^*_{j,T-(k+1),s} W^*_{T-(k+2),s} R^{k+2}) \\
&\mathsf{E}_{T-(k+2)}\{\exp(-a_j \sum_{s \in \mathcal{S}_j} \alpha^*_{j,T-(k+1),s} W_{T-(k+1),s} R^{k+1}) \Theta_{j,T-(k+1)}\}
\end{aligned}
$$

and $\Theta_{j,T-(k+2)}$

$$
\begin{aligned}
&= \exp(a_j \sum_{s \in \mathcal{S}_j} \alpha^*_{j,T-(k+1),s} W^*_{T-(k+2),s} R^{k+2}) \\
&\mathsf{E}_{T-(k+2)}\{\exp(-a_j \sum_{s \in \mathcal{S}_j} \alpha^*_{j,T-(k+1),s} W_{T-(k+1),s} R^{k+1}) \Theta_{j,T-(k+1)}\}.
\end{aligned}
$$

■

To prove Proposition 2.2 we use the following

REMARK 5.2

$$
V_{m,t} = R(-V_{m,t-1} + \sum_{s \in \mathcal{S}} \alpha_{m,t,s} W_{t-1,s}) - \sum_{s \in \mathcal{S}} \alpha_{m,t,s} W_{t,s},
$$

compare (5.1).

Proof. In $T-1$ the maximized utility of the investor with WD at site $s' \in \mathcal{S}$:

$$
\begin{aligned}
J(V_{mT-1} \;\; - \;\; & \sum_{s \in \mathcal{S}} \alpha_{mTs} W_{T-1s}, \sum_{s \in \mathcal{S}} \alpha_{mTs} W_{T-1s}) \\
= \;\; & \mathsf{E}_{T-1}\big(- \exp\big[- a_m\big\{ - \sum_{s \in \mathcal{S}} \alpha_{mTs} W_{Ts} \\
& + \big(\sum_{s \in \mathcal{S}} \alpha_{mTs} W_{T-1s} + V_{mT-1}\big) R\big\}\big]\big), \qquad (21)
\end{aligned}
$$

utility of the investor without WD at site $s' \in \mathcal{S}$

$$\begin{aligned} J(V_{mT-1} &- \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{mTs} W_{T-1s}, \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{mTs} W_{T-1s}) \\ &= \mathsf{E}_{T-1}\big(- \exp \big[- a_m \big\{ - \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{mTs} W_{Ts} \\ &\quad + \big(\sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{mTs} W_{T-1s} + V_{mT-1} \big) R \big\} \big] \big) \end{aligned} \tag{22}$$

With **A2**–**A4** we get

$$\exp \{ -a_m (1+r) \alpha_{m,T,s'} W_{T-1,s'} \} = \frac{\mathsf{E}_{T-1} \left\{ \exp \left(a_m \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T,s} W_{T,s} \right) \right\}}{\mathsf{E}_{T-1} \left\{ \exp \left(a_m \sum_{s \in \mathcal{S}} \alpha_{m,T,s} W_{T,s} \right) \right\}},$$

and

$$\begin{aligned} W_{T-1,s'} &= \frac{1}{a_m R \alpha_{m,T,s'}} \\ &\quad \log \frac{\mathsf{E}_{T-1} \left\{ \exp \left(a_m \sum_{s \in \mathcal{S}} \alpha_{m,T,s} W_{T,s} \right) \right\}}{\mathsf{E}_{T-1} \left\{ \exp \left(a_m \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T,s} W_{T,s} \right) \right\}}. \end{aligned}$$

The value function of the investor:

$$\begin{aligned} J^*_{m,T-1}(V_{m,T-1}) &= \underbrace{\exp \big(- a_m \sum_{s \in \mathcal{S}} \alpha^*_{m,T,s} W^*_{T-1,s} R \big) \mathsf{E}_{T-1} \big\{ \exp \big(a_m \sum_{s \in \mathcal{S}} \alpha^*_{m,T,s} W_{T,s} \big) \big\}}_{=1} \\ &\quad \exp(-a_m V_{m,T-1} R) \\ &= \exp(-a_m V_{m,T-1} R) \end{aligned}$$

In $T-2$ the maximized utility of the investor with WD at s'

$$J(V_{m,T-2} - \sum_{s \in \mathcal{S}} \alpha_{m,T-1,s} W_{T-2,s}, \sum_{s \in \mathcal{S}} \alpha_{m,T-1,s} W_{T-2,s}) = \mathsf{E}_{T-2} \left\{ J^*_{m,T-1}(V_{m,T-1}) \right\}$$
$$= \mathsf{E}_{T-2} \left\{ \exp\left(-a_m V_{m,T-1} R\right) \right\}$$
$$\stackrel{5.2}{=} \mathsf{E}_{T-2} \Big[\exp\{-a_m \big(\sum_{s \in \mathcal{S}} \alpha_{m,T-1,s} W_{T-2,s} - V_{m,T-2} \big) R^2$$
$$- \sum_{s \in \mathcal{S}} \alpha_{m,T-1,s} W_{T-1,s} R\} \Big] \tag{23}$$

Maximized utility of the investor in $T-2$ without WD at s'

$$J(V_{m,T-2} - \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T-1,s} W_{T-2,s}, \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T-1,s} W_{T-2,s})$$
$$= \mathsf{E}_{T-2} [\exp\{-a_m (\sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T-1,s} W_{T-2,s} - V_{m,T-2}) R^2$$
$$- \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T-1,s} W_{T-1,s} R\}] \tag{24}$$

Using (23), (24) and **A2**–**A4** we get

$$\exp\left\{-a_m \alpha_{m,T-1,s'} W_{T-2,s'} R^2\right\}$$
$$= \frac{\mathsf{E}_{T-2} \left[\exp\left\{a_m (\sum_{s \in \mathcal{S}} \alpha_{m,T-1,s} W_{T-1,s}) R\right\}\right]}{\mathsf{E}_{T-2} \left[\exp\left\{a_m \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T-1,s} W_{T-1,s} R\right\}\right]}$$

and thus

$$W_{T-2,s'} = \frac{1}{a_m \alpha_{m,T-1,s'} R^2} \log \frac{\mathsf{E}_{T-2} \left[\exp\left\{a_m (\sum_{s \in \mathcal{S}} \alpha_{m,T-1,s} W_{T-1,s}) R\right\}\right]}{\mathsf{E}_{T-2} \left[\exp\left\{a_m \sum_{s \in \mathcal{S} \setminus \{s'\}} \alpha_{m,T-1,s} W_{T-1,s} R\right\}\right]}$$

For $T-k$, $k \leq T$ it holds:

$$W_{T-k,s'} = \frac{1}{a_m \alpha_{m,T-k+1,s'} R^k} \log \frac{\mathsf{E}_{T-k}\{\exp(-a_m \sum_{s\in\mathcal{S}} \alpha_{m,T-k+1,s} W_{T-k+1,s} R^{k-1})\}}{\mathsf{E}_{T-k}\{\exp(-a_m \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T-k+1,s} W_{T-k+1,s} R^{k-1})\}} \tag{25}$$

$$J^*_{m,T-k}(V_{m,T-k}) = \exp(a_m V_{m,T-k} R^k) \tag{26}$$

We use backward induction over $T-k$ for the proof:

- for $T-1$ above.
- for one step backwards from $T-k$ it holds:

$$\begin{aligned}
&J(V_{m,T-(k+1)} - \sum_{s\in\mathcal{S}} \alpha_{m,T-k,s} W_{T-(k+1),s}, \sum_{s\in\mathcal{S}} \alpha_{m,T-k,s} W_{T-(k+1),s}) \\
&= \mathsf{E}_{T-(k+1)}\{J^*_{m,T-k}(V_{m,T-k})\} \\
&\overset{(26)}{=} \mathsf{E}_{T-(k+1)}\{\exp(a_m V_{m,T-k} R^k)\} \\
&= \mathsf{E}_{T-(k+1)}[\exp\{a_m(R^{k+1}(-V_{m,T-(k+1)} + \sum_{s\in\mathcal{S}} \alpha_{m,T-k,s} W_{T-(k+1),s}) \\
&- \sum_{s\in\mathcal{S}} \alpha_{m,T-k,s} W_{T-k,s}) R^k\}].
\end{aligned}$$

$$\begin{aligned}
&J(V_{m,T-(k+1)} - \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T-k,s} W_{T-(k+1),s}, \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T-k,s} W_{T-(k+1),s}) \\
&= \mathsf{E}_{T-(k+1)}[\exp\{a_m(R^{k+1}(-V_{m,T-(k+1)} + \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T-k,s} W_{m,T-(k+1),s}) \\
&- \sum_{s\in\mathcal{S}\setminus\{s'\}} \alpha_{m,T-k,s} W_{T-k,s}) R^k\}].
\end{aligned}$$

With **A2–A4**:

$$\exp(-a_m\alpha_{m,T-k,s'}W_{T-(k+1),s'}R^{k+1})$$

$$\mathsf{E}_{T-(k+1)}\{\exp(-a_m\sum_{s\in\mathcal{S}}\alpha_{m,T-k,s}W_{T-k,s}R^k)\}$$

$$= \mathsf{E}_{T-(k+1)}\{\exp(-a_m\sum_{s\in\mathcal{S}\setminus\{s'\}}\alpha_{m,T-k,s}W_{T-k,s}R^k)\}$$

and

$$W_{T-(k+1),s'} = \frac{1}{a_m\alpha_{m,T-k,s'}R^{k+1}}$$

$$\log\frac{\mathsf{E}_{T-(k+1)}\{\exp(-a_m\sum_{s\in\mathcal{S}}\alpha_{m,T-k,s}W_{T-k,s}R^k)\}}{\mathsf{E}_{T-(k+1)}\{\exp(-a_m\sum_{s\in\mathcal{S}\setminus\{s'\}}\alpha_{m,T-k,s}W_{T-k,s}R^k)\}}$$

For the value function of the investor in $T-(k+1)$ we have:

$$J^*_{m,T-(k+1)}(V_{m,T-(k+1)})$$

$$= \exp(-a_mV_{m,T-(k+1)}R^{k+1})\exp(a_m\sum_{s\in\mathcal{S}}\alpha_{m,T-k,s}W_{T-(k+1),s}R^{k+1})$$

$$\mathsf{E}_{T-(k+1)}\{\exp(-a_m\sum_{s\in\mathcal{S}}\alpha_{m,T-k,s}W_{T-k,s}R^k)\}$$

$$= \exp(-a_mV_{m,T-(k+1)}R^{k+1}).$$

■

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