SFB 649 Discussion Paper 2012-033

Simultaneous Statistical Inference in Dynamic Factor Models

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin

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April 30, 2012

Based on the theory of multiple statistical hypothesis testing, we elaborate simultaneous statistical inference methods in dynamic factor models. In particular, we employ structural properties of multivariate chi-squared distributions in order to construct critical regions for vectors of likelihood ratio statistics in such models. In this, we make use of the asymptotic distribution of the vector of test statistics for large sample sizes, assuming that the model is identified and model restrictions are testable. Examples of important multiple test problems in dynamic factor models demonstrate the relevance of the proposed methods for practical applications.

MSC 2010 classification numbers: Primary 62J15, 62M10; secondary 62M07, 62F05, 62F03.
JEL Classification: C12, C32, C52.
Key words: family-wise error rate, false discovery rate, likelihood ratio statistic, multiple hypothesis testing, multivariate chi-squared distribution, time series regression, Wald statistic.

1. DYNAMIC FACTOR MODELS

Dynamic factor models are multivariate time series models of the form

$$\mathbf{X}(t) = \sum_{s=-\infty}^{\infty} \Lambda(s)\, \mathbf{f}(t-s) + \varepsilon(t),\ 1 \leq t \leq T. \tag{1.1}$$

Thereby, $\mathbf{X} = (\mathbf{X}(t) : 1 \leq t \leq T)$ denotes a p-dimensional, covariance-stationary stochastic process in discrete time with mean zero, $\mathbf{f}(t) = (f_1(t), \ldots, f_k(t))^\top$ with $k < p$ denotes a k-dimensional vector of so-called "common factors" and $\varepsilon(t) = (\varepsilon_1(t), \ldots, \varepsilon_p(t))^\top$ denotes a p-dimensional vector of "specific factors", to be regarded as error or remainder terms. Both $\mathbf{f}(t)$ and $\varepsilon(t)$ are assumed to be centered and the error terms are modeled as noise in the sense that they are mutually uncorrelated at every time point and, in addition, uncorrelated with $\mathbf{f}(t)$ at all leads and lags. The error terms $\varepsilon(t)$ may, however, exhibit non-trivial (weak) serial autocorrelations. Processes with the latter property are occasionally referred to as "approximate" factor models in contrast to "strict" factor models where also the serial autocovariance matrix of the specific factors is assumed to be strictly diagonal. We will refer to T as the sample size.

The underlying interpretation of model (1.1) is that the dynamic behavior of the process $\mathbf{X}$ can already be described well (or completely) by a lower-dimensional "latent" process. The entry (i, j) of the matrix $\Lambda(s)$ quantitatively reflects the influence of the j-th common factor at lead or lag s, respectively, on the i-th component of $\mathbf{X}(t)$, where $1 \leq i \leq p$ and $1 \leq j \leq k$. Recently, Park et al. (2009) studied the case where factor loadings may depend on covariates and discussed applications in economics and neuroimaging.

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A special case of model (1.1), sometimes termed "conventional" factor model, results if the influence of the common factors on $\mathbf{X}$ is itself without dynamics, i. e., if the model simplifies to

$$\mathbf{X}(t) = \Lambda\,\mathbf{f}(t) + \varepsilon(t),\ 1 \leq t \leq T. \tag{1.2}$$

Peña and Box (1987) were concerned with methods for the determination of the (number of) common factors in a factor model of the form (1.2) and derived a canonical transformation allowing a parsimonious representation of $\mathbf{X}(t)$ in (1.2) in terms of the common factors. Statistical inference in conventional factor models has been studied, for instance, by Jöreskog (1969). For further references and developments regarding the theory of conventional and dynamic factor models we defer the reader to Breitung and Eickmeier (2005).

Statistical inference methods for dynamic factor models typically consider the time series in the frequency domain, cf., among others, Forni et al. (2000, 2009) and references therein, and analyze spectral decompositions of the autocovariance matrix of $\mathbf{X}$. Along similar lines, Geweke and Singleton (1981) developed a framework for statistical inference in dynamic factor models based on the likelihood principle by making use of central limit theorems for time series regression in the frequency domain by Hannan (1973). Their inferential considerations rely on the asymptotic normality of the maximum likelihood estimator $\hat{\vartheta}$ of the (possibly very high-dimensional) parameter vector ϑ in the resulting representation of the model. We will provide more details in Section 4. To this end, it is essential that the time series model (1.1) is identified in the sense of Geweke and Singleton (1981), which we will assume throughout the paper. If the model is not identified, the individual contributions of the common factors cannot be expressed unambiguously and, consequently, testing for significance or the construction of confidence sets for elements of ϑ is obviously not informative.

In the present work, we will extend the methodology by Geweke and Singleton (1981). Specifically, we will be concerned with simultaneous statistical inference in dynamic factor models under the likelihood framework by considering multiple test procedures for positively dependent test statistics, in our case likelihood ratio statistics (or, asymptotically equivalently, Wald statistics). The paper is organized as follows. In Section 2, we provide a brief introduction to multiple testing, especially under positive dependence. Section 3 is devoted to the analysis of structural properties of multivariate chi-squared distributions and a numerical assessment of type I error control for multiple tests with multivariate chi-square distributed test statistics. Finally, Section 4 exemplifies important simultaneous inference problems for dynamic factor models of the form (1.1). We conclude with a discussion in Section 5.

2. MULTIPLE TESTING UNDER POSITIVE DEPENDENCE

The general setup of multiple testing theory assumes a statistical model $(\Omega, \mathcal{F}, (\mathbb{P}_\vartheta)_{\vartheta\in\Theta})$ parametrized by $\vartheta \in \Theta$ and is concerned with testing a family $\mathcal{H} = (H_i, i \in I)$ of hypotheses regarding the parameter ϑ with corresponding alternatives $K_i = \Theta \setminus H_i$, where I denotes an arbitrary index set. We identify hypotheses with subsets of the parameter space throughout the paper. Let $\varphi = (\varphi_i, i \in I)$ a multiple test procedure for $\mathcal{H}$, meaning that each component φ_i, $i \in I$ is a (marginal) test for the test problem H_i versus K_i in the classical sense. Moreover, let $I_0 \equiv I_0(\vartheta) \subseteq I$ denote the index set of true hypotheses in $\mathcal{H}$ and $V(\varphi)$ the number of false rejections (type I errors) of φ, i. e., $V(\varphi) = \sum_{i\in I_0} \varphi_i$. The classical multiple type I error measure in multiple hypothesis testing is the family-wise error rate, FWER for short, and can (for a given $\vartheta \in \Theta$) be expressed as $\mathrm{FWER}_\vartheta(\varphi) = \mathbb{P}_\vartheta(V(\varphi) > 0)$. The multiple test φ is said to control the FWER at a pre-defined significance level α, if $\sup_{\vartheta\in\Theta} \mathrm{FWER}_\vartheta(\varphi) \leq \alpha$. A simple, but often conservative method for FWER control is based on the union bound and is referred to as Bonferroni correction in the multiple testing literature. Assuming that $|I| = m$, the Bonferroni correction carries out each individual test $\varphi_i, i \in I$, at (local) level α/m. The "Bonferroni test" $\varphi = (\varphi_i, i \in I)$ then controls the FWER. In case that joint independence of all m marginal test statistics can be assumed, the Bonferroni-corrected level α/m can be enlarged to the "Šidák-corrected" level $1-(1-\alpha)^{1/m} > \alpha/m$ leading to slightly more powerful (marginal) tests. Both the Bonferroni and the Šidák test are single-step procedures, meaning that the same local significance level is used for all m marginal tests.

An interesting other class of multiple test procedures are stepwise rejective tests, in particular step-up-down tests, introduced by Tamhane et al. (1998). They are most conveniently described in terms of p-values $p_1, \ldots, p_m$ corresponding to test statistics $T_1, \ldots, T_m$. It goes beyond the scope of this paper to discuss the notion of p-values in depth. Therefore, we will restrict attention to the case that every individual null hypothesis is simple, the distribution of every T_i, $1 \leq i \leq m$, under H_i is continuous and each T_i tends to larger values under alternatives. The test statistics considered in Section 4 fulfill these requirements, at least asymptotically. Then, we can calculate (observed) p-values by $p_i = 1-F_i(t_i)$, $1 \leq i \leq m$, where F_i is the cumulative distribution function (cdf) of T_i under H_i and t_i denotes the observed value of T_i. The transformation with the upper tail cdf brings all test statistics to a common scale, because each p-value is supported on $[0,1]$. Small p-values are in favor of the corresponding alternatives.

Definition 1 (Step-up-down test of order λ in terms of p-values, cf. Finner et al., 2012)**.** *Let $p_{1:m} < p_{2:m} < \ldots < p_{m:m}$ denote the ordered p-values for a multiple test problem. For a tuning parameter $\lambda \in \{1, \ldots, m\}$ a step-up-down test $\varphi^\lambda = (\varphi_1, \ldots, \varphi_m)$ (say) of order λ based on some critical values $\alpha_{1:m} \leq \cdots \leq \alpha_{m:m}$ is defined as follows. If $p_{\lambda:m} \leq \alpha_{\lambda:m}$, set $j* = \max\{j \in \{\lambda, \ldots, m\} : p_{i:m} \leq \alpha_{i:m} \text{ for all } i \in \{\lambda, \ldots, j\}\}$, whereas for $p_{\lambda:m} > \alpha_{\lambda:m}$, put $j* = \sup\{j \in \{1, \ldots, \lambda-1\} : p_{j:m} \leq \alpha_{j:m}\}$ $(\sup\emptyset = -\infty)$. Define $\varphi_i = 1$ if $p_i \leq \alpha_{j*:m}$ and $\varphi_i = 0$ otherwise $(\alpha_{-\infty:m} = -\infty)$.*

A step-up-down test of order $\lambda = 1$ or $\lambda = n$, respectively, is called step-down (SD) or step-up (SU) test, respectively. If all critical values are identical, we obtain a single-step test.

In connection with control of the FWER, SD tests play a pivotal role, because they can often be considered a shortcut of a closed test procedure, cf. Marcus et al. (1976). For example, the famous SD procedure of Holm (1979) employing critical values $\alpha_{i:m} = \alpha/(m-i+1)$, $1 \leq i \leq m$ is, under the assumption of a complete system of hypotheses, a shortcut of the closed Bonferroni test, see, for instance, Sonnemann (2008), and hence controls the FWER at level α.

In order to compare concurring multiple test procedures, also a type II error measure or, equivalently, a notion of power is required under the multiple testing framework. To this end, we define $I_1 \equiv I_1(\vartheta) = I \setminus I_0$, $m_1 = |I_1|$, $S(\varphi) = \sum_{i\in I_1} \varphi_i$ and refer to the expected proportion of correctly detected alternatives, i. e., $\mathrm{power}_\vartheta(\varphi) = \mathbb{E}_\vartheta[S(\varphi)/\max(m_1, 1)]$, as the multiple power of φ under ϑ. If the structure of φ is such that $\varphi_i = \mathbf{1}_{p_i \leq t^*}$ for a common, possibly data-dependent threshold t^*, then the multiple power of φ is isotone in t^*. For step-up-down tests, this entails that index-wise larger critical values lead to higher multiple power.

Gain in multiple power under the constraint of FWER control is only possible if certain structural assumptions for the joint distribution of $(p_1, \ldots, p_m)$ or, equivalently, $(T_1, \ldots, T_m)$ can be established, cf. Example 1 below. In particular, positive dependency among the $(p_1, \ldots, p_m)$ in the sense of multivariate total positivity of order 2 (MTP_2, see Karlin and Rinott, 1980) or positive regression dependency on subsets (PRDS, see Benjamini and Yekutieli, 2001) allows to enlarge the critical values $(\alpha_{i:m})_{1 \leq i \leq m}$. To give a specific example, Sarkar (1998) proved that the critical values $\alpha_{i:m} = i\alpha/m$, $1 \leq i \leq m$ can be used as the basis for an FWER-controlling closed test procedure, provided that the joint distribution of p-values is MTP_2. These critical values have originally been proposed by Simes (1986) in connection with a global test for the intersection hypothesis $H_0 = \bigcap_{i=1}^m H_i$ and are therefore often referred to as Simes' critical values. Hommel (1988) worked out a shortcut for the aforementioned closed test procedure based on Simes' critical values; we will refer to this multiple test as $\varphi^{\mathrm{Hommel}}$ in the remainder of this work.

Simes' critical values also play an important role in connection with control of the false discovery rate (FDR). The FDR is a relaxed type I error measure suitable for large systems of hypotheses. Formally, it is defined as $\mathrm{FDR}_\vartheta(\varphi) = \mathbb{E}_\vartheta[\mathrm{FDP}(\varphi)]$, where $\mathrm{FDP}(\varphi) = V(\varphi)/\max(R(\varphi), 1)$ with $R(\varphi) = V(\varphi) + S(\varphi)$ denoting the total number of rejections of φ under ϑ. The random variable $\mathrm{FDP}(\varphi)$ is called the false discovery proportion. The meanwhile classical linear step-up test by Benjamini and Hochberg (1995), φ^{LSU} (say), is an SU test with Simes' critical values. Under joint independence of all p-values, it provides FDR-control at (exact) level $m_0\alpha/m$, where $m_0 = m - m_1$, see, for instance, Finner et al. (2009). Independently of each other, Benjamini and Yekutieli (2001) and Sarkar (2002) proved that $\sup_{\vartheta \in \Theta} \mathrm{FDR}_\vartheta(\varphi^{\mathrm{LSU}}) \leq m_0\alpha/m$ if the joint distribution of $(p_1, \ldots, p_m)$ is PRDS on I_0 (notice that MTP_2 implies PRDS on any subset).

3. MULTIVARIATE CHI-SQUARED DISTRIBUTIONS

In order to formalize inference for several likelihood ratio statistics simultaneously, we have to generalize the definition of the multivariate chi-squared distribution as given in Definition 3.5.7 of Timm (2002) to allow for possibly different degrees of freedom in each marginal.

Definition 2 (Generalized multivariate chi-squared distribution)**.**
Let $m \geq 2$ and $\vec{\nu} = (\nu_1, \ldots, \nu_m)^\top \in \mathbb{N}^m$. Let $\mathbf{Z}_1 = (Z_{1,1}, \ldots, Z_{1,\nu_1})^\top$, $\mathbf{Z}_2 = (Z_{2,1}, \ldots, Z_{2,\nu_2})^\top, \ldots,$ $\mathbf{Z}_m = (Z_{m,1}, \ldots, Z_{m,\nu_m})^\top$ denote m vectors of standard normal variates with joint correlation matrix $R = (\rho(Z_{k_1,\ell_1}, Z_{k_2,\ell_2}) : 1 \leq k_1, k_2 \leq m, 1 \leq \ell_1 \leq \nu_{k_1}, 1 \leq \ell_2 \leq \nu_{k_2})$ such that for any $1 \leq k \leq m$ the variates $Z_{k,1}, \ldots, Z_{k,\nu_k}$ are jointly stochastically independent. Let $\mathbf{Q} = (Q_1, \ldots, Q_m)^\top$, where for all $1 \leq k \leq m : Q_k = \sum_{\ell=1}^{\nu_k} Z_{k,\ell}^2$. Then we call the distribution of $\mathbf{Q}$ a generalized multivariate (central) chi-squared distribution with parameters m, $\vec{\nu}$ and R and write $\mathbf{Q} \sim \chi^2(m, \vec{\nu}, R)$.

The following lemma shows that among the components of a generalized multivariate chi-squared distribution only non-negative correlations can occur.

Lemma 1. *Let $\mathbf{Q} \sim \chi^2(m, \vec{\nu}, R)$. Then, for any pair of indices $1 \leq k_1, k_2 \leq m$ it holds*

$$0 \leq \mathit{Cov}(Q_{k_1}, Q_{k_2}) \leq 2\sqrt{\nu_{k_1}\nu_{k_2}}. \tag{3.1}$$

Proof. Without loss of generality, assume $k_1 = 1$ and $k_2 = 2$. Simple probabilistic calculus now yields

$$\begin{aligned}\mathrm{Cov}(Q_1, Q_2) &= \mathrm{Cov}\left(\sum_{i=1}^{\nu_1} Z_{1,i}^2, \sum_{j=1}^{\nu_2} Z_{2,j}^2\right) \\ &= \sum_{i=1}^{\nu_1}\sum_{j=1}^{\nu_2} \mathrm{Cov}(Z_{1,i}^2, Z_{2,j}^2) = 2\sum_{i=1}^{\nu_1}\sum_{j=1}^{\nu_2} \rho^2(Z_{1,i}, Z_{2,j}) \geq 0.\end{aligned}$$

The upper bound in (3.1) follows directly from the Cauchy-Schwarz inequality, because the variance of a chi-squared distributed random variable with ν degrees of freedom equals 2ν. □

In view of the applicability of multiple test procedures for positively dependent test statistics that have been discussed in Section 2, Lemma 1 points into the right direction. However, unfortunately, pairwise positive correlations are not sufficient to prove the MTP_2 property (see, for instance, Example 3.2. in Karlin and Rinott, 1980). In fact, the MTP_2 property for multivariate chi-squared or, more generally, multivariate gamma distributions could up to now only be proved for special cases as, for example, exchangeable gamma variates (Example 3.5. in Karlin and Rinott (1980), see also Sarkar and Chang (1997) for applications of this type of multivariate gamma distributions in multiple hypothesis testing).

Therefore, we conducted an extensive simulation study of FWER and FDR control of multiple tests suitable under MTP_2 (or PRDS) in the case that the vector of test statistics follows a generalized multivariate chi-squared distribution. Specifically, we investigated the shortcut test φ^{Hommel} for control of the FWER and the linear step-up test φ^{LSU} for control of the FDR and considered the following correlation structures among the variates $(Z_{k,\ell^*} : 1 \leq k \leq m)$ for any given $1 \leq \ell^* \leq \max\{\nu_k : 1 \leq k \leq m\}$. (Since only the coefficients of determination enter the correlation structure of the resulting chi-square variates, we restricted our attention to positive correlation coefficients among the $Z_{k,\ell}$.)

1. Autoregressive, AR(1): $\rho_{ij} = \rho^{|i-j|}$, $\rho \in \{0.1, 0.25, 0.5, 0.75, 0.9\}$.
2. Compound symmetry (CS): $\rho_{ij} = \rho + (1-\rho)\mathbf{1}_{\{i=j\}}$, $\rho \in \{0.1, 0.25, 0.5, 0.75, 0.9\}$.
3. Toeplitz: $\rho_{ij} = \rho_{|i-j|+1}$, with $\rho_1 \equiv 1$ and $\rho_2, ..., \rho_{m^*}$ randomly drawn from the interval $[0.1, 0.9]$.
4. Unstructured (UN): The ρ_{ij} are elements of a normalized realization of a Wishart-distributed random matrix with m degrees of freedom and diagonal expectation the elements of which were randomly drawn from $[0.1, 0.9]^m$.

In all four cases, we have $\rho_{ij} = \mathrm{Cov}(Z_{i,\ell^*}, Z_{j,\ell^*})$, $1 \leq i, j \leq m^*$, where $m^* = |\{1 \leq k \leq m : \nu_k \geq \ell^*\}|$. The marginal degrees of freedom $(\nu_k : 1 \leq k \leq m)$ have been drawn randomly from the set $\{1, 2, \ldots, 100\}$ for every simulation setup. In this, we chose decreasing sampling probabilities of the form $\gamma/(\nu + 1)$, $1 \leq \nu \leq 100$, where γ denotes the norming constant, because we were most interested in the small-scale behavior of φ^{Hommel} and φ^{LSU} under dependency. For the number of marginal test statistics, we considered $m \in \{2, 5, 10, 50, 100\}$ and for the number of true hypotheses the respective values of m_0 provided in Tables 1 - 4. For all false hypotheses, we set the corresponding p-values to zero, because the resulting so-called "Dirac-uniform configurations" are assumed to be least favorable for φ^{Hommel} and φ^{LSU}, see, for instance, Finner et al. (2009) and Blanchard et al. (2011). For every simulation setup, we performed $M = 1,000$ Monte Carlo repetitions of the respective multiple test procedures and estimated the FWER or FDR, respectively, by relative frequencies or means, respectively. We present our results in Tables 1 - 4 in the appendix.

Remark 1. *For carrying out these large-scale simulation studies efficiently, we made use of the simulation platform provided by the μTOSS software for multiple hypothesis testing, see Blanchard et al. (2010).*

To summarize our findings, φ^{Hommel} behaved remarkably well over the entire range of simulation setups. Only in a few cases, it violated the target FWER level slightly, but one has to keep in mind that Dirac-uniform configurations correspond to extreme deviations from the null hypotheses which are not expected to be encountered in practical applications.

In line with the results by Benjamini and Yekutieli (2001) and Sarkar (2002), φ^{LSU} appeared to be extremely conservative for small values of m_0 (notice the factor m_0/m in the bound reported at the end of Section 2). One could try to diminish this conservativity either by pre-estimating m_0 and plugging the estimated value $\hat{m}_0$ into the nominal level, i. e., replacing α by $m\alpha/\hat{m}_0$, or by employing other sets of critical values. For instance, Finner et al. (2009) and Finner et al. (2012) developed non-linear critical values aiming at full exhaustion of the FDR level for any value of m_0 under Dirac-uniform configurations. However, both strategies are up to now only guaranteed to work well under the assumption of stochastically independent p-values and it would need deeper investigations of their validity under positive dependence. Here, we can at least report that we have no indications that φ^{LSU} may not keep the FDR level under our framework.

Example 1 (Communicated to the author by Klaus Straßburger)**.** *Let us emphasize here that the observed control of FWER and FDR is a specific property of positively dependent test statistics. To give a counterexample, consider $m = 2$ and two normally distributed test statistics T_1 and T_2, where $T_i \sim \mathcal{N}(\mu_i, 1)$, $i = 1, 2$, and $\rho(T_1, T_2) = -1$. Let $H_i : \{\mu_i \leq 0\}$ and, consequently, $K_i : \{\mu_i > 0\}$, $i = 1, 2$, and notice that $T_2 = -T_1$ under $\mu_1 = \mu_2 = 0$, with corresponding probability measure $\mathbb{P}_{(0,0)}$. A single-step multiple test at local level $\alpha_{loc.}$ for this problem is given by $\varphi = (\varphi_1, \varphi_2)$ with $\varphi_i = \mathbf{1}_{[\Phi^{-1}(1-\alpha_{loc.}),\infty)}(T_i)$, $i = 1, 2$, where Φ denotes the cumulative distribution function of the standard normal distribution.*

Now, in order to control the FWER at level α with φ, we have to choose $\alpha_{loc.} = \alpha/2$, because

$$\begin{aligned} FWER_{(0,0)}(\varphi) &= \mathbb{P}_{(0,0)}\left(T_1 \geq \Phi^{-1}(1-\alpha_{loc.}) \vee T_2 \geq \Phi^{-1}(1-\alpha_{loc.})\right) \\ &= \mathbb{P}_{(0,0)}\left(T_1 \geq \Phi^{-1}(1-\alpha_{loc.})\right) + \mathbb{P}_{(0,0)}\left(T_1 \leq -\Phi^{-1}(1-\alpha_{loc.})\right) = 2\alpha_{loc.}. \end{aligned}$$

4. EXEMPLARY MULTIPLE TEST PROBLEMS IN DYNAMIC FACTOR MODELS

In order to maintain a self-contained presentation, we first briefly summarize the essential techniques and results from Geweke and Singleton (1981).

Making use of (1.1), the autocovariance function of the observable process $\mathbf{X}$, $\Gamma_{\mathbf{X}}$ for short, and its spectral density matrix $S_{\mathbf{X}}$ (say), can be expressed by

$$
\begin{aligned}
\Gamma_{\mathbf{X}}(u) &= \mathbb{E}[\mathbf{X}(t)\mathbf{X}(t+u)^\top] = \sum_{s=-\infty}^{\infty} \Lambda(s) \sum_{v=-\infty}^{\infty} \Gamma_{\mathbf{f}}(u+s-v)\Lambda(v)^\top + \Gamma_\varepsilon(u), \\
S_{\mathbf{X}}(\omega) &= (2\pi)^{-1} \sum_{u=-\infty}^{\infty} \Gamma_{\mathbf{X}}(u)\exp(-i\omega u) \\
&= \tilde{\Lambda}(\omega) S_{\mathbf{f}}(\omega) \tilde{\Lambda}(\omega)' + S_\varepsilon(\omega), \ -\pi \leq \omega \leq \pi. \qquad (4.1)
\end{aligned}
$$

In (4.1), $\tilde{\Lambda}(\omega) = \sum_{s=-\infty}^{\infty} \Lambda(s)\exp(-i\omega s)$ and the prime stands for transposition and conjugation. The identifiability conditions mentioned in Section 1 can be plainly phrased by postulating that the representation in (4.1) is unique (up to scaling).

A localization technique now allows to apply the likelihood principle to the dynamic factor model (1.1), assuming that the sample size T is large. All further methods in this section rely on asymptotic considerations with respect to T. To this end, we consider a scaled version of the empirical (finite) Fourier transform of $\mathbf{X}$. Evaluated at harmonic frequencies, it is given by

$$
\tilde{\mathbf{X}}(\omega_j) = (2\pi T)^{-1/2} \sum_{t=1}^{T} \mathbf{X}(t)\exp(it\omega_j), \ \text{where} \ \ \omega_j = 2\pi j/T, \ 1 \leq j \leq T.
$$

Moreover, we choose B disjoint frequency bands $\Omega_1, \ldots, \Omega_B$, such that $S_{\mathbf{X}}$ can be assumed approximately constant within each of these bands. Under standard regularity assumptions and with n_b denoting the number of harmonic frequencies ω_j that fall into the band Ω_b, $1 \leq b \leq B$, Hannan (1973) showed that the n_b random vectors $(\tilde{\mathbf{X}}(\omega_j) : \omega_j \in \Omega_b)$ converge in distribution to a vector of n_b stochastically independent random vectors, each of which follows a complex normal distribution with mean zero and covariance matrix $S_{\mathbf{X}}(\omega^{(b)})$, where $\omega^{(b)}$ denotes the center of the band Ω_b. According to Goodman (1963), this entails, for a given realization $\mathbf{X} = \mathbf{x}$ of the process, the likelihood function

$$
\ell_b(\vartheta_b, \mathbf{x}) = \pi^{-p \times n_b} \, |S_{\mathbf{X}}(\omega^{(b)})|^{-n_b} \, \exp\left(- \sum_{j:\omega_j \in \Omega_b} \tilde{\mathbf{x}}(\omega_j)' \left[S_{\mathbf{X}}(\omega^{(b)}) \right]^{-1} \tilde{\mathbf{x}}(\omega_j) \right)
$$

in frequency band Ω_b. Therein, the parameter vector ϑ_b contains all $d = 2pk + k^2 + p$ distinct parameters in $\tilde{\Lambda}(\omega^{(b)})$, $S_{\mathbf{f}}(\omega^{(b)})$ and $S_\varepsilon(\omega^{(b)})$. Notice here that for computational purposes each of the (in general) complex elements in $\tilde{\Lambda}(\omega^{(b)})$ and $S_{\mathbf{f}}(\omega^{(b)})$ is represented by a pair of real components in ϑ_b, corresponding to its real part and its imaginary part.

For the optimization of the B local (log-) likelihood functions, an algorithm originally developed by Jöreskog (1969) for conventional factor models has been adapted. It delivers not only the numerical value of the maximum likelihood estimator $\hat{\vartheta}_b$, but additionally an estimate of the covariance matrix V_b (say) of $\hat{\vartheta}_b$. Standard arguments from likelihood theory (cf., e. g., Section 12.4 in Lehmann and Romano, 2005) yield that

$$
\hat{\vartheta}_b \overset{\text{as.}}{\sim} \mathcal{N}_d(\vartheta_b, \hat{V}_b), 1 \leq b \leq B, \qquad (4.2)
$$

where $\hat{V}_b$ denotes the estimated covariance matrix of $\hat{\vartheta}_b$.

The result in (4.2), in connection with the fact that the vectors $\hat{\vartheta}_b$, $1 \leq b \leq B$, are asymptotically jointly uncorrelated with each other, is very helpful for testing linear (point) hypotheses.

Such hypotheses are of the form $H: C\vartheta = \xi$ with a contrast matrix $C \in \mathbb{R}^{r \times Bd}$, $\xi \in \mathbb{R}^r$ and ϑ consisting of all elements of all the vectors ϑ_b. Geweke and Singleton (1981) proposed the usage of Wald statistics in this context. The Wald statistic for testing H is given by

$$W = (C\hat{\vartheta} - \xi)^\top (C\hat{V}C^\top)^{-1}(C\hat{\vartheta} - \xi), \tag{4.3}$$

where $\hat{V}$ is the block matrix built up from the band-specific matrices $\hat{V}_b$, $1 \leq b \leq B$. It is well-known that W is asymptotically equivalent to the likelihood ratio statistic for testing H. In particular, W is asymptotically χ^2-distributed with r degrees of freedom under the null hypothesis H, see Section 12.4.2 in Lehmann and Romano (2005). Wald statistics have the practical advantage that they can be computed easily, avoiding restricted maximization of the likelihood function.

In the remainder of this section, we discuss two exemplary simultaneous statistical inference problems in model (1.1) and demonstrate that they can be formalized by families of linear hypotheses regarding (components of) ϑ which in turn can be tested employing the statistical framework we considered in Sections 2 and 3.

Problem 1 (Which of the specific factors have a non-trivial autocorrelation structure?)**.** *Solving this problem is substantially more informative than just testing a single specific factor for trivial autocorrelations as considered by Geweke and Singleton (1981). Presence of many coloured noise components may hint at further hidden common factors and therefore, the solution to Problem 1 can be utilized for the purpose of model diagnosis in the spirit of a residual analysis. In the notational framework of Section 2, we have $m = p$, $I = \{1, \dots, p\}$ and for all $i \in I$ we can consider the linear hypothesis $H_i : C_{\text{Dunnett}}\mathbf{s}_{\varepsilon_i} = 0$. The contrast matrix C_{Dunnett} is the "multiple comparisons with a control" contrast matrix with $B-1$ rows and B columns, where in each row j the first entry equals $+1$, the $(j+1)$-th entry equals -1 and all other entries are equal to zero. The vector $\mathbf{s}_{\varepsilon_i} \in \mathbb{R}^B$ consists of the values of the spectral density matrix S_ε corresponding to the i-th noise component, evaluated at the B centers $(\omega^{(b)} : 1 \leq b \leq B)$ of the chosen frequency bins. Denoting the subvector of $\hat{\vartheta}$ that corresponds to $\mathbf{s}_{\varepsilon_i}$ by $\hat{\mathbf{s}}_{\varepsilon_i}$, the i-th Wald statistic is given by*

$$W_i = (C_{\text{Dunnett}}\hat{\mathbf{s}}_{\varepsilon_i})^\top \left[C_{\text{Dunnett}}\hat{V}_{\varepsilon_i}C_{\text{Dunnett}}^\top\right]^{-1}(C_{\text{Dunnett}}\hat{\mathbf{s}}_{\varepsilon_i}),$$

where $\hat{V}_{\varepsilon_i} = diag(\hat{\sigma}^2_{\varepsilon_i}(\omega^{(b)}) : 1 \leq b \leq B)$.

Under H_i, W_i asymptotically follows a χ^2-distribution with $B-1$ degrees of freedom. Considering the vector $\mathbf{W} = (W_1, \dots, W_p)^\top$ of all p Wald statistics corresponding to the p specific factors in the model, we finally have $\mathbf{W} \overset{as.}{\sim} \chi^2(p, (B-1, \dots, B-1)^\top, R)$ under the p hypotheses $H_1, \dots, H_p$, with some correlation matrix R. This allows to employ the multiple tests considered in Sections 2 and 3 for solving this problem.

Problem 2 (Which of the common factors have a lagged influence on $\mathbf{X}$?)**.** *In many economic applications, it is informative if certain factors (such as interventions) have an instantaneous or a lagged effect. By solving Problem 2, this can be answered for several of the common factors simultaneously, accounting for the multiplicity of the test problem. As done by Geweke and Singleton (1981), we formalize the hypothesis that common factor j has a purely instantaneous effect on $\mathbf{X}_i$, $1 \leq j \leq k$, $1 \leq i \leq p$ in the spectral domain by $H_{ij} : |\tilde{\Lambda}_{ij}|^2$ is constant across the B frequency bands. In an analogous manner to the derivations in Problem 1, the contrast matrix C_{Dunnett} can be* *used as the basis to construct a Wald statistic W_{ij}. The vector $\mathbf{W} = (W_{ij} : 1 \leq i \leq p, 1 \leq j \leq k)$ then asymptotically follows a multivariate chi-squared distribution with $B - 1$ degrees of freedom in each marginal under the corresponding null hypotheses and we can proceed as in Problem 1.*

Many other problems of practical relevance can be formalized analogously by making use of linear contrasts and thus, our framework applies to them, too. Furthermore, the hypotheses of interest may also refer to different subsets of $\{1, \ldots, B\}$. In such a case, the marginal degrees of freedom for the test statistics are not balanced, as considered in the general Definition 2 and in our simulations in Section 3.

5. CONCLUDING REMARKS AND OUTLOOK

First of all, we would like to mention that the multiple testing results with respect to FWER control achieved in Sections 3 and 4 also imply (approximate) simultaneous confidence regions for the parameters of model (1.1) because of the extended correspondence theorem, see Finner (1994). In such cases (in which focus is on FWER control), a promising alternative method for constructing a multiple test procedure is to deduce the limiting joint distribution of the vector $(Q_1, \ldots, Q_m)^\top$ of likelihood ratio statistics. For instance, one may follow the derivations by Katayama (2008) for the case of likelihood ratio statistics stemming from models with stochastically independent and identically distributed observations. Once this limiting joint distribution is obtained, simultaneous test procedures like the ones developed by Hothorn et al. (2008) are applicable. However, these methods are constructed by considering the global intersection hypothesis H_0 and therefore cannot be applied for FDR control. This is the reason why we focused on generic p-value based methods in Section 3.

Second, it may be interesting to assess the variance of the FDP in dynamic factor models, too. Among others, Finner et al. (2007) and Blanchard et al. (2011) have shown that this variance can be large in models with dependent test statistics and have consequently questioned if it is appropriate only to control the first moment of the FDP, because this does not imply a type I error control guarantee for the actual experiment at hand. A maybe more convincing concept in such cases is given by control of the false discovery exceedance, see Farcomeni (2009) for a good survey.

A topic relevant for economic applications is to what extent the results in the present paper can be transferred to more complicated models where factor loadings are modeled as a function of covariates like in Park et al. (2009). To this end, stochastic process techniques way beyond the scope of our setup are required. A first step may be the consideration of parametric models in which conditioning on the design matrix will lead to our framework.

Finally, if appropriate resampling schemes for empirically approximating the distribution of $\hat{\vartheta}$ in cases with small or moderate sample sizes could be worked out, a more accurate exhaustion of the multiple type I error level could be achieved. This is a topic devoted to future research.

APPENDIX

Table 1: Simulated FWER control of φ^{Hommel} under AR(1) and compound symmetry structure, respectively. The target FWER level was set to 5% in all simulations.

m	ρ	m_0	$\widehat{\text{FWER}}_{AR(1),\rho}(\varphi^{\text{Hommel}})$	$\widehat{\text{FWER}}_{CS,\rho}(\varphi^{\text{Hommel}})$
2	0.1	1	0.052	0.045
2	0.1	2	0.052	0.057
2	0.25	1	0.06	0.064
2	0.25	2	0.049	0.049
2	0.5	1	0.035	0.056
2	0.5	2	0.055	0.043
2	0.75	1	0.056	0.043
2	0.75	2	0.052	0.049
2	0.9	1	0.051	0.048
2	0.9	2	0.054	0.042
5	0.1	1	0.05	0.053
5	0.1	3	0.047	0.046
5	0.1	5	0.042	0.043
5	0.25	1	0.047	0.031
5	0.25	3	0.057	0.055
5	0.25	5	0.057	0.047
5	0.5	1	0.051	0.043
5	0.5	3	0.052	0.038
5	0.5	5	0.05	0.048
5	0.75	1	0.049	0.054
5	0.75	3	0.055	0.04
5	0.75	5	0.049	0.041
5	0.9	1	0.053	0.045
5	0.9	3	0.043	0.045
5	0.9	5	0.044	0.035
10	0.1	1	0.044	0.054
10	0.1	4	0.06	0.049
10	0.1	7	0.047	0.059
10	0.1	10	0.06	0.057
10	0.25	1	0.048	0.046
10	0.25	4	0.061	0.035
10	0.25	7	0.056	0.045
10	0.25	10	0.057	0.041
10	0.5	1	0.042	0.053
10	0.5	4	0.047	0.059

m	ρ	m_0	$\widehat{\mathrm{FWER}}_{AR(1),\rho}(\varphi^{\mathrm{Hommel}})$	$\widehat{\mathrm{FWER}}_{CS,\rho}(\varphi^{\mathrm{Hommel}})$
10	0.5	7	0.049	0.04
10	0.5	10	0.055	0.062
10	0.75	1	0.048	0.056
10	0.75	4	0.051	0.038
10	0.75	7	0.036	0.049
10	0.75	10	0.031	0.044
10	0.9	1	0.049	0.053
10	0.9	4	0.04	0.038
10	0.9	7	0.041	0.036
10	0.9	10	0.036	0.026
50	0.1	1	0.044	0.061
50	0.1	10	0.036	0.055
50	0.1	25	0.051	0.055
50	0.1	40	0.055	0.043
50	0.1	50	0.042	0.041
50	0.25	1	0.048	0.047
50	0.25	10	0.05	0.062
50	0.25	25	0.03	0.052
50	0.25	40	0.04	0.052
50	0.25	50	0.041	0.052
50	0.5	1	0.047	0.05
50	0.5	10	0.046	0.045
50	0.5	25	0.047	0.058
50	0.5	40	0.047	0.046
50	0.5	50	0.052	0.039
50	0.75	1	0.055	0.055
50	0.75	10	0.055	0.028
50	0.75	25	0.041	0.029
50	0.75	40	0.04	0.044
50	0.75	50	0.039	0.029
50	0.9	1	0.05	0.059
50	0.9	10	0.038	0.03
50	0.9	25	0.037	0.017
50	0.9	40	0.044	0.022
50	0.9	50	0.028	0.024
100	0.1	1	0.056	0.05
100	0.1	10	0.038	0.055
100	0.1	25	0.046	0.056
100	0.1	50	0.06	0.053

m	ρ	m_0	$\widehat{\mathrm{FWER}}_{AR(1),\rho}(\varphi^{\mathrm{Hommel}})$	$\widehat{\mathrm{FWER}}_{CS,\rho}(\varphi^{\mathrm{Hommel}})$
100	0.1	75	0.049	0.047
100	0.1	90	0.06	0.051
100	0.1	100	0.057	0.05
100	0.25	1	0.047	0.057
100	0.25	10	0.055	0.047
100	0.25	25	0.054	0.044
100	0.25	50	0.048	0.045
100	0.25	75	0.041	0.051
100	0.25	90	0.044	0.052
100	0.25	100	0.054	0.044
100	0.5	1	0.047	0.046
100	0.5	10	0.053	0.04
100	0.5	25	0.048	0.04
100	0.5	50	0.056	0.052
100	0.5	75	0.043	0.045
100	0.5	90	0.047	0.033
100	0.5	100	0.042	0.049
100	0.75	1	0.046	0.052
100	0.75	10	0.039	0.039
100	0.75	25	0.044	0.034
100	0.75	50	0.046	0.03
100	0.75	75	0.047	0.024
100	0.75	90	0.048	0.026
100	0.75	100	0.043	0.028
100	0.9	1	0.051	0.05
100	0.9	10	0.045	0.038
100	0.9	25	0.033	0.02
100	0.9	50	0.042	0.008
100	0.9	75	0.046	0.017
100	0.9	90	0.04	0.012
100	0.9	100	0.045	0.016

Table 2: Simulated FWER control of φ^{Hommel} under Toeplitz structure and for unstructured correlation matrices, respectively. The target FWER level was set to 5% in all simulations.

m	m_0	$\widehat{\text{FWER}}_{\text{Toeplitz}}(\varphi^{\text{Hommel}})$	$\widehat{\text{FWER}}_{UN}(\varphi^{\text{Hommel}})$
2	1	0.043	0.052
2	2	0.049	0.052
5	1	0.052	0.057
5	3	0.048	0.041
5	5	0.044	0.037
10	1	0.048	0.05
10	4	0.057	0.04
10	7	0.048	0.046
10	10	0.045	0.043
50	1	0.046	0.043
50	10	0.069	0.043
50	25	0.048	0.044
50	40	0.047	0.036
50	50	0.045	0.054
100	1	0.044	0.047
100	10	0.044	0.054
100	25	0.05	0.048
100	50	0.055	0.054
100	75	0.044	0.055
100	90	0.055	0.038
100	100	0.047	0.055

Table 3: Simulated FDR control of φ^{LSU} under AR(1) and compound symmetry structure, respectively. The target FDR level was set to 5% in all simulations.

m	ρ	m_0	$\widehat{\text{FDR}}_{AR(1),\rho}(\varphi^{\text{LSU}})$	$\widehat{\text{FDR}}_{CS,\rho}(\varphi^{\text{LSU}})$
2	0.1	1	0.026	0.0225
2	0.1	2	0.052	0.057
2	0.25	1	0.03	0.032
2	0.25	2	0.049	0.049
2	0.5	1	0.0175	0.028
2	0.5	2	0.055	0.043
2	0.75	1	0.028	0.0215
2	0.75	2	0.052	0.049
2	0.9	1	0.026	0.024
2	0.9	2	0.054	0.042
5	0.1	1	0.01	0.0106
5	0.1	3	0.028	0.0275
5	0.1	5	0.043	0.043
5	0.25	1	0.0094	0.0062
5	0.25	3	0.033	0.030
5	0.25	5	0.058	0.05
5	0.5	1	0.0102	0.0086
5	0.5	3	0.0308	0.025
5	0.5	5	0.051	0.049
5	0.75	1	0.0098	0.0108
5	0.75	3	0.034	0.030
5	0.75	5	0.052	0.041
5	0.9	1	0.0106	0.009
5	0.9	3	0.0302	0.026
5	0.9	5	0.048	0.038
10	0.1	1	0.0044	0.0054
10	0.1	4	0.0201	0.023
10	0.1	7	0.032	0.037
10	0.1	10	0.061	0.058
10	0.25	1	0.0048	0.0046
10	0.25	4	0.0201	0.020
10	0.25	7	0.0375	0.0336
10	0.25	10	0.057	0.043
10	0.5	1	0.0042	0.0053
10	0.5	4	0.022	0.022
10	0.5	7	0.033	0.029
10	0.5	10	0.055	0.068

m	ρ	m_0	$\widehat{\mathrm{FDR}}_{AR(1),\rho}(\varphi^{\mathrm{LSU}})$	$\widehat{\mathrm{FDR}}_{CS,\rho}(\varphi^{\mathrm{LSU}})$
10	0.75	1	0.0048	0.0056
10	0.75	4	0.021	0.019
10	0.75	7	0.032	0.038
10	0.75	10	0.034	0.045
10	0.9	1	0.0049	0.0053
10	0.9	4	0.017	0.017
10	0.9	7	0.035	0.033
10	0.9	10	0.037	0.03
50	0.1	1	0.00088	0.00122
50	0.1	10	0.0093	0.010
50	0.1	25	0.025	0.025
50	0.1	40	0.043	0.041
50	0.1	50	0.042	0.042
50	0.25	1	0.00096	0.00094
50	0.25	10	0.0094	0.0099
50	0.25	25	0.023	0.025
50	0.25	40	0.037	0.040
50	0.25	50	0.042	0.053
50	0.5	1	0.00094	0.001
50	0.5	10	0.0101	0.010
50	0.5	25	0.024	0.024
50	0.5	40	0.042	0.037
50	0.5	50	0.054	0.04
50	0.75	1	0.0011	0.0011
50	0.75	10	0.011	0.0096
50	0.75	25	0.026	0.021
50	0.75	40	0.040	0.040
50	0.75	50	0.04	0.034
50	0.9	1	0.001	0.0012
50	0.9	10	0.0097	0.0086
50	0.9	25	0.024	0.020
50	0.9	40	0.040	0.039
50	0.9	50	0.034	0.032
100	0.1	1	0.00056	0.00050
100	0.1	10	0.0045	0.0049
100	0.1	25	0.012	0.012
100	0.1	50	0.026	0.025
100	0.1	75	0.037	0.035
100	0.1	90	0.044	0.046

m	ρ	m_0	$\widehat{\mathrm{FDR}}_{AR(1),\rho}(\varphi^{\mathrm{LSU}})$	$\widehat{\mathrm{FDR}}_{CS,\rho}(\varphi^{\mathrm{LSU}})$
100	0.1	100	0.058	0.05
100	0.25	1	0.00047	0.00057
100	0.25	10	0.0049	0.0051
100	0.25	25	0.013	0.013
100	0.25	50	0.025	0.026
100	0.25	75	0.036	0.038
100	0.25	90	0.044	0.044
100	0.25	100	0.055	0.047
100	0.5	1	0.00047	0.00046
100	0.5	10	0.0051	0.0044
100	0.5	25	0.013	0.013
100	0.5	50	0.025	0.027
100	0.5	75	0.036	0.038
100	0.5	90	0.045	0.038
100	0.5	100	0.045	0.054
100	0.75	1	0.00046	0.00052
100	0.75	10	0.0047	0.0046
100	0.75	25	0.012	0.012
100	0.75	50	0.024	0.023
100	0.75	75	0.039	0.034
100	0.75	90	0.044	0.035
100	0.75	100	0.044	0.035
100	0.9	1	0.00051	0.00050
100	0.9	10	0.0050	0.0050
100	0.9	25	0.012	0.012
100	0.9	50	0.026	0.020
100	0.9	75	0.039	0.033
100	0.9	90	0.042	0.032
100	0.9	100	0.048	0.022

Table 4: Simulated FDR control of φ^{LSU} under Toeplitz structure and for unstructured correlation matrices, respectively. The target FDR level was set to 5% in all simulations.

m	m_0	$\widehat{\text{FDR}}_{\text{Toeplitz}}(\varphi^{\text{LSU}})$	$\widehat{\text{FDR}}_{UN}(\varphi^{\text{LSU}})$
2	1	0.0215	0.026
2	2	0.049	0.052
5	1	0.0104	0.011
5	3	0.034	0.033
5	5	0.045	0.037
10	1	0.0048	0.005
10	4	0.022	0.019
10	7	0.035	0.033
10	10	0.046	0.045
50	1	0.00092	0.00086
50	10	0.011	0.0096
50	25	0.025	0.023
50	40	0.037	0.038
50	50	0.047	0.057
100	1	0.00044	0.00047
100	10	0.0047	0.0053
100	25	0.012	0.012
100	50	0.025	0.026
100	75	0.034	0.037
100	90	0.044	0.044
100	100	0.049	0.057

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