Do Japanese Stock Prices Reflect Macro Fundamentals?

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Abstract

This paper investigates to what extent the fundamentals of the real economy are reflected in the stock prices of Japan. A Markov switching VAR model with switching variances is used to test the structural identification scheme. Identification of fundamental and nonfundamental shocks is shown to be supported by the data. Based on the appropriate structural restriction, the historical stock prices are decomposed into fundamental components and nonfundamental components. The decomposition shows that the linkage between Japanese stock prices and real activity shocks became strengthened since the bubble collapsed in the beginning of 1990s.

Keywords: Stock price, real activity, financial crisis, structural restrictions

JEL classifications: G12, E23

1 Introduction

There is an ongoing controversy regarding the extent to which stock prices reflect fundamental values. Earlier literature such as Shiller (1981) found that the U.S. stock prices were much more volatile than their subsequent changes in dividends. More recently, Binswanger (2004) shows evidence that stock prices are priced substantially above their fundamentals since the early 1980s for the U.S., Japan and Europe. On the contrary, other literature such as Chung and Lee (1998) found that the stock prices hardly deviate from their fundamental value in Hong Kong and Singapore.[1]

Among the developed countries, Japan is worth special investigation. From 1986 to 1991, the stock prices and the real estate prices were greatly inflated, a period well known as the Japanese asset price bubble. The bubble started collapsing since the beginning of the 1990s, contributing to the start of the so-called 'lost decades' and the end of the Japanese economic growth. This paper reinvestigates how the linkage between the Japanese stock prices and the real activities has changed before, in-between and after the collapse of the asset price bubble. The recent financial crisis period is also included in our sample and interesting findings regarding this period are revealed later.

Mixed results along the time line of Japan have been shown in existing literature. Chung and Lee (1998) found that Japanese stock prices were substantially overvalued from 1984 to 1990. When the market started to collapse from 1991, the stock prices were undervalued for several years and their deviation from the fundamental became much smaller. In contrast, Binswanger (2004) claims that the Japanese stock prices have been priced far above their fundamental values ever since the mid-1980s.

In order to disentangle the fundamental shocks and non-fundamental shocks, a long-run identification strategy in the spirit of Blanchard and Quah (1989) is often applied. Specifically, it is assumed that the nonfundamental

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[1] Chung and Lee (1998) use earnings and dividends as fundamental variables, while GNP and industrial production are used by Groenewold (2004) and Huang and Guo (2008) as fundamental variables.

shocks have no long-run effect on the output. This type of identification framework has been employed by Chung and Lee (1998), Rapach (2001), Binswanger (2004), Groenewold (2004), and Huang and Guo (2008). In a just-identified structural VAR model, these restrictions can only be assumed. However, as pointed by Uhlig (2005), the appropriateness of the structural information used for identification could be questionable.

In this paper, we follow Lanne, Lütkepohl, and Maciejowska (2010), and obtain over-identifying information from Markov switching variance models to test whether the assumed long run structural restrictions are appropriate or not. Markov switching variance VAR models provide over-identifying information from decomposition of covariance matrices across states to test the assumed structural restrictions, which is essential for the correct identification of fundamental and nonfundamental shocks.

Our results indicate that the assumed structural identification scheme is compatible with the data. Based on the confirmed identification of fundamental and nonfundamental shocks, the historical stock prices are decomposed into fundamental components and nonfundamental components. In contrast to Binswanger (2004), the decomposition shows that the linkage between Japanese stock prices and real activity shocks became strengthened since the bubble collapsed in the beginning of 1990s. After the outburst of the recent financial crisis, the stock price collapsed again, while the deviation from the fundamental value remained small. In line with Chung and Lee (1998), our results suggest that the deviation of Japanese stock prices from the fundamentals has not been substantial since the bubble burst in the beginning of 1990s.

This paper is structured as follows: Section 2 describes the data. Section 3 introduces how fundamental shocks are identified, and how the Markov switching VAR model with switching variances can help to test the assumed identification. Section 4 discusses the test results regarding the structural identification scheme, and the empirical findings on the extent to which fundamental shocks explain stock price fluctuations. Section 5 concludes.

2 The Data

Figure 1: Japanese Stock Prices and Industrial Production
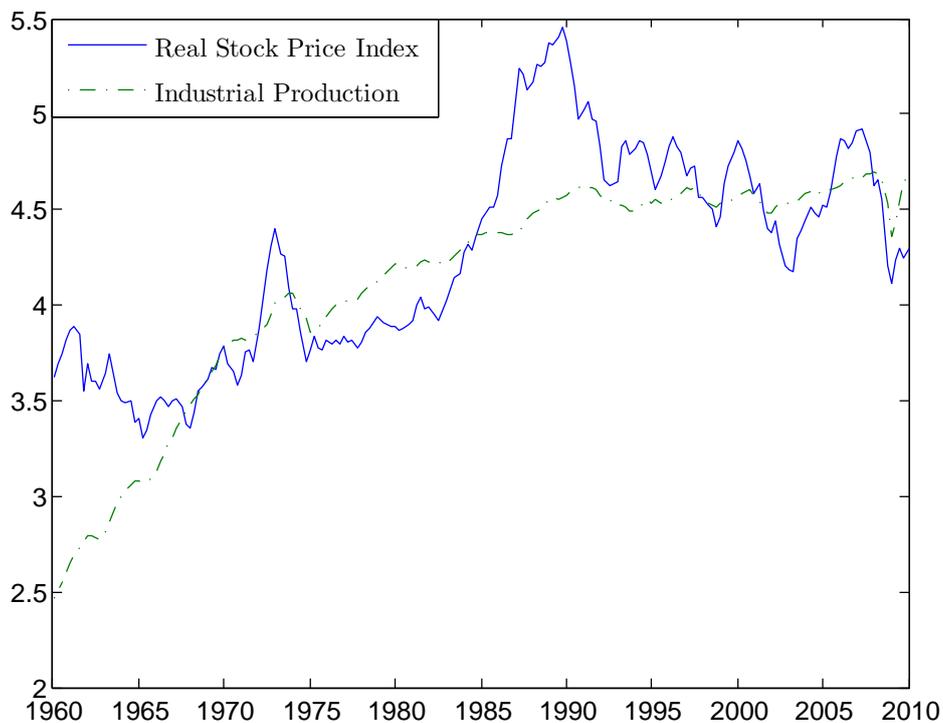
Notes: This graph depicts the series of the industrial production and the real stock prices of Japan in log levels from 1960 to 2010.

The data are obtained from the International Financial Statistics (IFS) of the International Monetary Fund. The series consist of seasonally adjusted industrial production y_t, a stock price index $Nikkei_t$, and Consumer Price Index (CPI).[2] All three series are normalized to a base year of 2005. The stock price series is converted to real terms by dividing by the Consumer Price Index. Figure 1 plots the deflated stock prices and the industrial production in log levels. The data range is from 1960 Q1 to 2010 Q1, implying that the period of the late 2000s financial crisis is also included.

To examine the stationarity of the data, ADF unit root tests are conducted. Results strongly suggest that the log-level series for both output and

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[2] The Nikkei index represents more than half of the total market capitalization in the Tokyo Stock Exchange.

real stock prices are of order $I(1)$. When testing for cointegration relationships for the unrestricted levels VAR model, the Saikkonnen and Lütkepohl test rejects the null hypothesis that there is no cointegration relation between output and real stock prices.[3] As a consequence, the empirical analysis is based on a VAR in first differences.

3   Identification of Fundamental and Nonfundamental Shocks

3.1   The Long-run Restriction à la Blanchard and Quah

Following earlier empirical literature, we adopt the following bivariate VAR model to study the interdependence of stock prices and the real activities:

$$\Delta x_t = \nu + A_1 \Delta x_{t-1} + A_2 \Delta x_{t-2} + \cdots + A_p \Delta x_{t-p} + u_t, \qquad (1)$$

where Δx_t is a 2×1 vector of the endogenous variables representing logs of industrial production and logs of real stock prices in first differences. A_i's are 2×2 parameter matrices, with $i = 1, \ldots, p$. u_t is a 2×1 vector of unobservable error terms with $E[u_t] = 0$ and $E[u_t u_t'] = \Sigma_u$.

The structural shocks ε_t hitting the system can not be identified in the above reduced form VAR model. One popular way to identify the shocks is to impose restrictions on the long-run impact matrix as in Blanchard and Quah (1989). The long-run impact matrix can be represented as follows:

$$\Psi = (I - A_1 - \ldots - A_p)^{-1} B \qquad (2)$$

where I stands for the identity matrix, and $u_t = B\varepsilon_t$, and $\Sigma_u = BB'$. B transforms the reduced form residuals into structural innovations.

Following Binswanger (2004) and Groenewold (2004), we set the upper right element, $\Psi_{1,2}$, of the long-run impact matrix to zero making it lower triangular. The other elements of the Ψ matrix, denoted by $*$, can take on any value.

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[3]Results of the ADF tests and the cointegration test are shown in Table 4 and Table 5 in Appendix B.

$$\Psi = \begin{bmatrix} * & 0 \\ * & * \end{bmatrix} \tag{3}$$

Under this identification scheme, the structural shocks, $\varepsilon_t = [\varepsilon_t^F, \varepsilon_t^{NF}]'$, can be interpreted as fundamental and non-fundamental shocks respectively. By assumption, fundamental shocks can have a permanent effect on the real economy and on the stock market, while non-fundamental shocks can only have a transitory effect on the real economy and a permanent effect on the stock price.

However, the structural identification scheme introduced above can only be assumed and can not tested in a linear VAR model. Therefore, in the next subsection we introduce a Markov switching model with time-varying variances. This type of model is capable of providing over-identifying information to test structural restrictions.

3.2 Testing the Identification Scheme of Fundamental Shocks

Many researchers including Uhlig (2005) have criticized that the assumed structural restrictions could be too restrictive. Following Lanne, Lütkepohl, and Maciejowska (2010), a Markov Switching model is used to validate the identification strategy. This model allows for heteroscedasticity of the residuals as follows:

$$\Delta x_t = \nu + A_1 \Delta x_{t-1} + A_2 \Delta x_{t-2} + \cdots + A_p \Delta x_{t-p} + u_t | s_t. \tag{4}$$

where the distribution of the residuals is assumed to be governed by a Markov process, s_t and it is assumed that the residuals are normally distributed conditional on the given state, i.e., $u_t | s_t \sim N(0, \Sigma_{s_t})$.

The discrete stochastic process s_t assumes M regimes with transition probabilities given by

$$p_{ij} = P(s_t = j | s_{t-1} = i), \quad i, j = 1, \ldots, M$$

with a $M \times M$ matrix of transitional probabilities. Note that the probabilities add up to one row-wise, hence $p_{iM} = 1 - p_{i1} - p_{i2} - \cdots - p_{iM-1}$.

In the above framework, if there exist at least two different covariance states, shocks can be identified without assuming further restrictions. Special features of (4) provide over-identifying information to test the appropriateness of structural restrictions, if the covariance matrices could be uniquely decomposed in the following way:

$$\Sigma_1 = BB', \quad \Sigma_2 = B\Lambda_2 B', \quad \ldots, \quad \Sigma_M = B\Lambda_M B', \quad (5)$$

where B is the contemporaneous impact matrix which is used to transform reduced form shocks into structural shocks. Λ_i can be interpreted as the relative-variance matrix of the structural shocks in Regime i versus Regime 1. In the empirical example, $M = 3$ is chosen. For State 1, Λ_1 is normalized as a 2×2 identity matrix. For the second and the third state, Λ_i is a 2×2 diagonal matrix with the following representation:

$$\Lambda_i = \begin{bmatrix} \lambda_{i1} & 0 \\ 0 & \lambda_{i2} \end{bmatrix} \quad (6)$$

If diagonal elements in either Regime 2 or Regime 3 are distinct, i.e., $\lambda_{i1} \neq \lambda_{i2}$, the transformation matrix B is identified without further structural assumptions. The decomposition in (5) is unique up to sign changes in the B matrix. In accordance with Lanne, Lütkepohl, and Maciejowska (2010), sign changes in the columns of B are no problem for our analysis of structural identification since it corresponds to whether negative structural shocks or positive structural shocks are of interest.

Whether the structural restrictions are compatible with the data is verified through a likelihood ratio test. The maximum log-likelihood from the just-identified Markov switching VAR model can be compared with the maximum log-likelihood from the over-identified Markov switching VAR model including the structural restrictions. If the likelihood ratio test is rejected, it is evidence against the presumed structural restrictions.

The Markov switching VAR models are solved by the Expectation Maximization algorithm. Details of the algorithm are given in the appendix. The next section describes the data and the empirical results on the relation between stock prices and industrial production of Japan.

4   Empirical Results

Based on the information criteria, a three-state one-lag Markov switching structural VAR model is selected for Japan. In the following, the test results regarding the long-run structural restrictions are first illustrated. Then details are revealed about the extent to which the Japanese stock prices have been driven by the fundamental shocks.

4.1   Estimates from the Markov Switching VAR models

Table 1: Estimates of the relative variances of shocks across states

	estimates	standard errors
λ_{21}	2.912	1.186
λ_{22}	2.411	0.802
λ_{31}	55.637	41.765
λ_{32}	3.109	2.046

Notes: This table presents the estimates of diagonal elements of the relative-variance matrix Λ_i for $i = 2, 3$, and their corresponding standard errors from the Markov switching VAR models without further structural restrictions. λ_{i1} is the first element along the diagonal of Λ_i, while λ_{i2} represents the second element along the diagonal of Λ_i. λ_{i1} can be interpreted as the relative variance of fundamental shocks in Regime i versus Regime 1.

Lanne, Lütkepohl, and Maciejowska (2010) have shown that, in a three-state Markov switching VAR model, the necessary condition in achieving over-identifying information is that diagonal elements of Λ_i either in the second or the third state should be distinct from each other. Table 8 reports the estimated diagonal elements of Λ_2 and Λ_3. Since Λ is normalized to be the identity matrix in State 1, the relative ratios of variances show that the volatility is increasing in states. Figures 2 plots the smoothed probabilities for the Markov switching VAR model. The first state is the one with low volatility. The second one stands for the medium-volatility regime. The 1975 recession in Japan has been captured as the medium-volatility regime. The third state is the most volatile one, which coincides with the time of

the late 2000s financial crisis. As shown in Table 8, the relative variance of the fundamental shocks in the state of the recent financial crisis relative to the low-volatility state is around 56.

Figure 2: Smoothed probabilities for different volatility regimes in Japan
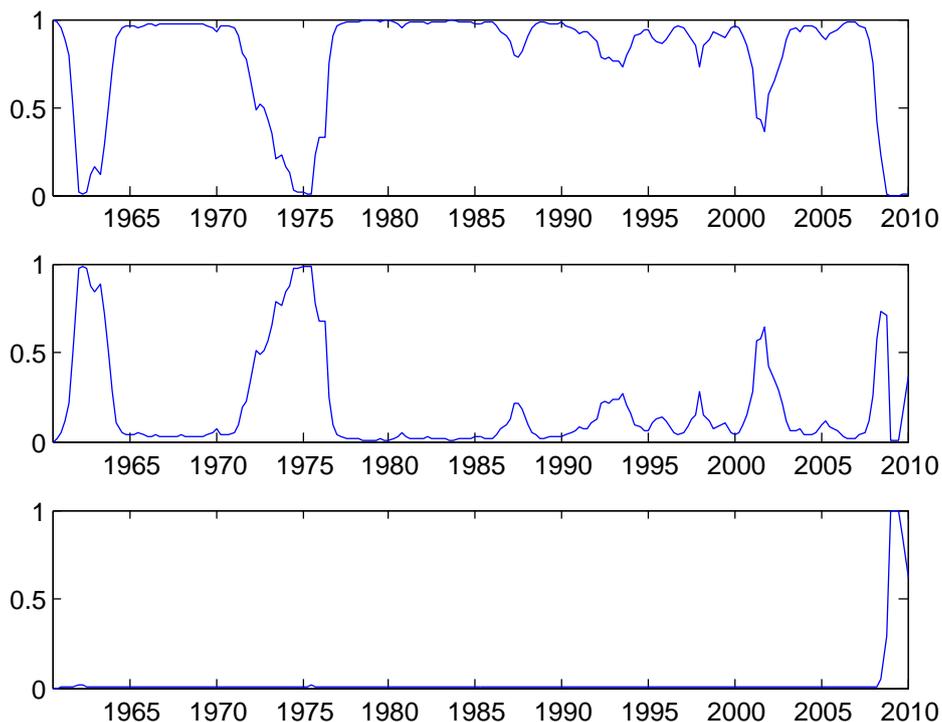
Notes: This graph depicts the smoothed probabilities estimated from the Markov Switching VAR model with three states and one lag with structural restrictions. The top panel shows the probability of the system being in a low-volatility regime. The panel in the middle represents the probability of being in a medium-volatility regime, while the bottom panel represents the probability of being in a high-volatility regime.

The standard errors of Λ_3 diagonal elements are noticeably large. It is very likely a result of the few observations for the recent financial crisis period. Due to concerns regarding robustness, we estimate also on the subsample that excludes the late 2000s financial crisis. A two-state two-lag model is selected, and the results regarding the test of the appropriateness of the structural identifications remain robust(see Appendix C).

4.2   Are the Structural Restrictions Appropriate?

In order to test whether the relative variance of the fundamental shocks in Regime 2 versus Regime 1 is indeed different from that of the non-fundamental shocks, a likelihood ratio test is performed. The likelihood ratio test statistics is 8.281 and the corresponding p-value obtained from a χ^2 distribution is 0.016. Hence there is evidence that $\lambda_{21} \neq \lambda_{22}$. Consequently, the decomposition in Equation (5) is unique up to sign changes in the B matrix.

Are the assumed structural restrictions on the long-run impact matrix Ψ in Equation (3) too restrictive? Let us now apply the likelihood ratio test to find out whether the imposed long run restriction is supported by the data or not. The likelihood ratio test compares the maximum log-likelihood achieved from the Markov switching VAR model without the long run structural restrictions to the maximum log-likelihood achieved from the Markov switching VAR model with the long run structural restrictions. As shown in Table2 , the test statistics is 2.449 with a corresponding p-value 0.118. Therefore, the identification of the structural innovations as fundamental shocks and non-fundamental shocks is compatible with the data.

Table 2: Likelihood ratio test for the structural restrictions

data	test statistic	p-value
1960-2010	2.449	0.118
Pre-crisis period	0.200	0.655

Notes: This table shows results of the likelihood ratio test that compares the maximum likelihood from the Markov switching VAR model without the structural restriction to the one from the Markov switching VAR model with the structural restriction imposed. P-values indicate that the long run restriction is compatible with the data.

4.3 The Role of Fundamental Shocks for Japanese Stock Prices

The above section demonstrates that the assumed long run restriction to disentangle fundamental shocks and non-fundamental shocks is validated by over-identifying information achieved from a three-state Markov switching variance model. The appropriate structural identification allows us to conduct further structural analysis on the extent to which Japanese stock prices are driven by the fundamental shocks. As the estimates from the structural model with and without switching variances are close, we present the following findings based on the linear structural VAR model for comparable analysis with former empirical literature.

Figure 3: Accumulated impulse responses
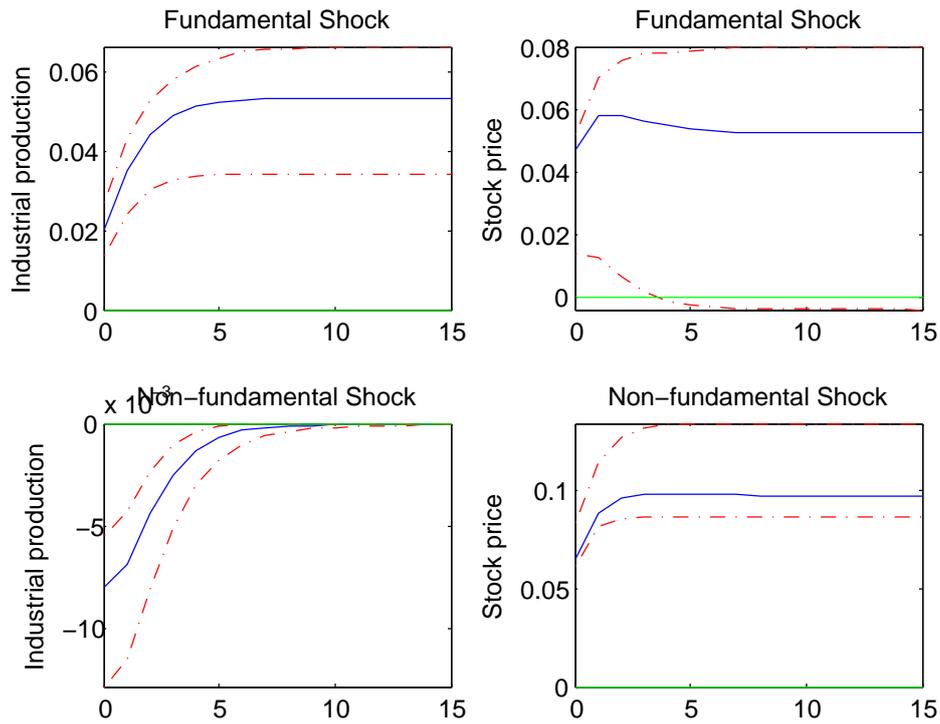
Notes: This graph depicts the accumulated impulse responses to one-standard-deviation structural shocks. Confidence intervals denoted by dashed lines are according to fixed design wild bootstrap at the 95% level.

Figure 3 presents the accumulated impulse responses of each variable to a one-standard-deviation structural shock. The responses to a fundamental shock are shown in the first row. Industrial production increases after a fundamental shock, converging to a permanently higher level after around five quarters. Real stock prices are also pushed up permanently after a fundamental shock.

The impulse responses to a nonfundamental shock are found in the second row. There is a temporary decline in industrial production after a nonfundamental shock. After around eight quarters, industrial production returns to its original level before the shock, as implied by the identifying long-run restriction. The short-run negative effects on industrial production may result from the changing sentiments of investors, who will shift funds into the stock market instead of financing new investment projects. The response of real stock prices to a nonfundamental shock is positive and permanent. In general, the impulse responses pictures seem much in line with the former empirical literature such as Rapach (2001) and Binswanger (2004).

What would the Japanese stock prices have been if they had only been driven by the fundamental shocks? To answer this question, a historical decomposition is conducted following the method proposed in Burbidge and Harrison (1985). Based on estimation on the full sample, the fundamental series is constructed by setting the value of nonfundamental shocks to zero and simulating the historical values of the Japanese stock prices in the presence of only fundamental shocks. The actual series shown in Figure 4 represents the historical stock prices in the presence of both the fundamental shocks and the nonfundamental shocks. The dashed line depicts the fundamentals values that represent the series influenced only by the fundamental shocks. In accordance with Binswanger (2004), it is important to look at the degree to which the fundamental series follow stock prices instead of the absolute value of the simulated series.

Figure 4: Fundamentals and Japanese stock prices: a historical decomposition

(a) The historical decomposition in Binswanger (2004)
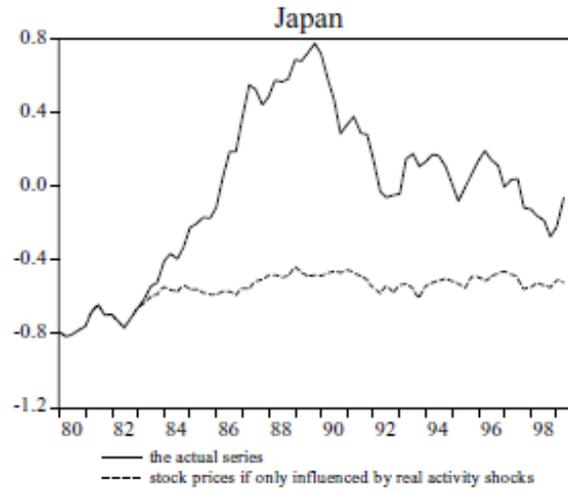
(b) The historical decomposition in our paper
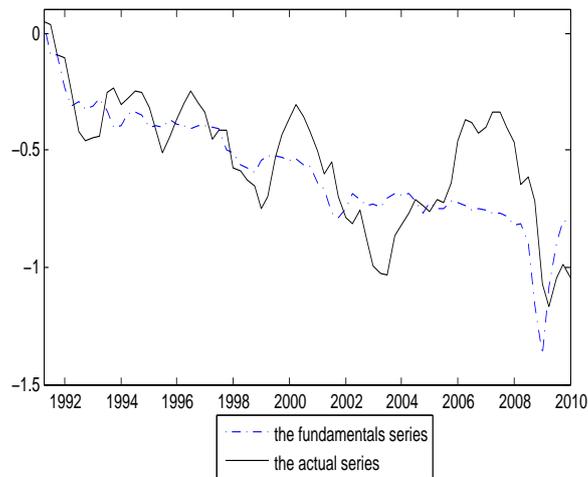
Notes: The upper panel in this figure shows the historical decomposition for the Japanese stock prices in Binswanger (2004). His decomposition starts from 1983 and ends in 1999. The lower panel presents the historical decomposition made in this paper for the Japanese stock prices from 1992 until 2010. For both panels, the solid lines represent the actual series, while the dashed lines refer to the decomposed series in which only fundamental shocks can influence stock prices, i.e. with the non-fundamental shocks set to zero.

One crucial step of the historical decomposition method is the choice of the starting value, as it is implicitly assumed that the real stock prices coincide with the fundamental series at the starting date. Since it is commonly believed that the Japanese stock price bubbles collapsed at the end of 1991, the stock price should be the closest to its fundamental component after the bubble burst.[4] Therefore the stock price at 1991 Q4 is chosen as the starting value for the simulation of the historical stock prices and also for the fundamental components.[5]

The lower panel in Figure 4 displays the graph of the historical decomposition for the Japanese stock prices from 1991 Q4 to 2010 Q1 based on estimation in this paper. The stock prices were moving very closely with the fundamentals in the 1990s. During the 2000s, the stock prices behaved more volatile. Especially for the several years before the start of the late 2000s financial crisis, the stock prices deviates the most from the fundamentals. Following the start of the crisis, both the fundamental and the stock prices declined sharply. However, the deviation between them remained small. In general, the linkage between the Japanese stock prices and the fundamentals has been rather strong after the asset price bubble burst.

Let us now compare the historical decomposition in this paper to the simulation presented in Binswanger (2004) and Chung and Lee (1998). As depicted the upper panel in Figure 4, Binswanger (2004) shows that the stock prices are floating far above the fundamentals from 1983 to 1999. In contrast, Chung and Lee (1998) demonstrate that though the stock prices were substantially overvalued from 1986 to 1990, the deviation of the stock prices from the fundamentals declined below zero and stayed small after the bubble collapsed in 1991. The the historical decomposition in our paper shown in the lower panel of Figure 4 seems more in line with those of Chung

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[4] See details of the Japanese asset bubble period in literature such as Goyal and Yamada (2004) and Shiller, Kon-Ya, and Tsutsui (1996).

[5] The historical decomposition remains generally robust when the starting date varies from the end of 1991 to 1994, a period known as the recovery period after the collapse of Japanese asset prices. The historical decomposition based on estimates from the linear structural VAR is close to the one based on estimates from the Markov switching structural VAR model.

Table 3: Variance decomposition of the stock prices over different periods

	Percentage of variance attributable to:		Percentage of variance attributable to:	
	Fundamental shock	Non-fundamental shock	Fundamental shock	Non-fundamental shock
	1960-1991		**1992-2010**	
1 quarter	22	78	72	28
5 quarters	21	79	72	28
10quarters	21	79	71	29
15 quarters	21	79	71	29
20 quarters	21	79	71	29

Notes: This table presents percentage of the 20-month forecast error variance explained respectively by fundamental shocks and nonfundamental shocks to real stock prices.

and Lee (1998), supporting their view that the dependence of stock prices on real activities became stronger after the Japanese stock price bubble burst.

A forecast error variance decomposition analysis confirms the results indicated by the historical decomposition. As shown in Table 3, based on the sub-sample from 1992 to 2010, the fundamental shocks explain around 70 percent of the stock prices fluctuations, while Binswanger (2004) shows that only 3 percent of stock price fluctuations are explained by the fundamentals from the mid-1980s to 1999. This is very likely due to a decade longer after-bubble period that we include in our data sample. Furthermore, choosing 1991 instead of the mid-1980s as the break point could also have led to the divergent results.

To sum up, both the historical decomposition and the forecast error variance decomposition analysis suggest that since the Japanese asset price bubble collapsed in 1991, the linkage between the stock price and fundamentals has been restored.

5 Conclusion

This paper has investigated the extent to which stock prices in Japan are explained by their fundamental values. A bivariate Markov switching VAR

model with Markov switching variances is employed to test the appropriateness of the long run structural restrictions, which assumes that the nonfundamental shocks have no long-run effect on output.

We found that the identification of fundamental shocks and nonfundamental shocks using long run structural restrictions is supported by the data. Based on the proper identification scheme, stock prices are decomposed into fundamental components and nonfundamental components for period from 1991 Q4 to 2010 Q1. In contrast to Binswanger (2004), but in line with Chung and Lee (1998), our results suggest that the linkage between stock prices and fundamental components has been strengthened since the collapse of the Japanese Asset Price Bubble in the beginning of 1990s. During the recent financial crisis, though the stock price dropped down sharply, the deviation between the stock prices and the fundamentals is not substantial.

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Appendices

A   The EM Algorithm

This is a technical appendix explaining the EM algorithm used in this paper based on Krolzig (1997). The same approach has been also applied by Lanne, Lütkepohl, and Maciejowska (2010) and Herwartz and Lütkepohl (2010).

Starting with the regression equation

$$\Delta x = (\bar{Z} \otimes I_K)\beta + u,$$

where Δx is a $(TK \times 1)$ vector or the vectorization of $\Delta X = [\Delta x_1, \dots, \Delta x_T]$, and where T is the sample size and K the number of variables. Here $\bar{Z} = [\mathbf{1}_T, \Delta X_{-1}, \dots, \Delta X_{-p}]$, where $\mathbf{1}_T$ is a $(T \times 1)$ vector of ones and $\Delta X_{-i} = [\Delta x_{1-i}, \dots, \Delta x_{T-i}]'$ is a $(T \times K)$ matrix of lagged regressors, for $i = 1, \dots, p$ and p being the number of lags of the MS-VAR model. The $(K(Kp + 1) \times 1)$ vector β contains the vectorized intercept and slope parameters, i.e. $\text{vec}[\nu, A_1, \dots, A_p]$ as defined in (1). Finally u is the $(TK \times 1)$ vectorization of the matrix of residuals, $U = [u_1, \dots, u_T]'$, where the distribution of each residual, $u_i, i = 1, \dots, T$ is given according to (4).

The EM algorithm is initiated by defining the starting values of the intercept, slope and contemporaneous impact matrix, B parameters as well as the transition probabilities and initial states. For the intercept and slope parameters the starting values are given by $\beta_0 = [\bar{Z}'\bar{Z} \otimes I_K]^{-1}(\bar{Z}' \otimes I_K)\Delta x$. The initial value of the contemporaneous impact matrix is $B_0 = (UU'/T)^{1/2}$, where U is obtained from $u = \Delta x - (\bar{Z} \otimes I_K)\beta_0$. The transition probabilities are set at $P_0 = \mathbf{1}_M \mathbf{1}_M'/M$, where $\mathbf{1}_M$ is an $(M \times 1)$ vector of ones and M are the number of states in the model. The initial states (defined below) are defined as $\xi_{0|0} = \mathbf{1}_M/M$. Finally, the starting values of the covariance matrices need to be determined as defined in the decomposition in (5). This is done by setting the values of the Λ_i matrices, $i = 2, \dots, M$. I use a loop of different starting values for these matrices by starting with $\Lambda_2 = 2 * I_K, \dots, \Lambda_M = 2^{M-1} * I_K$ and replacing the 2 with higher values and in the end seeing which starting value gives the highest log-likelihood.

The vector of conditional probabilities for the unobserved states is denoted as $\hat{\xi}_{t|t}$ and it indicates the probability of a given state in a given time period conditional on all observations up to time period t, ΔX_t and all intercept, slope, covariance parameters and transition probabilities stored in, θ. Hence

$$\hat{\xi}_{t|t} = \begin{bmatrix} P(s_t = 1|\Delta X_t, \theta) \\ P(s_t = 2|\Delta X_t, \theta) \\ \vdots \\ P(s_t = M|\Delta X_t, \theta) \end{bmatrix}. \tag{7}$$

It is also necessary to define the conditional densities of an observation given a particular state, all past observations and θ as

$$\eta_t = \begin{bmatrix} P\Delta x_t|s_t = 1, \Delta X_{t-1}, \theta) \\ P\Delta x_t|s_t = 2, \Delta X_{t-1}, \theta) \\ \vdots \\ P(\Delta x_t|s_t = M, \Delta X_{t-1}, \theta) \end{bmatrix} = \begin{bmatrix} \frac{1}{2\pi|\Sigma_1|^{1/2}}\exp\left\{-\frac{u_t'\Sigma_1^{-1}u_t}{2}\right\} \\ \frac{1}{2\pi|\Sigma_2|^{1/2}}\exp\left\{-\frac{u_t'\Sigma_2^{-1}u_t}{2}\right\} \\ \vdots \\ \frac{1}{2\pi|\Sigma_M|^{1/2}}\exp\left\{-\frac{u_t'\Sigma_M^{-1}u_t}{2}\right\} \end{bmatrix}. \tag{8}$$

Expectation Step

Now follows the expectation step where the filtered probabilities from (7) are calculated as

$$\hat{\xi}_{t|t} = \frac{\eta_t \odot \hat{\xi}_{t|t-1}}{\mathbf{1'}(\eta_t \odot \hat{\xi}_{t|t-1})}, \tag{9}$$

and

$$\hat{\xi}_{t|t-1} = P'\hat{\xi}_{t-1|t-1}, \tag{10}$$

for $t = 1, \ldots, T$. This generates an $(M \times 1)$ vector of conditional probabilities for each time period. Here $\odot$ denotes element-by-element multiplication and P is defined as in (6). Next using the values of the filtered probabilities, the smoothed probabilities, $P(s_t = i|\Delta X_T, \theta), i = 1, \ldots, M$ are estimated as

$$\hat{\xi}_{t|T} = [P(\hat{\xi}_{t+1|T} \oslash \hat{\xi}_{t+1|t})] \odot \hat{\xi}_{t|t}, \tag{11}$$

for $t = T - 1, \ldots, 0$. The symbol $\oslash$ denotes element-by-element division. Note that the filtered probabilities from the current iteration are used to estimate the smoothed probabilities.

Maximization Step

After the expectation step in the maximization step first the vector of transition probabilities $\hat{\rho}$ is estimated as

$$\hat{\rho} = \hat{\xi}^{(2)} \oslash (\mathbf{1}_M \otimes \hat{\xi}^{(1)}), \tag{12}$$

where $\hat{\xi}^{(2)} = \sum_{t=0}^{T-1} \hat{\xi}_{t|T}^{(2)}$ and

$$\hat{\xi}_{t|T}^{(2)} = \text{vec}(P) \odot \left[\left(\hat{\xi}_{t+1|T}^{(1)} \oslash \hat{\xi}_{t+1|t}^{(1)} \right) \otimes \hat{\xi}_{t|t}^{(1)} \right],$$

for $t = 0, \ldots, T-1$. Here $\otimes$ denotes the Kronecker product. Finally, $\hat{\xi}_{t|T}^{(1)}$ is the vector of smoothed probabilities from (9) and $\hat{\xi}_{t|t}^{(1)}$ is the vector of filtered probabilities from (7). Also note that $\hat{\xi}^{(1)} = (\mathbf{1}_M' \otimes I_M)\hat{\xi}^{(2)}$, where $\mathbf{1}_M$ is an $(M \times 1)$ vector of ones and I_M is the $(M \times M)$ identity matrix.

The B and Λ matrices are then estimated by optimizing

$$
\begin{aligned}
l(B, \Lambda_2, \ldots, \Lambda_M) &= T \log|\det(B)| + \frac{1}{2}\text{tr}\left((BB')^{-1}\hat{U}\hat{\Xi}_1\hat{U}' \right) \\
&+ \sum_{m=2}^{M} \left[\frac{\hat{T}_m}{2}\log(\det(\Lambda_m)) + \frac{1}{2}\text{tr}\left((B\Lambda_m B')^{-1}\hat{U}\hat{\Xi}_m\hat{U}' \right) \right]
\end{aligned} \tag{13}
$$

where $\hat{U}$ is obtained from $\hat{u} = \Delta x - (\bar{Z} \otimes I_K)\hat{\beta}$, $\hat{\Xi}_m = \text{diag}(\hat{\xi}_{m1|T}, \ldots, \hat{\xi}_{mT|T})$, the smoothed probabilities of regime m and $\hat{T}_m = \sum_{t=1}^{T} \hat{\xi}_{mt|T}$ is a summation of the smoothed probabilities. To avoid singularity a lower bound of 0.001 is imposed on the diagonal elements of the $\Lambda_m, m = 2, \ldots, M$ matrices. The updated covariance matrices are given from the decomposition

$$\hat{\Sigma}_1 = \hat{B}\hat{B}', \quad \hat{\Sigma}_2 = \hat{B}\hat{\Lambda}_2\hat{B}', \quad \ldots \quad \hat{\Sigma}_M = \hat{B}\hat{\Lambda}_M\hat{B}'.$$

Next the intercept and slope parameters are obtained as

$$\hat{\beta} = \left[\sum_{m=1}^{M} (\bar{Z}'\hat{\Xi}_m\bar{Z}) \otimes \hat{\Sigma}_m^{-1} \right]^{-1} \left[\sum_{m=1}^{M} (\bar{Z}'\hat{\Xi}_m) \otimes \hat{\Sigma}_m^{-1} \right] \Delta x. \tag{14}$$

Note, that to estimate $\hat{\beta}$ the covariances of the previous iteration were used. These parameters are then plugged back into (13) and new estimates of the covariance matrices are obtained which are then used in (14). All this is

iterated until convergence. The convergence criteria used is the absolute change in the log-likelihood given in (13), i.e.

$$\Delta = |l(\theta^{j+1}|\Delta X_T) - l(\theta^j|\Delta X_T)|, \tag{15}$$

where $l(\bullet)$ is the log-likelihood and θ^j denotes the parameters of the j-th iteration. Convergence is satisfied when $\Delta \leq 10^{-6}$ or after a specified maximum number of iterations.

The EM algorithm terminates as well after a similar convergence criteria as in (15). As shown in Hamilton (1994) the log-likelihood is given by $\log(\mathbf{1'}(\eta_t \odot \hat{\xi}_{t|t-1}))$.

The restricted MS-SVAR model is estimated in a similar way, recall that the long-run impact matrix, Ψ is related to the B matrix by $\Psi = A(1)^{-1}B$.

Standard Errors

Once the EM algorithm has converged and the point estimates of the parameters are obtained it is necessary to calculate their standard errors in order to carry out statistical tests. The optimal values of $P, \beta, B, \Lambda_m, m = 2, \ldots, M$ and $\xi_{0|0}$ are used in $\log(\mathbf{1'}(\eta_t \odot \hat{\xi}_{t|t-1}))$. Standard errors are then obtained by the inverse of the negative of the Hessian matrix.

B Tables for the Full Sample

Table 4: Augmented Dickey-Fuller test

variable	test statistic	1% critical value	5% critical value	10% critical value
output	-2.47	-3.96	-3.41	-3.13
stock price	-1.56	-3.43	-2.86	-2.57

Notes: This table shows results of the ADF test for the series of output and real stock prices. In both cases, the null hypothesis that there is a unit root is not rejected at 10% significance level since the test statistic is larger than the critical value.

Table 5: Test for cointegration

test statistic	p-value
10.28	0.11

Notes: This table shows results of the Saikkonen-Lütkepohl test. The null hypothesis that there is no cointegration relationship between output and real stock prices can not be rejected at 10% significance level.

Table 6: Estimates of the transition probabilities

	estimates	standard errors
p_{11}	0.963	0.031
p_{12}	0.037	0.027
p_{21}	0.155	0.106
p_{22}	0.812	0.092
p_{32}	0.134	0.347
p_{33}	0.866	0.479

Notes: This table presents the estimates of transition probabilities and their corresponding standard errors from the three-state Markov switching VAR models without further structural restrictions based on data from 1960 to 2010. p_{ij} represents the probability that the regime in the next period switches into j given that the current regime is i.

C Results for the Pre-crisis Period

Table 7: Estimates of the transition probabilities

	estimates	standard errors
p_{11}	0.971	0.025
p_{22}	0.865	0.081

Notes: This table presents the estimates of transition probabilities and their corresponding standard errors from the two-state Markov switching VAR models without further structural restrictions for the period from 1960 to 2007. p_{ij} represents the probability that the regime in the next period switches into j given that the current regime is i.

Table 8: Estimates of the relative variances of shocks across states

	estimates	standard errors
λ_{21}	3.596	1.170
λ_{22}	2.184	0.808

Notes: This table presents the estimates of diagonal elements of the relative-variance matrix Λ_2 and their corresponding standard errors from the Markov switching VAR models without further structural restrictions based on data from the pre-crisis period. λ_{21} can be interpreted as the relative variance of fundamental shocks in Regime 2 versus Regime 1, while λ_{22} can be interpreted as the relative variance of nonfundamental shocks in Regime 2 versus Regime 1.

Figure 5: Smoothed probabilities for different volatility regimes for the Pre-crisis Period
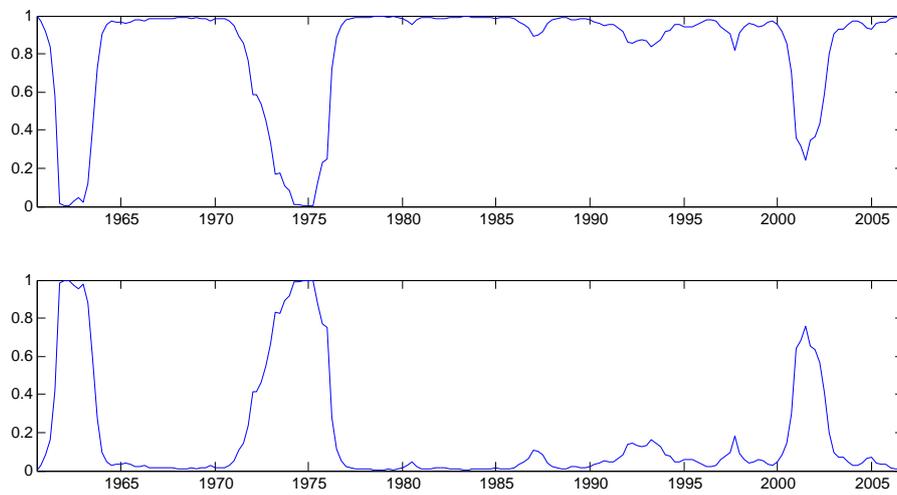
Notes: This graph depicts the smoothed probabilities estimated from the Markov Switching VAR model with two states and two lags based on data from 1960 to 2007. The top panel shows the probability of the system being in a low-volatility regime, while the bottom panel represents the probability of being in a high-volatility regime. It is noticeable that this graphs resemble closely with the first two subplots in Figure 2, which is based on estimation on the full sample.

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