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On the Dark Side of the Market: Identifying and Analyzing Hidden Order Placements

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Abstract

Trading under limited pre-trade transparency becomes increasingly popular on financial markets. We provide first evidence on traders' use of (completely) hidden orders which might be placed even inside of the (displayed) bid-ask spread. Employing TotalView-ITCH data on order messages at NASDAQ, we propose a simple method to conduct statistical inference on the location of hidden depth and to test economic hypotheses. Analyzing a wide cross-section of stocks, we show that market conditions reflected by the (visible) bid-ask spread, (visible) depth, recent price movements and trading signals significantly affect the aggressiveness of 'dark' liquidity supply and thus the 'hidden spread'. Our evidence suggests that traders balance hidden order placements to (i) compete for the provision of (hidden) liquidity and (ii) protect themselves against adverse selection, front-running as well as 'hidden order detection strategies' used by high-frequency traders. Accordingly, our results show that hidden liquidity locations are predictable given the observable state of the market.

Keywords: limit order market, hidden liquidity, high-frequency trading, non-display order, iceberg orders

JEL classification: G14, C24, C25, G17

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1 Introduction

Since the introduction and the growing dominance of electronic trading during the nineties, equity markets have trended toward higher transparency and more disclosure of trading information. However, displayed limit orders reveal trading intentions and may induce adverse selection effects, picking-off risks and "parasitic trading" (see, e.g., Harris, 1997). Consequently, the question of how much transparency should be optimally provided on a market is of ongoing importance. In particular, current developments in equity markets away from full transparency and back toward more opaque market structures have made this question again very topical in recent market microstructure research.

In modern trading, traders seek to conceal trading strategies and to avoid adverse price effects by hiding order sizes. Consequently, reserve ("iceberg") orders which require to display only a small fraction of the order size are increasingly popular and can be used on virtually all major exchanges and trading platforms worldwide. An even more extreme form of reducing pre-trade transparency is to trade in form of non-display ("hidden") orders which can be entirely hidden. Such orders do not even reveal the posted limit price and thus act as completely hidden liquidity supply in the limit order book (LOB). While there are a few empirical studies analyzing iceberg orders (see, e.g., Bessembinder, Panayides, and Venkataraman, 2009; Frey and Sandås, 2009), there is no empirical evidence on non-display orders. The important difference between a reserve order and a hidden order is that in case of the latter not only the size but, more importantly, also the *location* is unknown. This induces effects which are quite different from those caused by reserve orders and which are not well understood yet. For instance, the most interesting aspect behind hidden orders is that they can be placed inside of the bid-ask spread without affecting visible best ask and bid quotes. In fact, this mechanism creates enormous order activities in markets as market participants try to "ping" for hidden liquidity inside of the spread by posting "fleeting orders" which are canceled a few instants later if they do not get executed.

This paper aims at shedding light on the use of undisclosed orders in an electronic market where not only order volumes but also their locations are hidden. To our best knowledge, this study is the first one providing empirical evidence on liquidity supply which is completely "dark" and thus features some elements of the supply side of a dark pool. In contrast to prevailing empirical studies on the degree of order exposure in reserve orders, our focus is on the analysis of hidden order *locations* and thus the aggressiveness of hidden liquidity supply. Using data from the NASDAQ TotalView message stream allows us to retrieve information on hidden depth from one of the largest equity markets in the world. We propose an ordered response approach with censoring mechanism to retrieve conditional probabilities of hidden order locations given the state of the market and to provide insights into the distribution of hidden orders across different price levels. Performing statistical inference on the aggressiveness of hidden order placements (e.g., within the spread) allows us to test economic theory on the relation between the state of the market and traders' incentive to hide orders. Our findings based on a wide cross-section of NASDAQ stocks show that "dark" liquidity supply is significantly driven by market conditions and thus predictable in terms of the state of the (displayed) LOB. Empirical evidence supports the notion that hidden liquidity submitters balance their competition for liquidity supply versus the risk of non-execution. Under certain market conditions, there is significant competition for hidden liquidity supply inducing a narrowing of the "hidden" bid-ask spread. Conversely, in situations where the risk of being picked off becomes high, we observe a significant reduction in hidden order submitters' aggressiveness. Moreover, we provide novel insights into competition for hidden liquidity provision and hidden order placements in the presence of aggressive "hidden order detection strategies" used by algorithmic traders.

The current tendency of trading platforms toward more opaqueness is observable on all major markets. We can differentiate between three major types of "dark trading". The first group of markets, including various non-U.S. markets, such as the London Stock Exchange, Frankfurt Stock Exchange (XETRA), Australian Stock Exchange (ASX), Euronext, the Madrid Stock Exchange and the Toronto Stock Exchange, among others, offer the possibility of posting *only* iceberg orders (so-called reserve orders) where the trader is obliged to show only a small proportion ("peak") of the posted order size. The second category of trading platforms allows to use *both* reserve and hidden orders and thus offers the option to entirely hide an order. Prominent examples are NASDAQ, the New York Stock Exchange (NYSE), BATS (Best Alternative Trading System) – currently the third largest equity market in the U.S. – and the largest U.S. Electronic Communication Network (ECN) Direct Edge. According to the report by the Securities and Exchange Commission (2010), these markets cover approximately 75% of share volume in National Market System (NMS) stocks. The third group of modern trading systems are so-called dark pools where liquidity supply is hidden and no information on order matching and trading actions is provided to other market participants.

Recent empirical evidence shows that "dark trading" is not negligible and is increasingly popular. For instance, Bessembinder, Panayides, and Venkataraman (2009) report that 44% of order volume is hidden and 18% of incoming orders are reserve orders on Euronext Paris. Frey and Sandås (2009) show that reserve orders represent 9% of non-marketable orders with sizes of $12-20$ times the average in German XETRA trading. The Securities and Exchange Commission (2010) reports that 32 dark pools in the U.S. contribute approximately 8% of trading volume in NMS stocks. Figure 1 shows percentages of trading volume executed against hidden liquidity for 99 NASDAQ

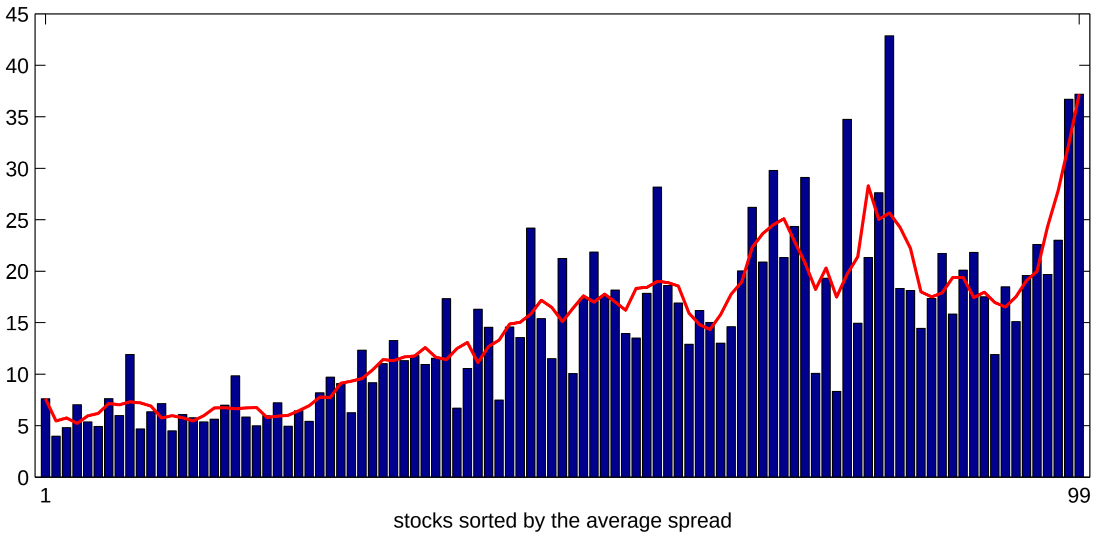

Figure 1: Percentage of trading volumes executed against hidden depth for 99 NASDAQ stocks representing a wide cross-section of the market. The stocks are sorted according to their average bid-ask spreads during the investigation period.

stocks used in our empirical analysis. Averaged across a wide range of the market, approximately 14% of the share volume originates from hidden depth. However, for some stocks, especially those revealing high spreads, it can be even greater than 40%.

The major motivation for hiding orders is to camouflage trading intention. The latter increases execution risk as the display of (large) orders may cause impatient traders to retreat (Moinas, 2010) and may lead to higher liquidity competition (Buti and Rindi, 2011). Moreover, posting limit orders induces front-running strategies (Harris, 1997), and the risk of adverse selection ("picking off risk"; see Harris, 1996). By hiding an order, execution risks can be reduced, while, on the other hand, the risk of non-execution rises as trading counterparties are not obviously attracted. Moreover, typically, hidden orders lose time priority to displayed orders. Hence, for a hidden order submitter it is crucial to balance the risk of non-execution vs. the risk of adverse selection.

Our empirical methodology is designed to provide insights into the placement of hidden orders and and to link them to the (observable) state of the market. Consequently, we are able to test implications from economic theory and to predict hidden order placements. The used data contains information on any order activity at NASDAQ and allows to completely reproduce the (displayed) LOB at each instant. As the data directly stems from the NASDAQ trading feed (and thus is publicly available even in real time), it naturally does not reveal direct information on hidden order locations. Nevertheless, as a crucial ingredient which (to our best knowledge) has not been exploited by any empirical study yet, it contains information on *executions* against hidden orders. Consequently, we are able to (ex post) identify whenever (at least partly) a hidden order has been executed. Likewise, market orders which are *not* executed against hidden depth and limit orders placed into the prevailing spread provide us implicit information on the *non-existence* of hidden orders on certain price levels.

To identify the locations of (executed) hidden orders, we employ two approaches. Firstly, we measure hidden order aggressiveness in terms of the distance between the order price and the best (visible) quote on the *own* side of the LOB. The larger this distance, the deeper a hidden order is placed within the spread and the higher is its aggressiveness. The second approach employs the distance to the best visible quote on the *opposite* side of the market. The lower this distance, the lower the transaction costs for a market order submitter on the opposite side. We show that both distance measures are necessary to fully capture hidden order placements. To fully exploit also (ex post) identifications on the *non-existence* of hidden orders on certain price grids, we set up an ordered response model with censoring mechanism yielding conditional probabilities of hidden order placements in terms of aggressiveness categories given the state of the market.

Using this setup, we analyze whether hidden order placements can be explained by the economic reasoning of balancing execution risk vs. exposure risk and thus can be predicted using the observable state of the LOB. In particular, we address three major research questions: (i) Does hidden liquidity supply compete with observable order flow and react to trading directions? (ii) Is there competition between hidden liquidity suppliers themselves? (iii) How does hidden supply react to "hidden order detection strategies"?

Analyzing hidden order placements for 99 stocks covering a wide cross-section of the NASDAQ market in 2010, we can summarize the following results: First, hidden order placements follow trade directions in order to increase execution probabilities and to reduce adverse selection. In particular, market participants submit hidden orders less aggressively when the price moves in their favorable direction. Second, the "hidden" spread is positively correlated with the observed spread. This is particularly true for stocks with comparably high (average) spreads. Third, there is significant competition for the provision of liquidity. This is true for hidden liquidity as traders use more aggressive hidden orders after observing competing hidden depth on the own side. Moreover, it is also true for the competition between hidden and disclosed liquidity. The latter is empirically supported by a strong (positive) correlation between undisclosed orders and the visible depth on the same side of the market. Fourth, hidden order submitters become more defensive when high-frequency traders actively "ping" for undisclosed volume in the spread. Overall, our findings clearly show that hidden orders are placed strategically in order to balance non-execution risks and adverse selection risks.

The remainder of this paper is organized in the following way: In Section 2, we review theoretical and empirical literature and formulate economic hypotheses. Section 3 briefly introduces the market environment and presents details on data construction and descriptive statistics. In Section 4, we introduce the econometric approach to model the aggressiveness of dark liquidity supply. In Section 5, we report and discuss the empirical findings. Section 6 concludes.

2 Economic Reasoning of Optimal Order Display

2.1 Market Microstructure Theory

A major motivation for posting a limit order is to minimize transaction costs by appropriately choosing the limit price and to signal trading intention to other market participants in order to attract counterparties which might be not in the market yet (according to Harris (1996), so-called "passive traders"). Compared to market orders, limit orders impose lower execution costs as they are executed at better prices (avoiding to cross the bid-ask spread), however bear the risk of non-execution if the market moves in opposite direction. This results into the fundamental trade-off between transaction costs (induced by a market order) and execution risks.

Hence, attracting a counterparty by maximizing the degree of order exposure is important to increase the execution probability and to decrease the execution time of the position. Such a strategy, however, induces also various adverse effects. Firstly, as empirically shown by Hautsch and Huang (2011), signaling trading intention may induce significant (adverse) price reactions. Secondly, according to Moinas (2010), displaying large orders may cause "defensive" market order traders to retreat from the market as soon as they interpret the signal as inside information. Thirdly, "parasitic" traders (Harris, 1997) may exploit the information value of a big order by using front-running strategies. Finally, posting a limit order induces the risk of being picked off and thus adverse selection (Harris, 1996). The latter occurs if limit orders cannot be canceled fast enough in a situation when prices move stronger in the favorable direction than expected. Consequently, the order becomes mis-priced. These effects induce the "exposure costs" (Buti and Rindi, 2011) of a displayed limit order.

These exposure costs can be alleviated by reducing the order exposure or – in the extreme case – completely hiding the order. A hidden order does not cause any price impact and prevents undercutting while still allowing to (aggressively) compete for the provision of liquidity, particularly if the hidden order undercuts (or overbids, respectively) the prevailing best limit price. However, a hidden order still runs adverse selection risks and, moreover, bears higher execution risks. This results into a trade-off between exposure costs and execution risks.

Based on these economic reasoning, several theories on the usage of undisclosed orders have been developed. Esser and Mönch (2007) propose a static framework in which the trader optimizes the peak size and limit price of reserve orders by continuously monitoring and balancing exposure risk against execution risk. Moinas (2010) presents a theoretical model where informed traders as well as large liquidity traders use reserve orders to mitigate the information leakage. Cebiroglu and Horst (2011) propose a model where traders decide on the peak size of the iceberg order by accounting for the exposure-induced market impact. Buti and Rindi (2011) present a dynamic framework where the trader chooses her optimal strategy by simultaneously deciding on trading direction, aggressiveness, size and peak proportion of the order. To our best knowledge, it is the only theoretical model that explicitly incorporates the possibility of hiding orders within the bid-ask spread into traders' trading options. In particular, Buti and Rindi (2011) consider the possibility of so-called hidden mid-point peg orders, i.e., hidden orders which are pegged to the midpoint of the national best bid and offer (NBBO).

2.2 Empirical Evidence on Undisclosed Orders

The empirical literature on reserve orders has been growing remarkably during the last decade, partially due to its proliferation in limit order markets and the increasing availability of data. Studying trading on Euronext Paris, Bessembinder, Panayides, and Venkataraman (2009) document that reserve orders induce lower implementation short fall costs but longer times to fill. De Winne and D'Hondt (2007) examine similar data and find that the detection of hidden depth increases order aggressiveness on the opposite side. Fleming and Mizrach (2009) examine data from BrockerTec, the leading interdealer ECN for U.S. Treasuries and documenting that the use of reserve orders varies considerably with the quantity of hidden depth increasing with price volatility. All studies show that the decision on using reserve orders is strongly related to prevailing market conditions, as characterized by the bid-ask spread, book depth and prevailing volatility.

Studying data from the Australian Stock Exchange (ASX), Aitken, Berkman, and Mak (2001) find that reserve orders do not have a different price impact than visible limit orders. According to their results, the use of reserve orders increases with volatility and the average order value, while it decreases in tick size and trading activity. Frey and Sandås (2009) analyze the Deutsche Börse's trading platform XETRA and show that the price impact of the reserve order depends on the executed fraction of its size with profitability increasing in the hidden proportion. Based on data from the Spanish Stock Exchange Pardo Tornero and Pascual (2007) find no significant price impact associated with the execution of hidden parts of reserve orders. These findings support the hypothesis that liquidity suppliers use reserve orders to compete for liquidity provision while preventing picking-off risks.

Tuttle (2006) shows that the overall market depth increased significantly after NASDAQ introduced undisclosed orders. Moreover, she provides evidence for hidden sizes being predictive for future market price movements while the visible size conveys only little information. Likewise, analyzing data from the Copenhagen Stock Exchange, Belter (2007) shows that non-displayed orders have more information content which, however, cannot be exploited to predict future returns. Anand and Weaver (2004) examine the abolition in 1996 and re-introduction in 2002 of reserve orders on the Toronto Stock Exchange and show that the spread and visible depth remain widely unchanged after both events. However, total depth, including both visible and hidden volume, significantly increases after the re-introduction. Both studies show that market quality is improved after the introduction of reserve orders and that informed traders tend to use them primarily to reduce the price impact.

2.3 Testable Hypotheses

Theoretical models on optimal order (non-)display, such as Buti and Rindi (2011), consider the optimization problem of a limit order submitter who simultaneously decides on limit price, order volume as well as degree of exposure. The market participant's objective is to maximize her expected profit conditional on the (observable) state of the LOB by optimally balancing exposure and execution risks. This induces testable hypotheses on the relation between the state of the market and the chosen degree of exposure.

In contrast to prevailing empirical studies evaluating order exposure in reserve orders, our focus is on the analysis of traders' decisions *where* to post a hidden order. In terms of its aggressiveness, a hidden order can be seen as an instrument categorized between a (displayed) limit order at the best available quote and a market order. Compared to a market order, it still allows to benefit from price improvements (as the bid-ask spread is not crossed completely) but faces non-execution risks as well as adverse selection risk. Hence, the economic reasoning behind the decision *where* to optimally place a hidden order is triggered by a balancing of (non-)execution risk, implied transaction costs and adverse selection risk. As discussed below in light of market microstructure theory, these considerations lead to testable hypotheses on the relationship between the state of the market and the aggressiveness of hidden order placements.

Asymmetric information based market microstructure theory (see, e.g., Easley, Kiefer, and O'Hara, 1997) suggests that wide bid-ask spreads reflect uncertainty on the fundamental value of the asset and on the presence of informed traders in the market. Consequently, the transaction costs implied by potential adverse selection increase, particularly if a hidden order is placed inside of the spread. To keep these risks and costs on a moderate level, liquidity suppliers should post their hidden orders not too deeply in the spread. Indeed, in such a situation they can benefit from a wide spread which naturally provides sufficient room to overbid or undercut best (displayed) quotes and thus to aggressively compete for liquidity supply while still being placed in sufficient distance from the opposite side of the market. Conversely, if spreads are narrow, the room for price improvements beyond best quotes is limited as the spread can only be a multiple of the minimum tick size. For instance, in the extreme case of a two-tick spread, traders who want to increase the execution probability by overbidding best quotes, are forced to place their order at the mid-quote. This high discreteness of possible price steps within the bid-ask spread forces liquidity suppliers who want to undercut best quotes have to become more aggressive than in a (hypothetical) situation of a continuous price grid. As a consequence of such "overbidding", hidden liquidity suppliers are more aggressive in small-spread states than in large-spread states. Consequently, we expect a positive correlation between the observable spread and the "hidden spread", defined as the difference between the best *hidden* ask and bid quotes:

Hypothesis 1 *The aggressiveness of hidden depth inside of the spread decreases with the size of the spread, i.e., observable and hidden spreads are positively correlated.*

Traders can use undisclosed orders to compete for the provision of liquidity while preventing others from undercutting their orders. Buti and Rindi (2011) demonstrate that undisclosed orders are part of equilibrium strategies of liquidity suppliers who maximize expected profits. In particular, when the depth on the own side of the market is high (relative to the other side), traders prefer to place more aggressive hidden orders inside of the spread to increase their execution probability. Moreover, relatively higher depth on the own side reflects price expectations in the favorable direction which in turn reduces the risk of (adversely) being picked up. Conversely, in case of a (relatively) high depth on the opposite side of the market, picking-up risks are higher as a high depth on the opposite side may reflect further price pressure. Hence, in such a situation, the downside of order aggressiveness in order to compete for liquidity supply, is much stronger than in the case of a high own-side depth. Confirming this reasoning, the theoretical setting by Buti and Rindi (2011) predicts a higher tendency of traders to post hidden orders within the bid-ask spread if the own-side depth is high and the opposite-side depth is low. This leads to the following hypotheses:

Hypothesis 2.A *The probability of hidden depth inside of the spread increases when the own-side depth increases relatively to the opposite-side depth.*

Hypothesis 2.B *The probability of hidden depth inside of the spread decreases when* *the opposite-side depth increases relatively to the own-side depth.*

Traders' order submission strategies depend not only on the current state of the LOB but also on recent price movements and trading signals. The dynamic equilibrium model on visible order flow proposed by Parlour (1998) shows a "crowding out" effect among market orders: the probability of incoming sell (buy) market orders is lower after observing a buy (sell) market order which is in line with the well-known strong persistence in trade directions. This effect implies that visible bid (ask) limit orders have a higher execution probability after a sell (buy) market order. This hypothesis is supported by Hall and Hautsch (2005) showing that price movements are positively (negatively) correlated with the aggressiveness of visible buy (sell) limit orders. We expect that liquidity suppliers take advantage of these trading signals by posting hidden orders deeper inside of the spread in order to increase execution probabilities. However, as argued above, in situations where liquidity suppliers aim at benefiting from price pressure built up on the opposite side of the market, their exposure to adverse selection risk increases and may dominate execution risk.

A similar reasoning applies in situations when market participants expect momentum in prevailing price movements. Then, it might be advantageous to reduce the risk of non-execution by placing aggressive hidden orders after observing price movements in favorable direction. However, we expect these effects being weaker than in case of trading signals as the predictability of price changes (even over short horizons) is typically much lower than the persistence in trading directions:

Hypothesis 3.A *The probability of hidden bid depth inside of the spread decreases (increases) when the prevailing trade is seller (buyer)-initiated. The converse effect applies for hidden ask depth.*

Hypothesis 3.B *The aggressiveness of hidden bid depth increases (decreases) after observing upward (downward) movements in prices.*

Traders' decision on using undisclosed orders might also depend on the asset's volatility. Foucault (1999) shows that volatility is an important parameter in order submission strategies. Indeed, higher volatility implies higher uncertainty on the value of the asset and thus increases picking-off risk. Buti and Rindi (2011) show that this mechanism is true not only for visible orders but also for hidden orders:

Hypothesis 4 *The aggressiveness of hidden depth is negatively correlated with prevailing asset price volatility.*

Hidden depth is a priori unobservable but is ex post identifiable as soon as it gets executed. This is, for instance, most clearly seen if a limit order posted *inside* of the spread gets immediate execution. Such information provides hidden liquidity providers hints on the possible prevailing competition for hidden liquidity supply. As a result of higher (hidden) liquidity competition, they post more aggressive orders to increase their execution probability. This is theoretically shown by Buti and Rindi (2011) who predict that detections of hidden depth encourage even more undisclosed order submissions as long as picking-off risks do not become too high. The reasoning is that market participants interpret the detection of hidden volume as a signal of high liquidity demand and compete for supplying it. Accordingly, we postulate the following hypothesis:

Hypothesis 5 *The aggressiveness of hidden bid (ask) depth increases after some hidden bid (ask) depth has been executed.*

In modern trading, high-frequency trading (HFT) plays an increasingly important role (see e.g., Angel, Harris, and Spatt, 2010; Securities and Exchange Commission, 2010) and might also influence the supply for hidden liquidity. In fact, HFT algorithms use front-running strategies (so-called "scalping") by posting a limit order in front of some other limit order which is expected to reveal information. Likewise, exploiting their low latency, mis-priced limit orders are picked up nearly instantaneously before they get canceled. Moreover, HFT trading algorithms also embed strategies for detecting hidden depth, such as "pinging", where visible (e.g., so-called Immediate-or-Cancel; IOC) limit orders are posted in the spread in order to test whether they might get executed. Our empirical results show that such effects create enormous order activities at NASDAQ. Pinging strategies, combined with scalping, induce severe picking-off risks for undisclosed orders and may make them quite inefficient. Indeed, Buti and Rindi (2011) theoretically show that when hidden depth can be perfectly detected there is no reason for traders using undisclosed orders to reduce exposure risks. Accordingly, we expect that hidden liquidity suppliers become less aggressive if high-frequency traders become very active in the market:

Hypothesis 6 *The aggressiveness of hidden depth decreases as HFT activities on the opposite side of the market increase.*

In the theoretical framework by Moinas (2010), informed traders use undisclosed orders to mitigate information leakage. Typically, information asymmetry is highest during the opening period as overnight information has to be processed. Accordingly, we expect a higher hidden order aggressiveness in this period compared to the rest of the trading day. Moreover, Esser and Mönch (2007) show that traders tend to display more of order sizes when they approach trading closure. This is driven by the typical requirement to close a position before the end of the trading session. Accordingly, trading intentions are revealed such that order execution probabilities are increased due to a higher time priority of visible orders. Buti and Rindi (2011) also argue that reserve orders are preferable to hidden orders in their framework when the time horizon becomes shorter. This leads to the following hypotheses:

Hypothesis 7.A *The aggressiveness of hidden depth is higher after market opening.*

Hypothesis 7.B *The aggressiveness of hidden depth is lower during market closure.*

3 Quantifying Hidden Order Locations

3.1 Institutional Background

As one of the largest electronic limit order markets in the world, the NASDAQ Single-Book platform provides an unified procedure for passing limit orders from ECNs (Brut and INET) and the traditional dealer-quote system. In particular, it treats a market maker's quote as a pair of limit orders on both sides of the market and aggregates them into a centralized order book. During continuous trading between 9:30 and 16:00 E.T., the system matches incoming orders against the best (in term of price) prevailing (possibly undisclosed) orders in the LOB. If there is insufficient volume to fully execute the incoming order, the remaining part will be consolidated into the book. Besides limit orders and market orders, NASDAQ offers market participants to use both reserve orders and hidden orders.[1] As a reward for traders disclosing their orders, the hidden part of undisclosed orders loses time priority compared to visible limit orders or peaks of reserve orders on the same price level. Market makers at NASDAQ may also provide hidden depth. The NASDAQ Stock Market trading rule (NASDAQ, 2008) requires the market maker to display at least one round lot size. In this case, the market maker's quotation corresponds to a pair of reserve orders.

3.2 Data

We conduct our study based on 99 stocks traded on NASDAQ during October 2010 corresponding to 21 trading days. To represent a wide cross-section across the market, we select stocks according to market capitalization. We first rank the 500 biggest NASDAQ stocks according to their market capitalizations as recorded by the Center for Research in Security Prices (CRSP) database on 30th September 2010. Furthermore, we restrict the sample by selecting a stock out of every percentile resulting in 99 stocks which are divided into three equal-size groups according to their average spreads and trade frequencies.

We retrieve historical NASDAQ market conditions from TotalView-ITCH data. NASDAQ TotalView^{SM} data, surpassing NASDAQ Level 2, is the current standard

[1] NASDAQ also provides so-called "discretionary orders" with a displayed price and size as well as a non-displayed discretionary price range. When the discretionary price range is hit by a matching order, the discretionary order converts into an IOC market order. This order type also allows to hide trading intention. However, we do not consider discretionary orders as undisclosed orders because (i) they *take* liquidity rather than providing it, and (ii) it is very difficult to identify them using TotalView-ITCH data as HFT algorithms generate an enormous number of IOC orders.

NASDAQ data feed for displaying the real-time full book depth for market participants. Historical data files record rich information on order activities, including limit order submissions, cancellations, executions of visible and hidden orders as well as a unique identification number for every (visible) limit order and peak of reserve orders.

We reconstruct the historical LOB using the algorithm proposed by Huang and Polak (2011). Their algorithm continuously updates the LOB according to all reported messages and represents the exact state of the LOB as shown to TotalView subscribers in real time. Furthermore, we identify the attribute of a limit order (cancelled or filled) and compute its lifetime by tracking it through its order ID.[2] Finally, we aggregate sequences of executions of buy (sell) limit or hidden orders occurring in less than 0.1 seconds into one sell (buy) market order. If a limit order is recorded immediately after such a sequence, it is also aggregated with the entire sequence being considered as a marketable limit order. Finally, to avoid erratic effects during the market opening and closure, our sample period covers only the periods between 9:45 and 15:45.

Table 1 summarizes major characteristics of the selected stocks. They cover a wide universe of stocks with market capitalization ranging from 900 million to 260 billion US dollar. We find a clear evidence for a high popularity of undisclosed orders in NASDAQ trading. On average, approximately 15% of the trading volume and 20% of all trades are executed against hidden depth. The average size of executed hidden depth is slightly smaller than that of visible depth. This is partially due to active HFTs who use high-speed hidden depth detecting algorithms to compete for trading against hidden volume. Moreover, note that only a small proportion of existing hidden depth gets executed (see e.g., Bessembinder, Panayides, and Venkataraman, 2009; Frey and Sandås, 2009). Hence, the share of (undetected) hidden depth is much greater than the magnitudes reported in the table. Furthermore, we show that the proportion of trading volume executed against hidden depth increases as the (average) spread becomes wider. Hence, traders of high-spread stocks are more likely to benefit from price improvements due to the existence of hidden depth.

Table 2 reports summary statistics on limit order executions and cancellations. On average, approximately 95% of all limit orders are cancelled without getting (partially) executed. This strikingly high number is robust across the sample with the cross-sectional standard deviation being very low. In fact, the stock with the smallest proportion of cancellations still reveals a percentage of 91%. Conversely, we observe the most extreme situation of a stock revealing 99% of all limit orders to be cancelled. Moreover, the median lifetime of cancelled orders is less than 10 seconds. For limit orders placed inside of the spread, the average time until cancellation is just around 3 seconds. This effect is obviously driven by a strong influence of HFT-induced ping-

[2]LOB reconstruction and limit order tracking is performed by the software "LOBSTER" which can be accessed at `http://lobster.wiwi.hu-berlin.de`.

Table 1

Cross-Sectional Summary Statistics on the Characteristics of the Selected Stocks.

The sample consists of 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. We divide them into three equal-size groups according to the average spread (AvgSpr) and the number of trades (AvgTrd). For each group, we report summary statistics of the following variables: MktCap is the market capitalization accroding to CRSP at 30 September, 2010. AvgSpr (in ¢) is the average spread in dollar cent. AvgTrd is the average number of daily trades. AvgHit is the average number of daily trades (partly or totally) traded against hidden depth. AvgHit (in %) is the average percentage of daily trades (partly or totally) traded against hidden volume. AvgVol is the average daily trading volume (in thousand shares). AvgHVol is the average daily trading volume traded against hidden depth. AvgHVol (in %) is the average daily percentage of executed hidden volume relative to overall trading volume.

		MktCap (in bil. $)	AvgSpr (in ¢)	AvgTrd	AvgHit	AvgHit (in %)	AvgVol ($\times 10^3$Shr)	AvgHVol ($\times 10^3$Shr)	AvgHVol (in %)
Entire	Mean	7.83	5.38	1861	429	20.1	3.93	0.59	14.6
Sample	Median	2.16	3.77	1083	178	18.7	2.05	0.21	13.5
	Std.	28.19	6.01	2616	959	7.9	5.87	1.59	8.1
	Min.	0.89	1.07	98	12	9.1	0.12	0.01	3.9
	Max.	259.90	34.91	20583	8446	46.0	41.37	14.37	42.8
AvgSpr	Small	9.00	1.36	2199	316	14.0	5.84	0.37	6.9
Groups	Medium	3.81	3.68	1776	414	20.0	3.08	0.54	15.0
(means)	Large	10.69	11.10	1608	558	26.2	2.88	0.86	21.8
AvgTrd	Low	1.50	8.85	461	91	20.4	0.70	0.10	16.3
Group	Medium	2.91	4.05	1121	202	18.2	2.10	0.24	12.7
(means)	High	19.09	3.23	4002	994	21.6	9.00	1.42	14.7

ing strategies aiming at detecting hidden orders inside of the spread. Interestingly, large visible limit orders have much longer execution times than small orders. This is indicated by the volume-weighted execution time of 142 seconds being substantially higher than the median lifetime of executed limit orders (12.9 seconds). This evidence is in line with extant empirical studies of the market impact of limit orders (see, e.g., Eisler, Bouchaud, and Kockelkoren, 2011; Hautsch and Huang, 2011) showing supportive evidence of large traders' economic motivation for using undisclosed orders. Finally, cancellation rates of aggressive limit orders turn out to be lower as they have higher execution probabilities.

3.3 Identifying Undisclosed Orders

It is in the nature of things, that information on hidden order placements is not provided by an exchange. Therefore, from classical transaction data sets, as, e.g., the Trade and Quote (TAQ) database released by the New York Stock Exchange (NYSE), it is impossible to infer on hidden orders. This difficulty is the major reason for the

Table 2

Cross-Sectional Summary Statistics on Limit Order Executions and Cancellations

The sample consists of 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. We divide them into three equal-size groups according to the average spread (AvgSpr) and the number of trades (AvgTrd). For each group, we report cross-sectional summary statistics for the following variables: NumLO is the average daily number of limit orders (including peaks of reserve orders). NumCanc is the average daily number of limit order cancellations before getting (partially) executed. MedCTim is the median of the lifetime of canceled visible limit orders. MedETim is the median of the lifetime of executed limit orders. VWETim is the volume-weighted execution time of limit orders. NumALO is the average daily number of limit orders placed inside of the spread (aggressive limit orders). NumACan (in %) is the average daily percentage of canceled aggressive limit orders placed inside of the spread. AvgATim is the average lifetime of canceled aggressive limit orders.

		NumLO	NumCanc	MedCTim	MedETim	VWETim	NumALO	NumACan	AvgATim
		($\times 10^3$)	(in %)	(sec.)	(sec.)	(sec.)	($\times 10^3$)	(in %)	(sec.)
Entire	Mean	57.92	94.7	9.7	12.9	142.0	3.84	76.3	3.11
Sample	Median	29.41	94.8	9.2	10.7	103.7	2.52	79.7	2.12
	Std.	84.50	1.9	6.3	9.5	152.2	5.81	14.9	4.79
	Min.	5.01	90.9	0.0	0.8	39.9	0.07	29.4	0.02
	Max.	650.66	99.2	33.2	60.3	981.2	49.9	98.7	41.0
AvgSpr	Small	79.93	93.7	10.0	15.8	154.8	1.85	61.1	1.27
Groups	Medium	47.14	94.5	11.2	10.6	114.2	4.06	79.6	3.23
(means)	Large	46.69	95.9	7.8	12.4	156.9	5.62	88.1	4.84
AvgTrd	Low	14.67	96.0	12.5	19.9	216.2	2.42	85.1	2.1
Group	Medium	33.50	94.3	9.3	12.2	109.5	2.50	75.5	2.04
(means)	High	125.6	93.9	7.2	6.7	100.1	6.61	68.2	5.12

lacking empirical evidence on hidden order placements. Message data, as provided by TotalView, however, contain information on *any* activities affecting the *visible* part of the LOB. In particular, it specifically reports *executions* against hidden orders which allow us to identify the exact position of hidden depth in the LOB. As illustrated below, these details can be utilized to conduct statistical inference on undisclosed order submissions.

In general, we distinguish between trading scenarios where we can *distinctly* (ex post) identify the location of hidden volume and situations where we can isolate at least *partial* information on the existence of undisclosed volume. Figure 2 illustrates an example of the first scenario where the best (visible) quotes in the LOB are 24.86 (bid) and 24.91 (ask) before a buy limit order with limit price 24.91 is posted. As there is a hidden ask order at price 24.90 inside of the spread, the incoming order is firstly partially filled by this order resulting in a type "P" trade message (denoting executions against hidden depth in the NASDAQ ITCH 4.0 format). Next, the remaining part of the buy order is executed against the visible depth at the best ask resulting in an "E"

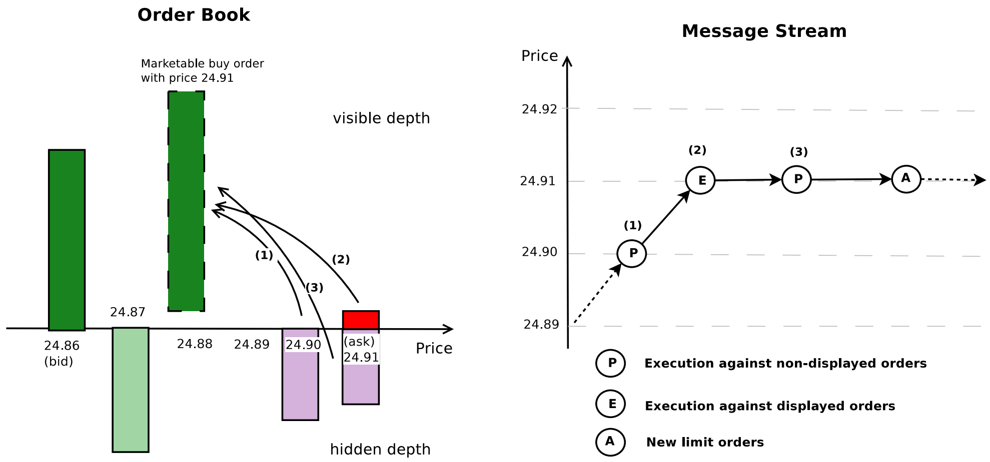

Figure 2: Left: Stylized trading scenario in an LOB where a buy market order is executed against hidden volume on the ask side and is uniquely identified. Bid orders are marked by green, whereas ask orders are marked by red. All orders above the horizontal axis are visible, whereas orders below the axis are hidden. The numbered arrows indicate the matching process. Right: Sequence of generated messages (in NASDAQ ITCH 4.0 format) resulting from this transaction.

message. Finally, hidden depth at the best (visible) ask gets executed resulting in a further "P" message. The remaining (non-executed) part of the incoming order enters the book as a new buy limit order submission (type "A" message) at 24.91.[3]

This example shows that due to the existence of hidden depth, the market order submitter faces a better execution price than expected from the visible LOB. If the trader is able to predict the existence of hidden depth within the spread, she can incorporate these transaction cost savings in her trading strategy. Moreover, it is illustrated that the visible depth has execution priority over the hidden depth at the same price, no matter *when* the order has been placed. Hence, if further depth on the best ask level cumulates, the time-to-fill of any hidden order becomes longer. Finally, since in this scenario, the execution of the hidden part is uniquely identified, we can *exactly* locate the undisclosed order.

Figure 3 shows a scenario which allows extracting at least incomplete information on hidden order placements. Suppose a buy limit order is submitted inside of the spread with price 24.88. The fact that the limit order does not get executed (otherwise we would have been observed a "P" message), reflects that there cannot be any hidden ask volume posted on a price level lower than 24.89. Hence, this observation reveals information about the *non-existence* of hidden depth. We refer to such an observation

[3]As in general, visible and hidden volumes are indicated by more than one order at the same price, we typically observe a sequence of simultaneous "P" and "E" messages.

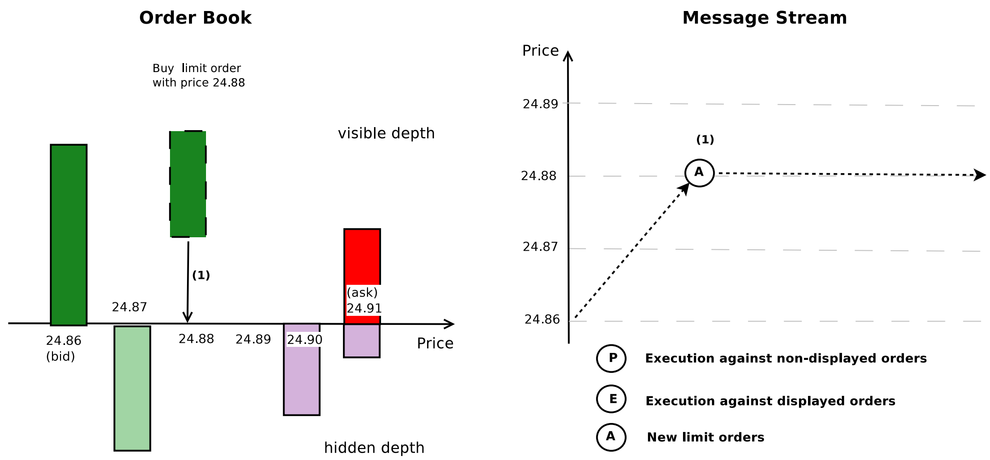

Figure 3: Left: Stylized trading scenario in an LOB where a limit order placed into the spread reveals (partial) information about the hidden depth. Bid orders are marked by green, whereas ask orders are marked by red. All orders above the horizontal axis are visible, whereas orders below the axis are hidden. Right: Sequence of generated messages (in NASDAQ ITCH 4.0 format) resulting from this submission.

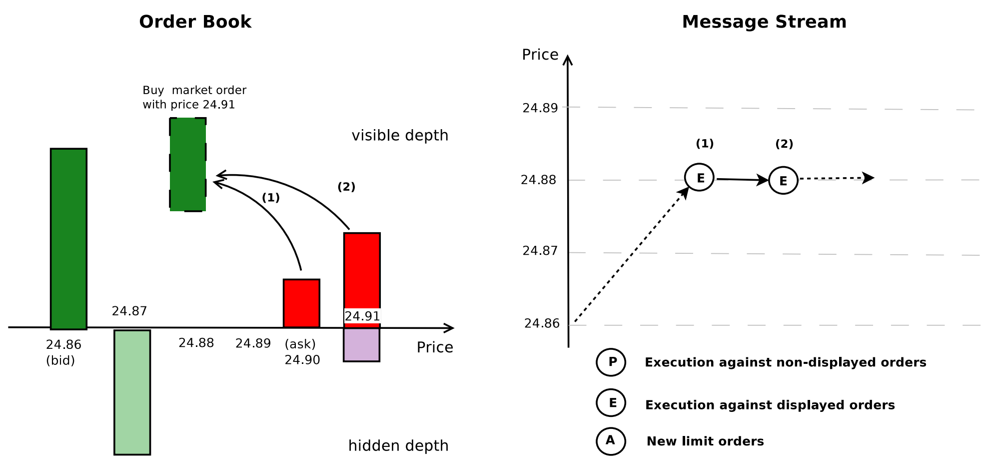

Figure 4: Left: Stylized trading scenario in an LOB where a buy market order is executed against visible volume only and thus reveals (partial) information about the hidden depth. Bid orders are marked by green, whereas ask orders are marked by red. All orders above the horizontal axis are visible, whereas orders below the axis are hidden. Right: Sequence of generated messages (in NASDAQ ITCH 4.0 format) resulting from this submission.

as *censored* as it only provides a lower (upper) bound for the location of hidden ask (bid) volume.

Finally, as illustrated by Figure 4, there might be a scenario where a marketable order is executed against two (or several) levels of visible depth. The fact that not even a part of the order is executed against hidden volume indicates the *non-existence* of hidden ask depth on any level up to (including) price level 24.90. Hence, also this observation is *censored* in the sense that it only yields a location (upper or lower) bound.

Summarizing, we infer price information on undisclosed orders based on the following three scenarios:

i. Submission of a marketable order when the spread is larger than one tick. If the order gets executed at a price better than the corresponding best (visible) quote, we can exactly identify the hidden order location and thus obtain an "uncensored" observation. Otherwise, we have a "censored" observation.

ii. Submission of a limit order inside of the spread. If it is not executed, we certainly know that there is no undisclosed order with better limit price. This results into a "censored" observation.

iii. Submission of a marketable order with size greater than the depth at the corresponding best (visible) quote. As this order may be split across several levels, we can infer on hidden depth at-the-market or behind-the-market. The observation can be uncensored or censored depending on whether it is partially filled by hidden depth or not.

3.4 Measuring the Aggressiveness of Undisclosed Orders

Biais, Hillion, and Spatt (1995) classify the aggressiveness of a limit order by measuring its (price) distance to the prevailing best quotes. This scheme has been widely employed in the empirical literature on limit orders (e.g., Griffiths, Smith, Turnbull, and White, 2000) and reserve orders (e.g., Bessembinder, Panayides, and Venkataraman, 2009). Following these approaches, we measure distances of hidden order placements relative to best quotes on the own and opposite side of the market.

Let p^a and p^b denote the best ask and bid quote and p^o represents the limit price of the undisclosed order. A natural way is to measure the distance between the undisclosed order and the best quote on the order's own side,

$$s = \begin{cases} p^o - p^b & \text{for undisclosed buy orders,} \\ p^a - p^o & \text{for undisclosed sell orders.} \end{cases}$$

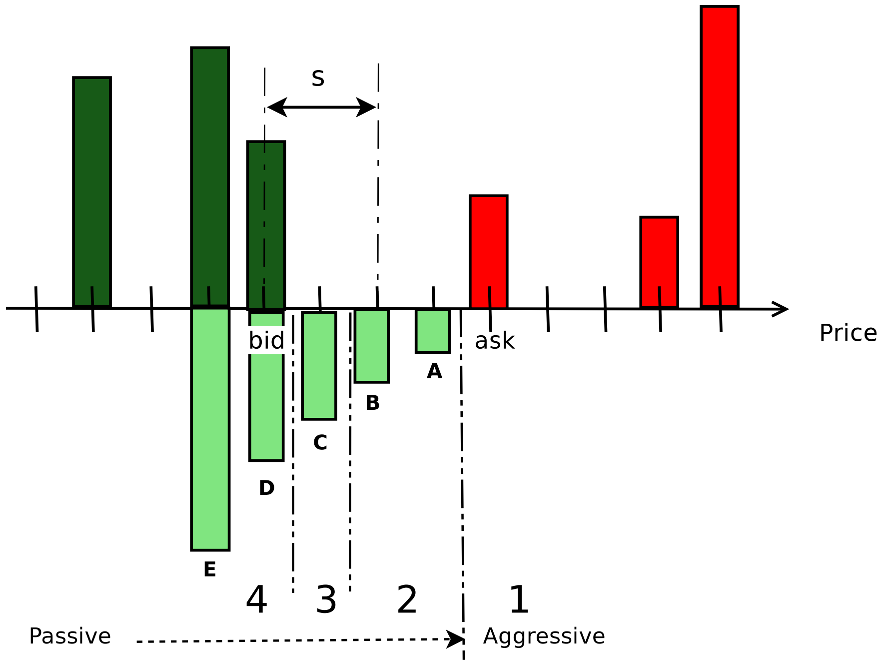

Figure 5: Graphical illustration of the hidden order aggressiveness measure s and corresponding classifications for the case of large-spread stocks (4 categories).

Hence, the larger s, the deeper the order is placed within the spread. Conversely, if $s \leq 0$, the undisclosed order is placed in the book (i.e., outside of the spread) and can be either a reserve order or a hidden order. Accordingly, s measures aggressiveness from the liquidity supplier's perspective (therefore the label "s"). Due to the fact that most observations only reveal incomplete, i.e., "censored", information, it is most natural to measure hidden order aggressiveness in terms of categories. As discussed in the following sections, this allows for straightforward and computationally tractable econometric modelling avoiding severe assumptions on the functional form. Depending on the underlying (average) size of the spread, we choose different categorization schemes. In particular, we divide the set of hidden order locations into 2, 3 and 4 categories for small-spread, medium-spread and large-spread stocks, respectively. Table 3 gives the chosen categories depending on s. The choice of the groups is motivated, on the one hand, by the need to have a sufficient number of observations in each category and, on the other hand, to use a preferably fine categorization *within* the spread. Figure 5 illustrates the resulting scheme for the case of large-spread stocks.

As bid-ask spreads are not constant over time, the distance measure s is not sufficient to fully capture hidden order locations. It is rather necessary to measure orders' aggressiveness also in terms of the distance to the opposite side of the market. Accord-

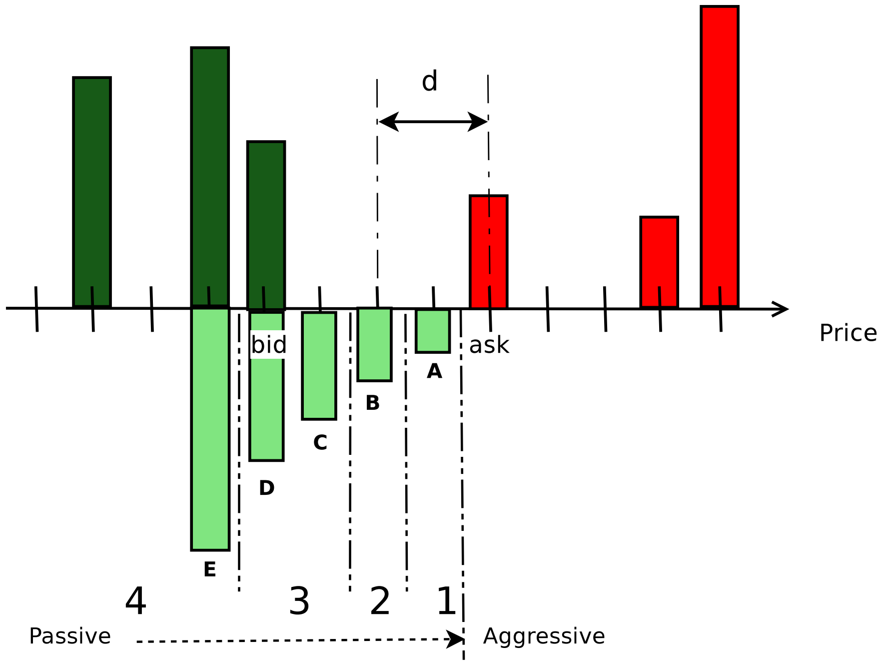

Figure 6: Graphical illustration of the hidden order aggressiveness measure d and corresponding classifications for the case of large-spread stocks (4 categories).

ingly, we define

$$d = \begin{cases} p^a - p^o & \text{for undisclosed buy orders,} \\ p^o - p^b & \text{for undisclosed sell orders.} \end{cases}$$

Hence, d represents undisclosed orders' aggressiveness from the liquidity *demander's* perspective (therefore the label "d"), see Figure 6. The smaller d, the lower the actual transaction costs for a market order submitter who gets executed against this undisclosed order. Note that d cannot become negative as any placement *behind* the opposite side of the market would immediately result into an execution. As shown by Table 3, we categorize d in a similar way to s. However, as d highlights the implied transaction costs induced by execution against undisclosed orders, we choose a categorization which is particularly fine close to the opposite side.

Note that the categorizations underlying the two measures can be partially overlapping. For instance, category 2 in Figure 5 overlaps with the categories 1 and 2 in Figure 6, while category 3 in Figure 6 overlaps with categories 3 and 4 in Figure 5. As shown in the empirical part of this paper, this overlapping structure is particularly advantageous as it enables us to capture manifold (non-linear) changes of the hidden depth distribution by means of relatively simple models.

Table 3 summarizes information on undisclosed orders. Firstly, the number of order submissions revealing (at least partial) information on hidden depth is huge, especially

Table 3

Cross-sectional summary statistics on observations on undisclosed orders

The sample consists of 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. We divide them into three equal-size groups according to the average spread. The aggressiveness of undisclosed orders is measured by s and d as described in Section 3.4. We employ two, three and four categories for small-spread, medium-spread and large-spread stocks, respectively. Censored observations are defined as in Section 3.3. For each group we show cross-sectional statistics on total numbers (over all trading days).

Cate-	Distance	# Observation ($\times 10^3$)			Censored Obs. (%)			% Buy Orders (%)		
gory	(ticks)	max.	mean	min.	max.	mean	min.	max.	mean	min.
Aggressiveness measured by the distance to the own side quote (s)										
Spread group: small										
"1"	$s_t > 0$	147.65	29.32	1.71	96.5	83.3	26.2	52.7	49.8	45.5
"2"	$s_t \leq 0$	15.33	3.42	0.46	50.6	19.9	5.5	62.7	50.2	43.3
Spread group: medium										
"1"	$s_t > 1$	357.27	50.02	7.20	98.8	96.0	92.6	57.0	50.0	44.1
"2"	$s_t = 1$	91.33	21.22	3.91	97.0	87.1	65.1	56.9	49.9	44.2
"3"	$s_t \leq 0$	63.17	7.895	1.12	83.3	65.0	41.2	58.4	48.5	43.6
Spread group: large										
"1"	$s_t > 3$	623.82	76.58	12.13	99.8	97.8	90.6	56.1	50.5	44.7
"2"	$s_t = 2, 3$	193.84	17.04	0.40	98.6	86.4	38.2	56.8	51.2	47.7
"3"	$s_t = 1$	107.46	9.48	0.33	90.2	64.7	23.0	56.0	49.0	43.6
"4"	$s_t \leq 0$	121.38	10.03	0.77	94.9	81.3	60.0	57.3	49.3	42.1
Aggressiveness measured by the distance to the opposite side quote (d)										
Spread group: small										
"1"	$d_t = 1$	156.96	31.12	2.07	94.1	75.2	22.4	54.4	49.8	45.1
"2"	$d_t > 1$	15.19	1.62	0.10	98.0	87.5	69.6	71.5	51.3	37.2
Spread group: medium										
"1"	$d_t = 1$	327.35	57.75	9.89	97.5	93.2	86.3	55.6	50.0	45.0
"2"	$d_t = 2$	80.48	11.27	1.97	95.8	88.1	77.9	54.9	48.9	40.9
"3"	$d_t > 2$	99.99	10.11	0.94	93.7	81.3	52.5	61.0	50.4	39.2
Spread group: large										
"1"	$d_t = 1$	561.97	61.63	12.96	99.8	97.2	92.1	55.4	50.4	42.5
"2"	$d_t = 2$	174.53	17.44	2.89	99.7	93.9	82.7	55.8	50.0	39.6
"3"	$d_t = 3, 4$	162.94	16.73	1.51	99.3	89.6	68.4	56.3	49.9	41.1
"4"	$d_t > 4$	147.06	17.34	0.64	98.0	82.0	46.4	77.4	52.0	41.7

Table 4

Definitions of LOB control variables hidden orders on the buy side

"Aggressive limit orders" are defined as limit orders undercutting the prevailing best quote. "Fleeting orders" are defined as limit orders that are canceled within one second after the submission.

SPR	$\equiv$ log(best ask/best bid)
DPS	$\equiv$ log(depth at best bid (same side of the market))
DPO	$\equiv$ log(depth at best ask (opposite side of the market))
TYP	$\equiv$ 1 if the prevailing trade is seller-initiated; -1 otherwise
RET	$\equiv$ log return over the prevailing 5 minutes
VOL	$\equiv$ market price range (maximum - minimum) over the prevailing 5 minutes
HVS	$\equiv$ log(1+volume of executed hidden bid depth during the prevailing 1 minute)
HVS_5	$\equiv$ log(1+volume of executed hidden bid depth during the prevailing 5 minutes)
HRS	$\equiv HVS - HVS_5$
HVO	$\equiv$ log(1+volume of executed hidden ask depth during the prevailing 1 minute)
HVO_5	$\equiv$ log(1+volume of executed hidden ask depth during the prevailing 5 minutes)
HRO	$\equiv HVO - HVO_5$
ALS	$\equiv$ log(1+ number of aggressive buy limit orders that are not canceled during the prevailing 3 minutes)
ALO	$\equiv$ log(1+ number of aggressive sell limit orders that are not canceled during the prevailing 3 minutes)
HFS	$\equiv$ log(1+ number of fleeting buy orders during the prevailing 3 minutes)
HFO	$\equiv$ log(1+ number of fleeting sell orders during the prevailing 3 minutes)
OPN	$\equiv$ 1 trading before $10:30$; 0, otherwise.
CLS	$\equiv$ 1 trading after $15:00$; 0, otherwise.

for large-spread stocks. Secondly, more than 90% of all observations are censored in the sense of reflecting only an upper bound of aggressiveness of hidden depth. Thirdly, the number of observations on the buy and sell side are very similar.

Finally, note that we label the underlying categories in a consistent way with the least aggressive categories being associated with the highest label and the most aggressive category being associated with the lowest label. Hence, hidden order aggressiveness declines with category labels.

3.5 Capturing Market Conditions

To test our postulated hypotheses and to relate the usage of undisclosed orders to prevailing market conditions, we construct different variables representing various states of the market. Table 4 gives the exact definitions of constructed variables used for hidden order submissions on the buy side. For statistical inference on the sell side, we modify some of the variables as follows:

DPS $\equiv$ log(depth at best ask)

DPO $\equiv$ log(depth at best bid)

TYP $\equiv$ 1 if the prevailing trade is buyer-initiated; -1 otherwise

RET $\equiv$ negative log return over the prevailing 5 minutes

HVS $\equiv$ log(1+volume of executed hidden ask depth during the prevailing 1 minute)

HVS_5 $\equiv$ log(1+volume of executed hidden ask depth during the prevailing 5 minutes)

HVO $\equiv$ log(1+volume of executed hidden bid depth during the prevailing 1 minute)

HVO_5 $\equiv$ log(1+volume of executed hidden bid depth during the prevailing 5 minutes)

ALS $\equiv$ log(1+ number of aggressive sell limit orders that are not canceled during the prevailing 3 minutes)

ALO $\equiv$ log(1+ number of aggressive buy limit orders that are not canceled during the prevailing 3 minutes)

HFS $\equiv$ log(1+ number of fleeting sell orders during the prevailing 3 minutes)

HFO $\equiv$ log(1+ number of fleeting buy orders during the prevailing 3 minutes)

The prevailing LOB state is represented by the visible bid-ask spread (SPR), reflecting the (displayed) transaction costs of immediate trading, the visible depth on the best level on the *same* side (DPS) and the visible depth on the best level on the *opposite* side (DPO). To capture the impact of prevailing trade signals, we include a dummy variable (TYP) representing the most recent trading direction and the prevailing five-minute mid-quote return (RET) capturing short-term price movements. Moreover, local price volatility (VOL) is included in terms of the (max/min) range of trade prices during the last 5 minutes.

Information on prevailing hidden depth is incorporated by the short-run executed hidden depth on the own side and the opposite side (HVS, HVO), representing how successfully traders detected pending hidden depth. To capture temporal effects, we also compute the executed hidden depth during the last minute relative to that executed during the last five minutes (HRS, HRO). Moreover, HFT activities are captured by two variables, HFS and HFO, which are the number of fleeting orders on the own side and the opposite side, respectively. Defined as in Hasbrouck and Saar (2009), a "fleeting order" is a limit order that is canceled within one second after the submission and thus is posted to "test" for the existence of hidden volume within the spread. Using the intensity of fleeting orders as a proxy for HFT activities is inspired by Hendershortt, Jones, and Menkveld (2010). To differentiate between fleeting orders and "normal" limit orders, we also include the number of aggressive limit orders that have *not* been canceled (ALS, ALO) and thus represent the frequency of quote updating by low frequency traders. Finally, OPN and CLS are dummy variables representing the opening and closure period. To be able to aggregate estimates across the market, all variables (expect for dummies, i.e., TYP, OPN and CLS) are normalized to have zero mean and unit standard deviation.

4 Econometric Modelling

The chosen categorizations straightforwardly motivate modeling hidden order locations based on an ordered response model. This has several advantages: Firstly, censored observations are straightforwardly taken into account. Secondly, relating market variables (as constructed in the previous section) to order categories rather than to plain distances s and d, requires imposing less assumptions on functional form (e.g., linearity) and allows reducing the impact of extreme observations (e.g., executions against hidden depth deeply in the book). Thirdly, given the high number of observations (combined with a significant cross-sectional dimension), a reduction of the computational burden is crucial to make the approach tractable. In fact, exploiting the Gaussianity and global concavity of objective functions in an ordered probit model allows to significantly reduce computation time in contrast to, for instance, a (censored) count data model (e.g., negative binomial model) for the variables s or d.

Therefore, we propose modelling hidden order placements using a censored ordered probit model. In order to test our hypotheses, it is sufficient to utilize only order messages which provide information (censored or non-censored) on the location of undisclosed volume. Consequently, the model is not estimated based on the continuous time series of *all* order book messages but only based on those observations revealing some information on hidden order locations. Moreover, we do not require a dynamic (e.g., autoregressive) approach as all information on the current and prevailing state of the market is captured by corresponding regressors.

4.1 An Ordered Probit Model with Censoring

Let y_t denote the discrete ordered label representing the underlying categories of undisclosed order placements as described in Section 3.4. It is driven by a continuous latent variable y_t^* with the link function given by

$$y_t = \begin{cases} 1, & \text{if } y_t^* \leq \gamma_1, \\ 2, & \text{if } \gamma_1 < y_t^* \leq \gamma_2, \\ \vdots & \\ J-1, & \text{if } \gamma_{J-2} < y_t^* \leq \gamma_{J-1}, \\ J, & \text{if } \gamma_{J-1} < y_t^*, \end{cases} \tag{1}$$

where J is the number of categories and γ_j, $j = 1, \ldots, J-1$, denote unknown thresholds. Furthermore, y_t^* is given by

$$y_t^* = \boldsymbol{\beta}'\boldsymbol{x}_t + \varepsilon_t \tag{2}$$

with $\boldsymbol{x}_t$ being a $(K \times 1)$ vector of regressors as defined in Section 3.5, $\boldsymbol{\beta}$ is a vector of unknown parameters and ε_t denotes an i.i.d. standard normally distributed variable. If the response variable y_t is observed (i.e., in the case of non-censoring), the likelihood function is given by

$$L_t^{\mathrm{U}} = \begin{cases} \Phi(\gamma_1 - \boldsymbol{\beta}'\boldsymbol{x}_t) & \text{if } y_t = 1, \\ \Phi(\gamma_j - \boldsymbol{\beta}'\boldsymbol{x}_t) - \Phi(\gamma_{j-1} - \boldsymbol{\beta}'\boldsymbol{x}_t) & \text{if } y_t \in \{2, \ldots, j, \ldots, J-1\}, \\ 1 - \Phi(\gamma_{J-1} - \boldsymbol{\beta}'\boldsymbol{x}_t) & \text{if } y_t = J, \end{cases} \tag{3}$$

where $\Phi(\cdot)$ denotes the cumulative density function of the standard normal distribution. In cases, where y_t is not directly observable but only a censored outcome $\widetilde{y}_t$ linked to y_t (according to the scenarios described in Figure 3 and 4) by

$$\widetilde{y}_t = j, \text{ if } y_t \in \{j+1, \ldots, J\}, \tag{4}$$

the likelihood function is given by

$$L_t^{\mathrm{C}} = \begin{cases} 1 - \Phi(\gamma_j - \boldsymbol{\beta}'\boldsymbol{x}) & \text{if } \tilde{y}_t = j \text{ and } j = 1, \ldots, J-2, \\ 1 - \Phi(\gamma_{J-1} - \boldsymbol{\beta}'\boldsymbol{x}) & \text{if } \tilde{y}_t \geq J-1. \end{cases} \tag{5}$$

Then, the resulting log likelihood function is given by

$$l = \sum_{t \in \zeta^{\mathrm{U}}} \log(L_t^{\mathrm{U}}) + \sum_{t \in \zeta^{\mathrm{C}}} \log(L_t^{\mathrm{C}}), \tag{6}$$

where ζ^{U} and ζ^{C} denote the index sets of uncensored and censored observations, respectively.

The marginal effects of the regressors are straightforwardly computed by

$$
\begin{aligned}
\boldsymbol{q}_1 &= \frac{\partial F_1}{\partial \boldsymbol{x}} = -\phi(\gamma_1 - \boldsymbol{\beta}'\boldsymbol{x})\boldsymbol{\beta}, \\
\boldsymbol{q}_2 &= \frac{\partial F_2}{\partial \boldsymbol{x}} = -(\phi(\gamma_2 - \boldsymbol{\beta}'\boldsymbol{x}) - \phi(\gamma_1 - \boldsymbol{\beta}'\boldsymbol{x}))\boldsymbol{\beta}, \\
&\vdots \\
\boldsymbol{q}_J &= \frac{\partial F_J}{\partial \boldsymbol{x}} = \phi(\gamma_{J-1} - \boldsymbol{\beta}'\boldsymbol{x})\boldsymbol{\beta},
\end{aligned}
\tag{7}
$$

which are commonly evaluated at the sample mean $\bar{\boldsymbol{x}}$.

In case of the dummy variables x_d, we calculate the marginal effects as

$$
\Delta F_j = \mathbb{P}(y = j|\boldsymbol{x}_{(x_d=1)}, \boldsymbol{\gamma}, \boldsymbol{\beta}) - \mathbb{P}(y = j|\boldsymbol{x}_{(x_d=0)}, \boldsymbol{\gamma}, \boldsymbol{\beta}), \tag{8}
$$

where $\boldsymbol{x}_{(x_d=i)}$ is a vector with the dummy variable x_d set to i and all other elements being equal to $\boldsymbol{x}$. Appendix A gives the asymptotic distribution of the maximum likelihood estimators $\widehat{q}_j$ and $\Delta\widehat{F}_j$.

4.2 Cross-Sectional Aggregation

We estimate the econometric model on a stock-by-stock basis. For the sake of brevity and ease of presentation, we aggregate the corresponding estimates across stocks. To explicitly account for differences in estimation precision, we assess the cross-sectional statistical significance relying on a Bayesian framework attributable to DuMouchel (1994) and implemented by Bessembinder, Panayides, and Venkataraman (2009). Assume that a parameter estimate associated with stock i, $\hat{\beta}_i$, is normally distributed with

$$
\widehat{\beta}_i \sim i.i.d.N(\beta_i, s_i^2)
$$

and

$$
\beta_i \sim i.i.d.N(\beta, \sigma^2),
$$

where s_i^2 is the estimated variance of parameter i and the variance σ^2 estimated by maximum likelihood. Then, the aggregated estimate β is obtained by summing up the weighted estimates for all stocks as

$$
\widehat{\beta} = \sum_{i=1}^{N} w_i \widehat{\beta}_i, \quad w_i = \frac{(s_i^2 + \hat{\sigma}^2)^{-1}}{\sum_{j=1}^{N}(s_j^2 + \hat{\sigma}^2)^{-1}}. \tag{9}
$$

Assuming independence across stocks, the variance of the aggregated estimate is given by

$$
\mathrm{V}(\hat{\beta}) = \frac{1}{\sum_{j=1}^{N}(s_j^2 + \hat{\sigma}^2)^{-1}}.
$$

5 Empirical Evidence

We estimate separate models for both ask and bid hidden orders for categorizations based on both distance measures s and d. Covering 99 stocks over the cross-section of the market, we estimate 396 models in total. Table 5 presents the ordered probit estimates aggregated across all stocks. Recall that lower category labels are associated with a higher hidden order aggressiveness, thus *negative* coefficients reflect that undisclosed orders are set (marginally) *deeper* in the spread. To assess the explanatory power, we report the pseudo-R^2 proposed by McKelvey and Zaviona (1975),

$$R^2_{MZ} = \frac{\sum_{t=1}^{T}(\widehat{y}_t^* - \bar{y}^*)^2}{\sum_{t=1}^{T}(\widehat{y}_t^* - \bar{y}^*)^2 + T}, \tag{10}$$

where $\widehat{y}_t^*$ is the fitted value of the latent variable y_t^* and $\bar{y}^* = 1/T \sum_{t=1}^{T} \widehat{y}_t^*$.

To provide also insights into the cross-sectional variation of estimates we show histograms of the significant estimates (5% significance level) in Figures 10 to 13 in Appendix B. Note that the Bayesian cross-sectional aggregates, as discussed in Section 4.2, are generally close to the averages of significant estimates as these estimates get more weight in eq. (9). Finally, (Bayesian averaged) estimates of marginal effects for the individual groups of low-, medium- and large-spread stocks are given in Tables 6 and 7.

Below we will discuss the individual results in light of the testable hypotheses formulated in Section 2.3. As estimates of parameters and marginal effects are not always straightforward to interpret, we partly illustrate the resulting effects graphically. For the sake of brevity, we will discuss the findings for undisclosed orders on the buy (bid) side only. The corresponding effects on the ask side are closely symmetric.

5.1 Hidden Order Placements in Dependence of Spread Sizes

We find that the size of the (displayed) bid-ask spread (SPR) has a significant impact on the probability of hidden order placements inside of the spread. However, there are fundamental differences between small-spread stocks and large-spread stocks. As indicated by the marginal effects, for small-spread stocks, a widening of the spread significantly reduces the aggressiveness of hidden order placements within the spread. In particular, for small-spread stocks, one standard deviation spread increase implies a decrease of the probability of hidden depth inside of the spread (category $y = 1$) by approximately 6.7%. Hence, the high discreteness of price grids inside of the spread seem to cause an "overshooting" of aggressiveness when spreads are particularly small. As argued in Section 2.3 and supporting hypothesis (1), in this situation, order submitters are forced to post hidden orders deeper in the spread as it would be necessary in case of a more continuous grid. For large-spread stocks, we find similar effects though they are

Table 5

Ordered probit estimates

Ordered probit estimates of hidden order locations on the bid and ask side depending on categorized distance measures s and d as discussed in Section 3.4. The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. Reported estimates and t-statistics (in parentheses) are cross-sectional aggregates across all stocks using the Bayesian framework of DuMouchel (1994). The reported R^2 is McKelvey and Zaviona's (1975) pseudo R^2.

	Undisclosed bid limit orders				Undisclosed ask limit orders			
	Neg. distance s		Distance d		Neg. distance s		Distance d	
SPR	0.04	(1.1)	1.37	(29.0)	0.06	(1.7)	1.34	(32.2)
DPS	−0.13	(−7.2)	0.01	(2.0)	−0.12	(−7.0)	0.02	(2.5)
DPO	0.00	(0.1)	−0.06	(−6.5)	−0.01	(−0.9)	−0.06	(−6.6)
TYP	0.06	(4.7)	0.10	(7.1)	0.07	(4.9)	0.11	(7.9)
RET	−0.09	(−11.0)	−0.10	(−11.0)	−0.08	(−12.9)	0.10	(−12.2)
VOL	0.01	(1.7)	0.01	(1.9)	0.03	(2.6)	0.02	(2.4)
HVS	−0.40	(−28.3)	−0.36	(−32.9)	−0.43	(−27.4)	−0.39	(−35.7)
HRS	0.17	(14.7)	0.14	(15.7)	0.18	(17.4)	0.16	(16.4)
HVO	−0.01	(−0.9)	−0.02	(−3.2)	−0.00	(−0.3)	−0.01	(−1.1)
HRO	0.03	(4.8)	0.03	(6.6)	0.03	(4.4)	0.03	(4.5)
ALS	0.04	(6.1)	0.01	(2.4)	0.04	(8.4)	0.02	(3.0)
ALO	0.03	(5.2)	0.02	(3.7)	0.04	(7.3)	0.03	(5.0)
HFS	0.07	(6.7)	0.03	(2.6)	0.08	(6.6)	0.03	(2.7)
HFO	0.15	(10.6)	0.11	(11.1)	0.18	(14.2)	0.15	(14.5)
OPN	−0.01	(−0.3)	0.11	(2.8)	−0.04	(−1.0)	0.07	(1.8)
CLS	0.14	(5.7)	0.17	(5.9)	0.13	(5.2)	0.14	(4.7)
Pseudo-R^2	0.29		0.67		0.31		0.68	

less distinct than in the small-spread case. Indeed, we observe partly opposite marginal effects based on both distance measures s and d. These are explained by the fact that the underlying aggressiveness categories are partly overlapping. In fact, "translating" the estimated marginal effects in Tables 6 and 7 into a graphical illustration results in Figure 7 showing the effects of a widening of the bid-ask spread on a hypothetical hidden order location distribution. According to the estimated marginal effects, we observe that a widening of the spread leads to a stronger cumulation of hidden volume *inside* of the spread but simultaneously relatively close to the *own side*. At the same time, the distance between hidden orders and the opposite side of the market increases as the spread widens. Hence, traders use hidden orders to compete for the provision of liquidity with own-side liquidity suppliers (and thus to increase execution probabilities) while still balancing adverse selection risks by remaining sufficiently "passive". Observing similar effects on the opposite side of the market, we can conclude that a widening of the spread leads to a U-shaped concentration of hidden depth inside of the

Table 6

Marginal effects: Aggressiveness of undisclosed orders according to their distance to the own side (distance measure s as shown in Section 3.4)

The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. Reported estimates are cross-sectional aggregates using the Bayesian framework of DuMouchel (1994) for the underlying groups of small-, medium- and large-spread stocks. The marginal effects are evaluated at the sample mean. Significant estimates (5% level) are highlighted in boldfat. All values are given in percentages.

	Small spread		Medium spread			Large spread			
	$\mathbb{P}[y=1]$	$\mathbb{P}[y=2]$	$\mathbb{P}[y=1]$	$\mathbb{P}[y=2]$	$\mathbb{P}[y=3]$	$\mathbb{P}[y=1]$	$\mathbb{P}[y=2]$	$\mathbb{P}[y=3]$	$\mathbb{P}[y=4]$
Panel A: Undisclosed buy limit orders									
SPR	**-6.68**	**6.76**	0.22	0.44	-0.69	**0.21**	**4.76**	**2.06**	**-7.30**
DPS	**5.54**	**-5.68**	**0.32**	**0.65**	**-1.02**	**0.10**	**1.39**	**0.50**	**-2.30**
DPO	-0.23	0.46	-0.05	-0.04	0.07	-0.02	**-0.54**	**-0.20**	**0.87**
TYP	**-2.10**	**2.24**	**-0.15**	**-0.38**	**0.60**	**0.06**	**1.01**	0.15	**-1.40**
RET	**0.66**	**-0.78**	**0.37**	**0.76**	**-1.17**	**0.11**	**1.82**	**0.64**	**-2.86**
VOL	0.04	-0.03	-0.07	-0.19	0.32	-0.01	-0.32	-0.02	0.37
HVS	**5.41**	**-5.64**	**2.02**	**4.04**	**-6.28**	**0.39**	**5.97**	**2.96**	**-9.63**
HRS	**-1.04**	**1.10**	**-0.82**	**-1.76**	**2.75**	**-0.15**	**-2.47**	**-1.41**	**4.35**
HVO	**-0.52**	**0.80**	0.06	**0.21**	**-0.32**	0.04	1.11	0.26	**-1.50**
HRO	-0.13	0.13	**-0.09**	**-0.20**	**0.34**	-0.07	**-1.00**	**-0.27**	**1.54**
ALS	-0.17	0.18	**-0.14**	**-0.32**	**0.51**	-0.04	**-0.52**	-0.05	**0.94**
ALO	**-0.64**	**0.69**	**-0.14**	**-0.29**	**0.45**	-0.01	-0.14	-0.05	0.24
HFS	**-0.67**	**0.69**	**-0.22**	**-0.61**	**1.01**	-0.04	-0.74	-0.16	1.15
HFO	**-0.74**	**0.83**	**-0.73**	**-1.55**	**2.39**	**-0.20**	**-3.09**	**-1.37**	**5.09**
OPN	0.51	**-0.63**	0.01	0.11	-0.21	0.02	0.17	-1.43	1.44
CLS	0.13	-0.19	**-0.86**	**-1.79**	**2.79**	**-0.09**	**-2.22**	**-1.89**	**4.91**
Panel B: Undisclosed sell limit orders									
SPR	**-6.01**	**6.05**	0.09	0.40	-0.49	**0.25**	**4.28**	**1.07**	**-5.76**
DPS	**5.34**	**-5.45**	**0.35**	**0.75**	**-1.20**	**0.10**	**1.45**	**0.54**	**-2.26**
DPO	-0.74	0.76	0.06	0.20	-0.31	**-0.04**	**-0.61**	-0.14	**0.87**
TYP	**-2.63**	**2.92**	**-0.23**	**-0.52**	**0.95**	**0.10**	**1.28**	**0.20**	**-1.66**
RET	**0.50**	**-0.52**	**0.39**	**0.82**	**-1.25**	**0.11**	**1.58**	**0.47**	**-2.39**
VOL	-0.22	0.24	-0.13	-0.31	0.49	-0.00	-0.11	0.07	0.04
HVS	**5.68**	**-5.86**	**2.49**	**4.78**	**-7.47**	**0.44**	**6.61**	**2.92**	**-10.26**
HRS	**-1.19**	**1.21**	**-0.98**	**-1.95**	**3.13**	**-0.17**	**-2.84**	**-1.52**	**4.72**
HVO	**-0.28**	**0.30**	0.02	0.04	-0.06	0.05	0.91	0.15	**-1.25**
HRO	-0.11	0.11	**-0.14**	**-0.31**	**0.46**	**-0.05**	**-0.79**	-0.15	**1.14**
ALS	-0.17	0.17	**-0.25**	**-0.49**	**0.81**	**-0.01**	**-0.66**	**-0.26**	**1.13**
ALO	**-0.59**	**0.65**	**-0.05**	**-0.21**	**0.30**	-0.00	**-0.49**	**-0.15**	**0.77**
HFS	**-0.66**	**0.68**	**-0.20**	**-0.52**	**0.81**	-0.04	**-1.31**	**-0.54**	**2.29**
HFO	**-1.04**	**1.09**	**-0.97**	**-2.01**	**3.11**	**-0.24**	**-3.49**	**-1.49**	**5.57**
OPN	**0.82**	**-0.86**	-0.52	-0.84	1.02	0.10	1.87	0.60	-2.93
CLS	0.34	-0.41	**-1.01**	**-2.13**	**3.35**	**-0.13**	**-2.24**	**-0.95**	**3.76**

Table 7
Marginal effects: Aggressiveness of undisclosed orders according to their distance to the opposite side (distance measure d as shown in Section 3.4)

The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. Reported estimates are cross-sectional aggregates using the Bayesian framework of DuMouchel (1994) for the underlying groups of small-, medium- and large-spread stocks. The marginal effects are evaluated at the sample mean. Significant estimates (5% level) are highlighted in boldfat. All values are given in percentages.

	Small spread		Medium spread			Large spread			
	$\mathbb{P}[y=1]$	$\mathbb{P}[y=2]$	$\mathbb{P}[y=1]$	$\mathbb{P}[y=2]$	$\mathbb{P}[y=3]$	$\mathbb{P}[y=1]$	$\mathbb{P}[y=2]$	$\mathbb{P}[y=3]$	$\mathbb{P}[y=4]$
Panel A: Undisclosed buy limit orders									
SPR	**-27.33**	**28.35**	**-4.10**	**-8.93**	**13.46**	**-0.20**	**-5.06**	**-5.55**	**11.42**
DPS	**-0.11**	**0.11**	-0.01	-0.04	0.06	-0.00	0.00	0.00	-0.00
DPO	**1.08**	**-1.22**	**0.10**	**0.20**	**-0.33**	0.00	-0.02	-0.03	0.04
TYP	**-1.47**	**4.34**	**-0.25**	**-0.54**	**0.85**	-0.01	0.02	0.12	-0.12
RET	0.05	-0.07	**0.29**	**0.64**	**-0.98**	0.06	**0.59**	**0.68**	**-1.35**
VOL	0.09	-0.09	-0.01	0.01	-0.02	-0.01	-0.00	-0.00	0.00
HVS	**6.88**	**-6.89**	**1.26**	**2.58**	**-3.92**	**0.08**	**1.26**	**1.45**	**-2.92**
HRS	**-0.38**	**0.51**	**-0.44**	**-0.92**	**1.46**	-0.00	**-0.38**	**-0.46**	**1.04**
HVO	0.03	-0.04	0.05	**0.16**	**-0.24**	0.02	**0.15**	**0.21**	**-0.42**
HRO	-0.06	0.06	**-0.07**	**-0.16**	**0.25**	-0.01	**-0.08**	**-0.14**	**0.28**
ALS	-0.00	0.02	-0.05	-0.03	0.05	-0.00	0.00	-0.01	0.02
ALO	**-0.26**	**0.33**	**-0.03**	**-0.12**	**0.16**	0.01	0.03	0.04	-0.10
HFS	**-0.18**	**0.19**	-0.04	**-0.15**	**0.22**	-0.00	-0.00	-0.00	-0.27
HFO	**-0.28**	**0.31**	**-0.38**	**-0.85**	**1.32**	-0.06	**-0.27**	**-0.39**	**0.84**
OPN	**0.33**	**-0.37**	0.05	0.42	-0.73	**-0.08**	**-1.79**	**-1.78**	**3.91**
CLS	0.09	-0.10	**-0.26**	**-0.60**	**0.98**	**-0.02**	**-1.22**	**-1.36**	**2.82**
Panel B: Undisclosed sell limit orders									
SPR	**-25.00**	**27.46**	**-4.44**	**-8.56**	**13.55**	**-0.31**	**-5.67**	**-6.28**	**13.07**
DPS	**-0.27**	**0.27**	-0.01	-0.06	0.09	0.00	-0.01	-0.04	0.04
DPO	**1.02**	**-1.11**	**0.15**	**0.25**	**-0.41**	-0.00	-0.03	-0.05	0.05
TYP	**-1.78**	**2.34**	**-0.27**	**-0.62**	**1.07**	0.00	0.06	0.06	-0.08
RET	**0.29**	**-0.31**	**0.25**	**0.60**	**-0.92**	**0.01**	**0.57**	**0.62**	**-1.30**
VOL	-0.03	0.08	-0.01	-0.07	0.09	0.00	-0.00	-0.00	0.01
HVS	**6.38**	**-6.71**	**1.61**	**2.81**	**-4.61**	**0.03**	**1.64**	**1.68**	**-3.74**
HRS	**-0.81**	**0.85**	**-0.49**	**-1.02**	**1.66**	**-0.00**	**-0.45**	**-0.55**	**1.26**
HVO	-0.03	0.03	0.02	0.05	-0.07	-0.00	0.09	0.13	-0.22
HRO	-0.05	0.06	**-0.06**	**-0.16**	**0.23**	-0.02	-0.11	-0.16	0.30
ALS	-0.02	0.02	-0.01	-0.07	**0.11**	-0.00	-0.04	-0.08	0.15
ALO	**-0.24**	**0.28**	-0.01	**-0.10**	**0.14**	-0.00	-0.01	-0.03	0.05
HFS	**-0.31**	**0.33**	-0.05	-0.15	**0.21**	0.00	0.06	0.03	-0.13
HFO	**-0.58**	**0.64**	**-0.60**	**-1.11**	**1.82**	**-0.00**	**-0.60**	**-0.64**	**1.50**
OPN	**0.49**	**-0.66**	0.27	0.58	-1.05	**-0.02**	-0.63	-0.82	1.47
CLS	0.27	-0.30	-0.21	**-0.44**	0.74	**-0.02**	**-1.02**	**-1.19**	**2.55**

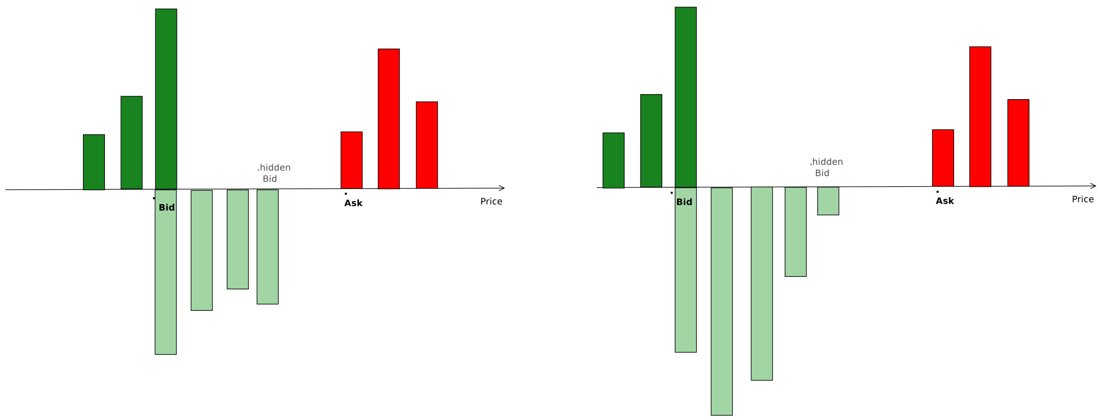

Figure 7: Stylized illustration of the effect of a widening of bid-ask spreads on hidden order placements for large-spread stocks. **Left:** scenario of a narrow spread; **right:** scenario of a wide spread. This illustration shows the effect of an increasing hidden order aggressiveness in terms of the distance to the same-side quote, coming along with a decreasing hidden order aggressiveness in terms of the distance to the opposite-side quote.

spread and thus a positive correlation between displayed spread and hidden spread.

5.2 (How) Does Hidden Liquidity Compete with Visible Liquidity Provision?

Our estimates associated with DPS indicate that the probability of hidden depth inside of the spread is positively related to the own side visible depth. Hence, we find a clear confirmation of Hypothesis (2.A) in the sense that same-side (visible) liquidity triggers competition for hidden liquidity supply. In particular, according to the estimates in Table 6, the probability of using aggressive hidden bid limit orders increases by approximately 5.5% as the visible depth at the best bid increases by one standard deviation. Confirming Buti and Rindi (2011) traders obviously increase their aggressiveness in order to compete for the provision of liquidity and thus to increase execution probabilities. This is particularly true for large-spread stocks offering sufficient room for undercutting (or overbidding, respectively) prevailing displayed quotes within the spread. In the case of small-spread stocks we observe partly contradicting marginal effects based on the two underlying distance measures. The resulting effects of own-side depth increases are similar to the effects illustrated in Figure 7 for spread increases. Hence, in case of small spreads offering not much room for (hidden) quote improvements, a higher (visible) own-side depth does not necessarily lead to hidden depth locations deeper in the spread but rather to a stronger clustering of hidden orders close to the own-side quote. This is still in line with Hypothesis (2.A).

Hypothesis (2.B) is only confirmed in the case of large-spread stocks. We observe

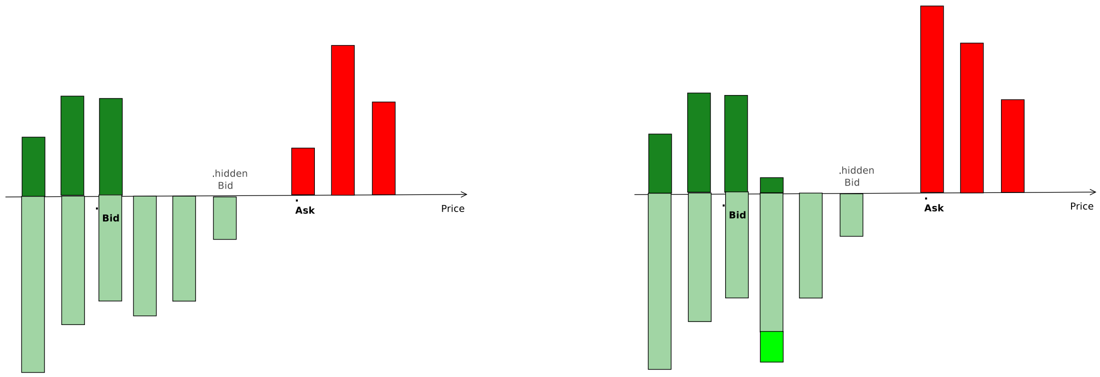

Figure 8: Stylized illustration of the effect of an increase of visible ask depth on undisclosed buy order placements for large-spread stocks. **Left:** low visible depth ; **right:** high visible depth. This illustration shows the effect of a decreasing hidden order aggressiveness in terms of the distance to the opposite-side quote, coming along with no significant effects on the hidden order aggressiveness in terms of the distance to the same-side quote.

that only the effects based on the distance s are significant while their counterparts based on d are insignificant. As illustrated by Figure 8, this may be induced by the hidden depth distribution shifting to the own side and/or the use of reserve orders rather than hidden orders. This effect is in line with the notion that hidden liquidity suppliers aim at reducing adverse selection risk if the price pressure on the opposite side becomes too high. The finding is in line with Buti and Rindi (2011)'s theoretical prediction that an increase of opposite-side visible depth triggers an increased use of reserve orders instead of hidden orders. Pardo Tornero and Pascual (2007) and De Winne and D'Hondt (2007) find similar evidence for the Spanish Stock Exchange and Euronext Paris where, however, only reserve orders but not hidden orders can be used.

In case of medium-spread and small-spread stocks, we observe converse effects with rising hidden aggressiveness if the opposite-side depth is increased. "Translating" the estimated marginal effects into a graphical illustration according to Figure 9 shows an increasing use of reserve orders and a rising hidden order aggressiveness. In this situation, adverse selection risk is obviously less dominant and seems to be overcompensated by liquidity suppliers' incentive to increase execution probabilities.

5.3 Hidden Order Placements After Price Movements and Trading Signals

We show that the aggressiveness of hidden bid depth decreases when the prevailing trade is seller-initiated (TYP). In particular, the hidden bid depth shifts away from the ask side. This reduces liquidity suppliers' risk of being picked off by (eventually better

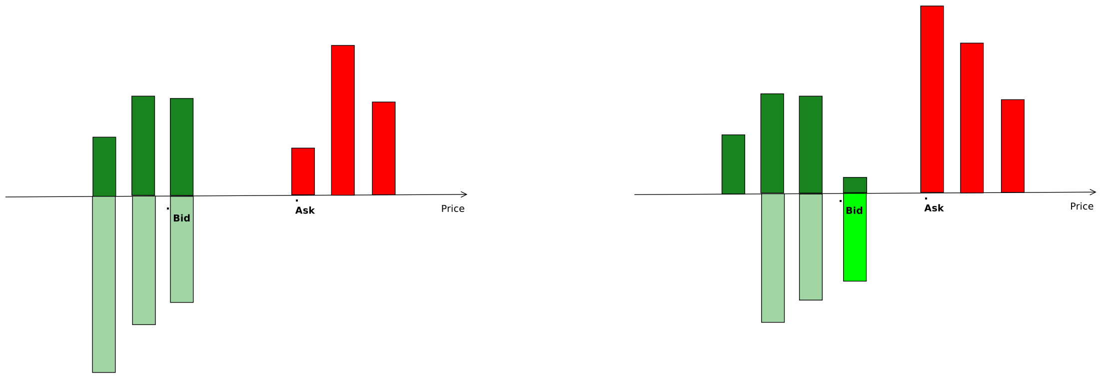

Figure 9: Stylized illustration of the effect of an increase of visible ask depth on undisclosed buy order placements for medium-spread stocks. **Left:** low visible depth ; **right:** high visible depth. This illustration shows the effect of increasing hidden order aggressiveness in terms of the distance to the opposite-side quote, coming along with no significant effects on the hidden order aggressiveness in terms of the distance to the same-side quote.

informed) sellers but increases their risk of non-execution. Conversely, in case of a buy market order, hidden liquidity supply on the bid side increases and moves toward the ask side. Hence, liquidity suppliers follow trading directions in the sense that they post more aggressively and thus increase execution probabilities without facing too high adverse selection risk (as long as buy pressure dominates).

In this sense, Hypothesis (3.A) is confirmed. Besides economic reasoning, a pure mechanical effect may further drive the results. In particular, as a sell trade itself absorbs pending aggressive undisclosed buy limit orders, the aggressiveness of hidden bid depth temporarily decreases. This effect, however, is only true in case of trades arriving *instantaneously* before the observation of interest. But as our estimates utilize all order messages revealing information on hidden orders (occurring on average 30 times more frequently than trades), these mechanical effects apply only infrequently.

Analyzing the effects of recent price movements (RET) on hidden order placements, we find similar effects and supportive evidence in favor of Hypothesis (3.B). Accordingly, the aggressiveness of undisclosed bid orders increases as prices have been moved upwards. Specifically, the probability of hidden orders inside of the spread increases by approximately 2.9% when the return increases by one standard deviation. Moreover, the estimates in Table 5 show that hidden bid depth moves closer to the ask side. Again, this supports liquidity suppliers' motivation to reduce the risk of non-execution. Conversely, in case of prevailing negative price movements, hidden liquidity placements on the bid side become less aggressive with the hidden depth distribution shifting away from the ask side. As postulated in Section 2.3, this is explained by protection against picking-off risks in case prices continue moving downwards.

Interestingly, no clear confirmation of Hypothesis (4) is found. We do not find significant impacts of prevailing return volatility. According to our estimates, hidden order aggressiveness even tends to increase in volatile market periods. However, in most cases, these effects are insignificant.

5.4 Competition for Hidden Liquidity Provision

Our estimates show clear evidence for competition between hidden liquidity providers. According to the estimates associated with the effects of own-side hidden liquidity supply (HVS), we support Hypothesis (5). In particular, Table 6 reports that the probability of hidden bid depth inside of the spread increases by approximately 10% as the execution of hidden bid volume during the last minute increases by one standard deviation. This effect is supported by the estimates in Table 7 indicating that hidden liquidity shifts closer to the opposite side of the market. Hence, according to the reasoning motivating Hypothesis (5), liquidity suppliers are encouraged to provide further hidden volume if they realize liquidity demand from the opposite side and competition on their own side.

According to Buti and Rindi (2011) these effects prevail as long as adverse selection risk does not become too high. Indeed, Testing this hypothesis by controlling for prevailing hidden depth execution relative to that during the last five minutes (HRS), we find negative effects. In particular, hidden order aggressiveness tends to decline if hidden depth demand becomes extraordinarily high. In this situation, price pressure from the opposite side becomes too strong and makes adverse selection risk too high.

Studying the effect of hidden order detections on the *opposite* side of the market, we find slight evidence for the effect that trading against hidden sell orders also increases the hidden order aggressiveness on the bid side. This might be explained by the fact that buy market orders make buy hidden orders (relatively) less aggressive and move away hidden ask quotes. This, in turn, gives hidden liquidity suppliers on the bid side more room for quote improvements and thus the reduction of non-execution risks.

5.5 Hidden Order Placements and HFT

Analyzing the effects of HFT (approximated by the intensity of fleeting orders) on hidden order submissions, we find strong empirical support for Hypothesis (6). Indeed, the more opposite-side traders try to detect hidden liquidity by "pinging activities" (HFO), the lower is the hidden order aggressiveness. Especially for large-spread stocks, an one-standard-deviation increase of HFT activities on the ask side implies a decrease of the probability of hidden bid depth placements inside of the spread by more than 5%. Consequently, the distribution of hidden depth moves away from the opposite quote. Hence, liquidity suppliers interpret the rapid cancellations of limit orders as signals for hidden liquidity detection strategies rather than true liquidity supply. These results are in line with empirical evidence reported by Hasbrouck and Saar (2009) and the predictions by Buti and Rindi (2011) showing that hidden order placements become non-attractive if hidden depth is easily detected.

Note that the effects on the distribution of the entire hidden depth, i.e., also that *behind* the market (as reported in Table 7) are substantially smaller than those on hidden depth inside of the spread (as revealed by Table 6). This finding also supports the theoretical prediction by Buti and Rindi (2011) that reserve orders, rather than hidden orders, are dominantly used when parasitic traders utilize front running strategies.

5.6 Intraday Patterns

We find no clear confirmation of Hypothesis (7.A) postulating a higher hidden order aggressiveness during or after the opening period. Actually, our findings for small-spread stocks support the hypothesis, while it is rejected for large-spread stocks. However, clear evidence for Hypothesis (7.B) is shown. Indeed, for large-spread stocks, we find that the probability for hidden bid depth placements within the spread in the hour before market closure is approximately 5% lower than during the rest of the day. This supports the economic reasoning that displayed orders are preferred if the time horizon becomes shorter and the importance of time priority rises.

6 Conclusions

Many stock exchanges around the world choose to reduce market transparency by allowing traders to hide a portion of their order size. As a consequence, trading under limited pre-trade transparency becomes increasingly popular in financial markets. Previous studies in the literature examine opaque markets with only partially undisclosed orders. This study sheds light on traders' use of completely undisclosed orders in electronic trading, based on a sample of 99 stocks traded on NASDAQ during October 2010.

Employing NASDAQ TotalView message data, we retrieve information on hidden depth from visible order activities and propose an ordered response approach with censoring mechanism for modelling hidden order locations conditional on the state of market. Our findings show that hidden liquidity supply is significantly correlated with market conditions and thus is predictable in terms of the state of the prevailing (visible) LOB and order flow. Our empirical evidence is in line with theoretical predictions and suggests that hidden liquidity suppliers post orders by strategically balancing non-execution risk and picking-off risk. We show the following effects: First, hidden spreads are positively correlated with observable spreads. Second, hidden liquidity competes with displayed liquidity and hidden liquidity on the own side of the market. Third, hidden order placements follow recent price movements and trading signals. Fourth, hidden order placement is reduced if hidden order execution strategies are prevailing.

Our findings might serve as valuable input to calibrate and further develop theoretical models. The proposed empirical model can be extended in various directions yielding an even more precise assessment of hidden order placement.

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A Asymptotic Distribution of Marginal Dummy Effects

The asymptotic covariance of marginal dummy effects is straightforwardly computed using the delta method. Let $\boldsymbol{\theta} = [\boldsymbol{\beta}', \gamma_1, \ldots, \gamma_{J-1}]'$ be the vector of $(K+J-1)$ unknown parameters, $\widehat{\boldsymbol{\theta}}$ denotes the maximum likelihood estimator with $V \equiv \text{Asy. Var}[\widehat{\boldsymbol{\theta}}]$ being its $(K+J-1) \times (K+J-1)$ asymptotic covariance matrix. Then, the asymptotic covariance matrix of the corresponding marginal effects is given by

$$\text{Asy. Var}[\widehat{\boldsymbol{q}}_j] = \left[\frac{\partial \widehat{\boldsymbol{q}}_j}{\partial \widehat{\boldsymbol{\theta}}'}\right] V \left[\frac{\partial \widehat{\boldsymbol{q}}_j}{\partial \widehat{\boldsymbol{\theta}}'}\right]', \tag{A.1}$$

where the derivatives of $\widehat{\boldsymbol{q}}_j$ with respect to $\widehat{\boldsymbol{\theta}}'$ are

$$\begin{aligned}
\frac{\partial \widehat{\boldsymbol{q}}_1}{\partial \widehat{\boldsymbol{\theta}}'} &= \Big[\phi_1(I_K - z_1\widehat{\boldsymbol{\beta}}\boldsymbol{x}'),\ \phi_1 z_1 \widehat{\boldsymbol{\beta}},\ \mathbf{0},\ \cdots,\ \mathbf{0}\Big], \\
\frac{\partial \widehat{\boldsymbol{q}}_2}{\partial \widehat{\boldsymbol{\theta}}'} &= \Big[(\phi_1 - \phi_2)I_K + (\phi_1 z_1 - \phi_2 z_2)\widehat{\boldsymbol{\beta}}\boldsymbol{x}',\ -\phi_1 z_1 \widehat{\boldsymbol{\beta}},\ \phi_2 z_2 \widehat{\boldsymbol{\beta}},\ \cdots,\ \mathbf{0}\Big], \\
&\ \vdots \\
\frac{\partial \widehat{\boldsymbol{q}}_J}{\partial \widehat{\boldsymbol{\theta}}'} &= \Big[\phi_{J-1}(I_K + z_{J-1}\widehat{\boldsymbol{\beta}}\boldsymbol{x}'),\ \mathbf{0},\ \mathbf{0},\ \cdots,\ -\phi_{J-1} z_{J-1} \widehat{\boldsymbol{\beta}}\Big],
\end{aligned} \tag{A.2}$$

with I_K denoting a $K \times K$ identity matrix, $\mathbf{0}$ is a $(K \times 1)$ zero vector and z_j and ϕ_j given by $z_j \equiv \gamma_j - \widehat{\boldsymbol{\beta}}'\boldsymbol{x}$ and $\phi_j \equiv \phi(z_j)$. Then,

$$\text{Asy. Var}[\Delta \widehat{F}_j] = \left[\frac{\partial \Delta \widehat{F}_j}{\partial \widehat{\boldsymbol{\theta}}'}\right] V \left[\frac{\partial \Delta \widehat{F}_j}{\partial \widehat{\boldsymbol{\theta}}'}\right]', \tag{A.3}$$

where

$$\frac{\partial \Delta \widehat{F}_j}{\partial \widehat{\boldsymbol{\theta}}'} = \left.\frac{\partial \widehat{F}_j}{\partial \widehat{\boldsymbol{\theta}}'}\right|_{\boldsymbol{x}(x_d=1)} - \left.\frac{\partial \widehat{F}_j}{\partial \widehat{\boldsymbol{\theta}}'}\right|_{\boldsymbol{x}(x_d=0)}$$

with

$$\begin{aligned}
\frac{\partial \widehat{F}_1}{\partial \widehat{\boldsymbol{\theta}}'} &= \big[\phi_1 \boldsymbol{x}',\ \phi_1,\ \mathbf{0},\ \cdots,\ \mathbf{0}\big], \\
\frac{\partial \widehat{F}_2}{\partial \widehat{\boldsymbol{\theta}}'} &= \big[(\phi_2 - \phi_1)\boldsymbol{x}',\ -\phi_1,\ \phi_2,\ \cdots,\ \mathbf{0}\big], \\
&\ \vdots \\
\frac{\partial \widehat{F}_J}{\partial \widehat{\boldsymbol{\theta}}'} &= \big[\phi_{J-1} \boldsymbol{x}',\ \mathbf{0},\ \mathbf{0},\ \cdots,\ -\phi_{J-1}\big].
\end{aligned}$$

B Histograms of Significant Ordered Probit Estimates

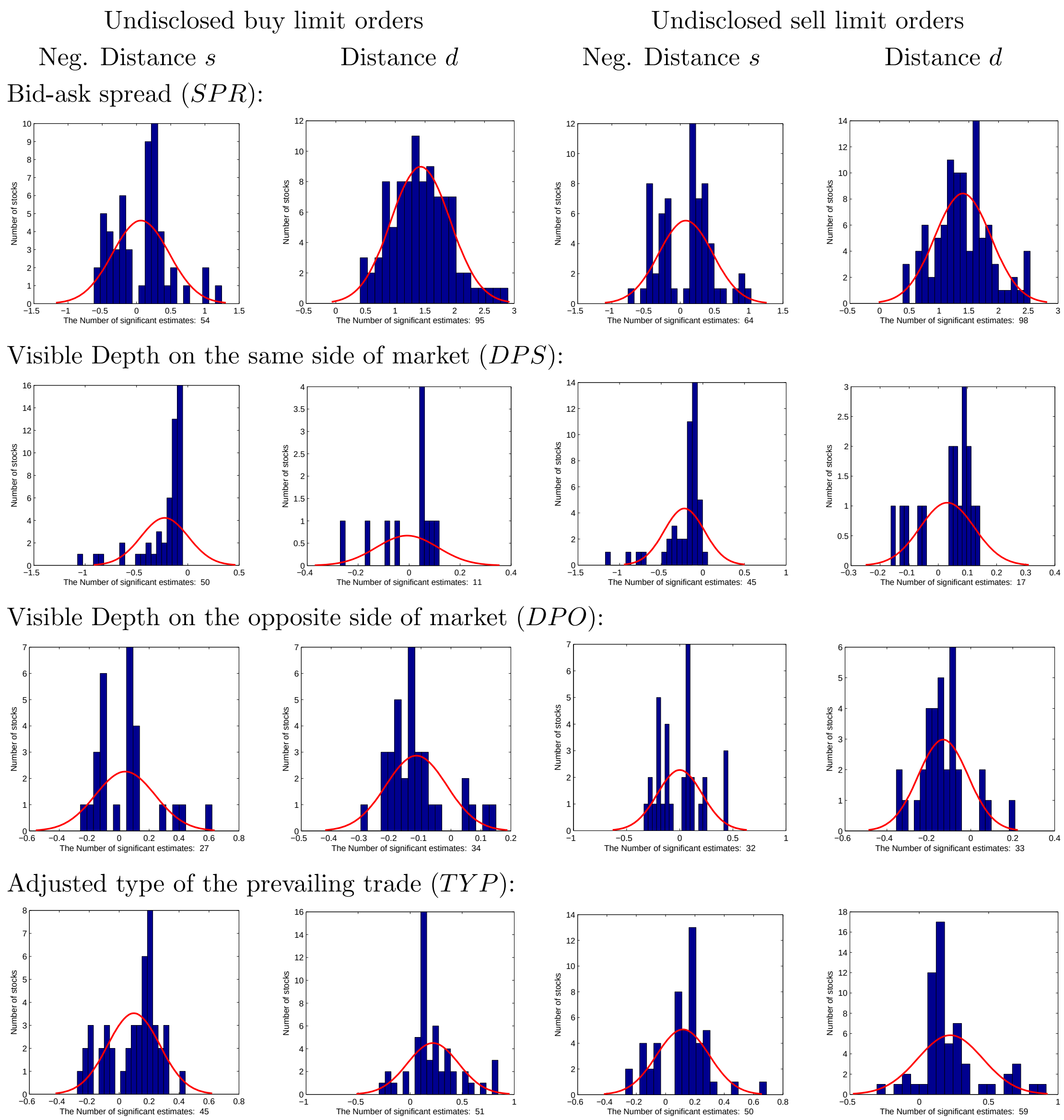


Figure 10: Histogram of significant estimates. Ordered probit estimates of hidden order locations on the bid and ask side depending on categorized measures s and d as presented in Section 3.4. The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. The histogram shows estimates which significantly different from zero on the 5%-level. TYP is adjusted such that it equals 1 whenever the prevailing trade consumes the own-side liquidity, -1 otherwise.

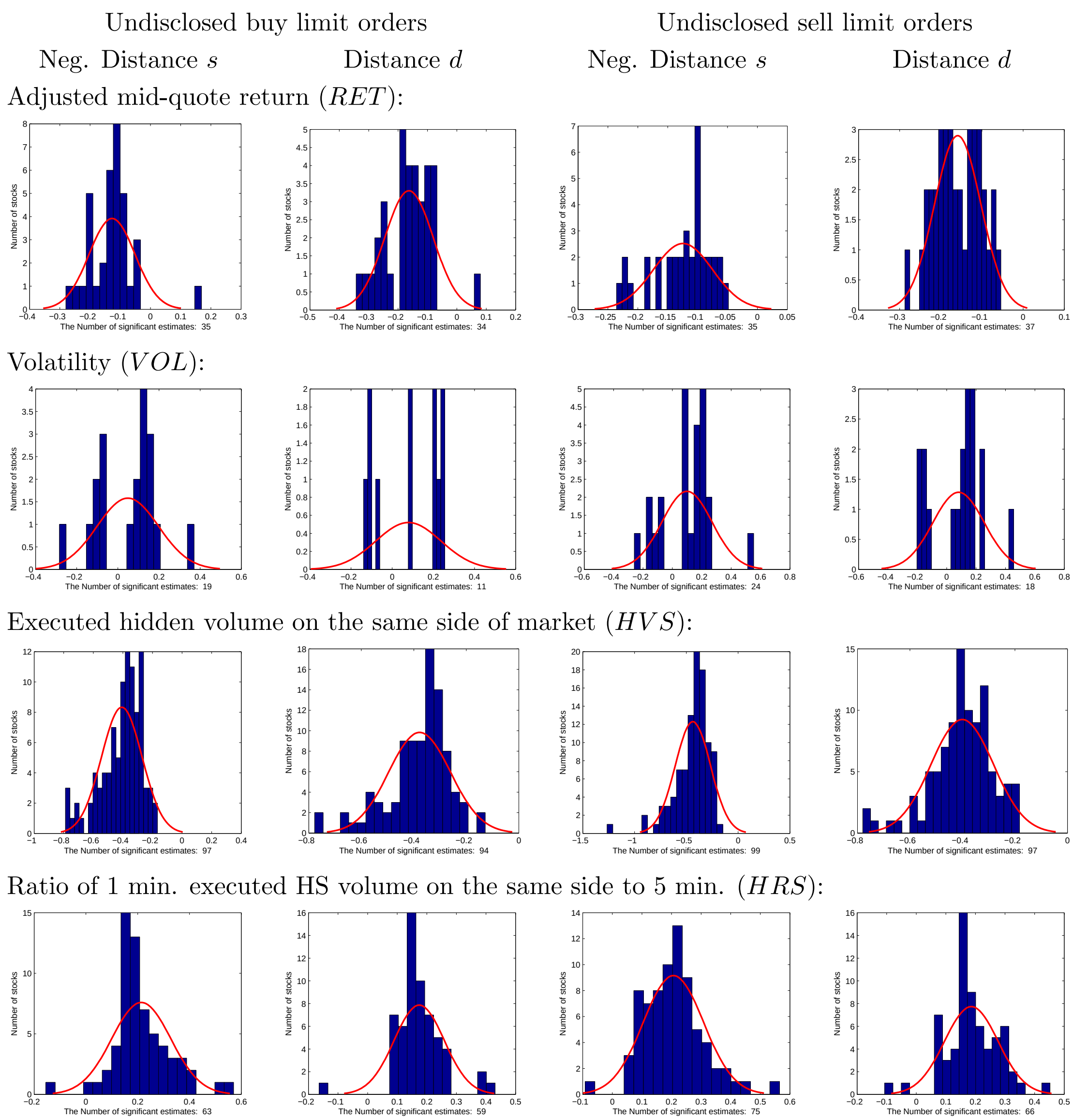
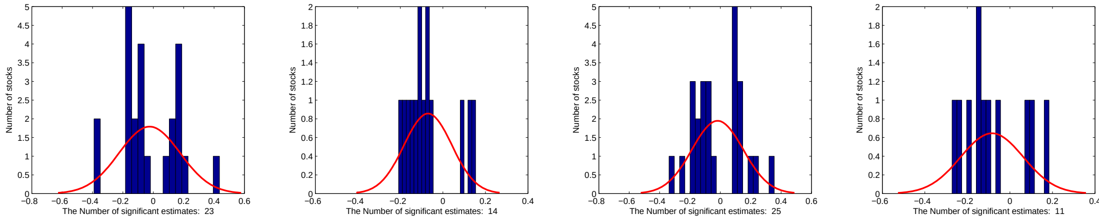


Figure 11: Histogram of significant estimates. Ordered probit estimates of hidden order locations on the bid and ask side depending on categorized measures s and d as presented in Section 3.4. The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. The histogram shows estimates which are significantly different from zero at 5%-level. RET is adjusted such that it is positive whenever mid-quotes move away from the own side of the market.

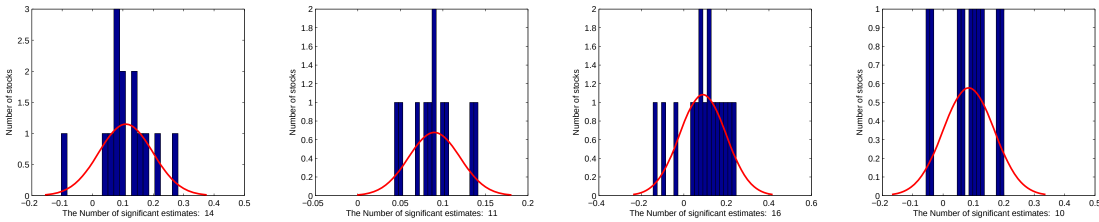
Undisclosed buy limit orders | Undisclosed sell limit orders

Neg. Distance s | Distance d | Neg. Distance s | Distance d

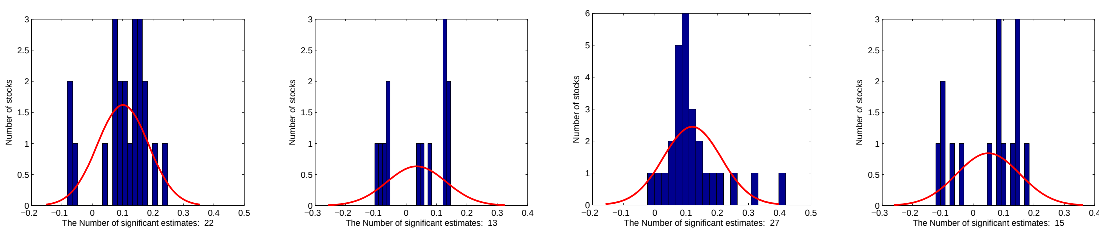
Executed hidden volume on the opposite side of market (HVO)



Ratio of 1 min. executed HO volume on the opposite side to 5 min. (HRO):



Number of "low frequency" limit order updating the same-side quote (ALS)



Number of "low frequency" limit order updating the opposite-side quote (ALO):

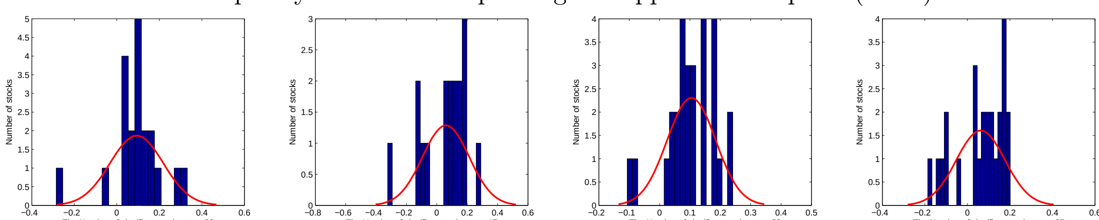


Figure 12: Histogram of significant estimates. Ordered probit estimates of hidden order locations on the bid and ask side depending on categorized measures s and d as presented in Section 3.4. The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 corresponding to 21 trading days. The histogram shows estimates which are significantly different from zero at 5%-level. "Low frequency" limit orders are defined as orders submitted during the prevailing 3 minutes and have not been canceled yet.

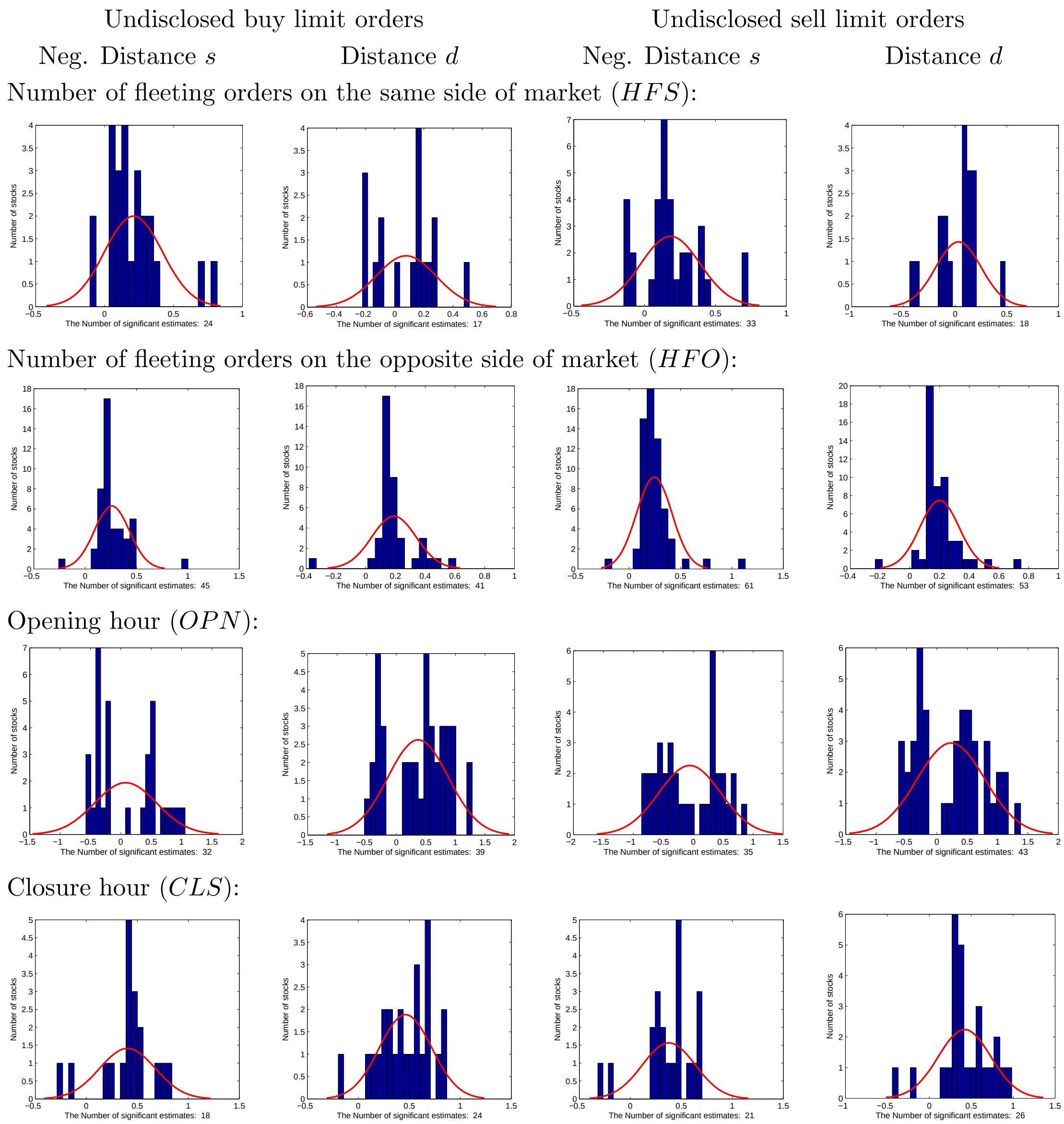


Figure 13: Histogram of significant estimates. Ordered probit estimates of hidden order locations on the bid and ask side depending on categorized measures s and d as discussed in Section 3.4. The order aggressiveness is declining with the category label, thus negative coefficients are associated with increasing aggressiveness. Based on 99 NASDAQ stocks during October 2010 correspondint to 21 trading days. The histogram shows estimates which are significantly different from zero at 5%-level. A fleeting order is defined as a limit order canceled at latest one second after the submission.

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