Quantitative forward guidance and the predictability of monetary policy - A wavelet based jump detection approach -

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Abstract
The publication of a projected path of future policy decisions by central banks is a controversially debated method to improve monetary policy guidance. This paper suggests a new approach to evaluate the impact of the guidance strategy on the predictability of monetary policy. Using the example of Norway, the empirical investigation is based on jump probabilities of interest rates on central bank announcement days before and after the introduction of quantitative guidance. Within the standard semimartingale framework, we propose a new methodology to detect jumps. We derive a representation of the quadratic variation in terms of a wavelet spectrum. An adaptive threshold procedure on wavelet spectrum estimates aims at localizing jumps. Our main empirical result indicates that quantitative guidance significantly improves the predictability of monetary policy.

Keywords: Central bank communication, interest rate projections, semimartingales, Locally Stationary Wavelet processes, jump detection.
JEL classification: E58, C14, C58

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1 Introduction

Guiding expectations about future policy decisions has become a standard practice of central banks around the world. Beside setting a very short term interest rate, i.e. the key rate, the expectations management about future key rate settings is an important instrument of monetary policy. However, specific techniques to manage expectations are remarkably different, see Blinder et al. (2008). Most central banks, including the Bank of England and the European Central Bank, give only qualitative signals about the likely direction of the next few policy decisions. In contrast, a small but increasing number of central banks implement a quantitative strategy by publishing numerical projections of key rates up to three years into the future.\footnote{Central banks that guide quantitatively are the Reserve Bank of New Zealand (since 1997), the Norges Bank (since 2005), the Swedish Riksbank (since 2007), the Czech National Bank (since 2008), the Sedlabanki Islands (since 2007) and, in a slightly different way, the Federal Reserve (since 2012).}

From both a theoretical and empirical perspective, there is a controversy about quantitative guidance. Mishkin (2004) addresses the concern that markets misinterpret the projections as a commitment to future policy decisions. Morris and Shin (2002) and Rudebusch and Williams (2008) show that central banks’ projections may crowd out private forecasts which results in even worse outcomes relative to the case of no central bank guidance. While empirical papers such as Detmers and Nautz (2012), Ferrero and Secchi (2009) and Moessner and Nelson (2008) find significant responses of market interest rates to a published policy path, advantages of quantitative guidance compared to qualitative guidance are not established yet.

A cross-country, event-study by Kool and Thornton (2012) suggests that adjustments of interest rates on monetary policy announcement days do not depend on the guidance regime. In the same vein, the GARCH approach of Andersson and Hofmann (2009) indicates equally likely policy surprises irrespective of whether forward guidance involves the publication of an own interest rate path or not.

The current paper proposes a jump detection approach to investigate whether a change in the guidance strategy from qualitative to quantitative guidance increases the predictability of monetary policy. We follow the analogy of Das (2002) and Piazzesi (2005), and identify monetary policy surprises through jumps in interest rates on policy announcement days. Due to relatively short sample periods of quantitative guidance, combined with occasional, extraordinary market responses to cen-
Central bank announcements, it appears beneficial to avoid inference on magnitudes of policy surprises. Parameter estimates of conventional event-study approaches are usually dominated by a small fraction of extreme values. Applying standard distributional assumptions renders it difficult to find any statistically significant difference between sample estimates. Therefore, we focus on a binary jump variable. The relative number of jumps within a qualitative and quantitative guidance period serves as a statistic to test the main hypothesis of less policy surprises during the period of key rate projections.

Estimation and testing for jumps in financial data usually refers to semimartingales which constitute a wide class of continuous time, stochastic processes. In the literature on non-parametric inference on semimartingales, several approaches have been proposed to separate a diffusion and jump part given discrete observations. Estimates of both components refer to the quadratic variation made up as the sum of the integrated volatility and the jump variation, see e.g. Aït-Sahalia and Jacod (2012) and Dumitru and Urga (2012). However, most methods like the bi-power variation of Barndorff-Nielsen and Shephard (2004) are not meant to localize the exact timing of jumps. To localize jumps, Lee and Mykland (2008) and Andersen et al. (2007) propose a t-test-type approach performed on each return standardized by the bi-power variation. Similarly, Mancini (2009) and the extension of Podolskij and Ziegel (2010) rely on the realized volatility and bi-power variation and utilize a thresholding technique on squared return series to locate jump points. Instead of thresholding returns directly, Fan and Wang (2007) propose a more comprehensive approach and use coefficients of a discrete, decimated wavelet transform. With the wavelet transform the information on jump locations and variation is stored at high-resolution levels (fine scales) while useful information for integrated volatility is captured by low-resolution levels (coarse scales). The variance of the integrated volatility estimator benefits from the orthogonality of the decimated transform. However, the orthogonality came at the price of localizing jumps at dyadic locations only. Furthermore, no straightforward expression for increments in the quadratic variation can be derived.

The methodology proposed here extends the wavelet approach of Fan and Wang (2007) to the discrete, non-decimated wavelet transform. We establish a pointwise decomposition of the quadratic variation, which we finally use to detect jump points. The decomposition is considered in the context of Locally Stationary Wavelet (LSW)
process of Nason et al. (2000) and van Bellegem and von Sachs (2008). Given the link between a discretely sampled semimartingale and the class of LSW processes, we derive a representation of the quadratic variation in terms of a wavelet spectrum. Since the local regularity of a semimartingale translates to its wavelet spectrum, jump points of an observed process can be detected via wavelet spectrum estimates. A smooth version of the adaptive threshold estimator of von Sachs and MacGibbon (2000) applied to the spectrum localizes jumps of the underlying semimartingale.

Empirical evidence refers to the Norwegian example. The case of Norway provides a sufficiently long history of six years of key rate projections. Furthermore, it allows observations of a 2001 to 2011 sample to be partitioned into a qualitative and quantitative guidance period. The main results on daily short and long term interest rates indicate: First, quantitative guidance does not further improve the predictability of current policy decisions. Second, switching from qualitative to quantitative guidance significantly decreases revisions of markets’ expectations about future policy decisions. Since a qualitative guidance strategy already implies a high level of short term guidance, we conclude that quantitative guidance significantly enhances the transparency about the decision making process of monetary policy and improves the longer-term predictability of central banks.

The remainder of the paper proceeds as follows. Section 2 highlights the rational of central banks’ forward guidance and provides examples of qualitative and quantitative guidance. The Norwegian interest rate data is introduced in the context of target and path surprises in Section 3. The economic hypotheses are formulated in Section 4. Section 5 introduces the jump detection approach and the estimator of jump probabilities. Section 6 contains the empirical part. We show estimates of wavelet spectra and jump probabilities as well as results of the hypothesis tests. Section 7 concludes.

2 Monetary policy guidance: the Norwegian example

Central Banks’ main instrument to reach the goal of price stability is a very short term interest rate, i.e. the key rate. It is, however, not the short end of the term

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2In Norway, the key rate is the overnight deposit rate. Decisions on the key rate are made on monetary policy announcement days every sixth week, see Norges Bank (2009).
structure but a longer rate that affects economic conditions. Therefore, the transmission of monetary policy to longer-term rates is of crucial importance. Relations between short and longer-term interest rates are usually discussed in the context of the expectations hypothesis. Its simplest, linearized form is given by:

$$R_t^{(n)} = \frac{1}{n} \left( r_t + \sum_{i=1}^{n-1} E_t(r_{t+i}) \right),$$

where $R_t^{(n)}$ denotes the interest rate at time $t$ and maturity $n > 1$, $r_t$ is the short term rate with $n = 1$ and $E_t$ is the expectations operator with respect to information available at $t$. The expectations hypothesis highlights that steering the very short end of the term structure, $r_t$, has only minor effects on longer-term yields, $R_t^{(n)}$. What matters is the path of expected future short rates, $\sum_{i=1}^{n-1} E_t(r_{t+i})$. Therefore, central banks have implemented communication strategies to manage expectations about a future policy path.

Most central banks adopt a qualitative guidance strategy. The technique provides hints on the most likely direction of the next few key rate changes. An example of qualitative guidance is given in a talk of the Norges Bank’s governor, Gjedrem (2003): "We have experienced a period of monetary policy easing. This period is not over." While this statement is precise about the sign of the next policy move, its magnitude and implications for subsequent key rate changes remains vague. From 2001 to 2005 such qualitative hints within speeches, interviews and press conferences constitute the Norges Bank’s guidance strategy.
A relatively new strategy, that has gained in importance in recent years, is quantitative guidance. Central banks that guide quantitatively are much more explicit about their future policy assessments. They provide a path of numerical values of future key rates on pre-announced publication days. In November 2005 the Norges Bank has replaced its qualitative guidance strategy by quantitative guidance. Since then numerical values of a projected path of future key rates are published at every third monetary policy announcement day. Figure 1 illustrates the Norges Bank’s history of quantitative guidance. Each dashed line belongs to a projected path that reflects quarterly averages of the key rate up to three years into the future.\(^3\)

The main goal of quantitative guidance is to enhance the efficiency of monetary policy. As highlighted by Svensson (2006), given a path of expected future key rates along with forecasts of major macroeconomic variables, a central bank’s reaction function becomes more explicit. Consequently, under stable central banks’ preferences, quantitative guidance should reduce the need of market adjustments on policy announcement days, thus, enhance the predictability of monetary policy.

### 3 Target and path surprises

We distinguish between the short term and long term predictability of monetary policy by referring to the target and path surprise framework introduced by Gürkaynak et al. (2005). With respect to the expectations hypothesis (1), target and path surprises are based on the driving forces behind short and longer-term interest rates. A target surprise is defined as an unexpected key rate change. According to (1), the key rate is a major component of shorter-term interest rates. Since decisions on the Norges Bank’s key rate are made every sixth week, we define jumps in 1 month Norwegian money market rates on monetary policy announcement days as an indicator of unexpected policy decisions. In contrast to shorter-term rates, longer-term rates are mainly driven by expectations about future key rates. Therefore, the path surprise measures the degree to which market participants revise their expected monetary policy path following the actual decision or aspects of policy guidance. We interpret jumps in a 1 year money market rate and a 3 and 5 year Norwegian government bond rate on monetary policy announcement days as evidence about

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\(^3\)Projections are published in Monetary Policy Reports. Further information about the underlying model and additional criteria are provided in Brubakk et al. (2006) and Qvigstad (2006).
revisions in market’s expectations about future policy decisions.

We assess target and path surprises for a sample period from March 2001 to December 2011. The starting date of our sample is marked by the introduction of inflation targeting in Norway. The sample includes a total of 2721 daily observations, including 94 policy announcement days. Figure 2 gives an example of the data. The figure depicts day to day changes of the 1 month and 3 year Norwegian interest rates in conjunction with estimates of the respective return probability density function. Norwegian interest rates reflect the well known and often cited stylized facts of financial return series: the sample mean is close to zero, the marginal distribution is slightly skewed and heavily tailed, volatility is clustered while the autocorrelation of adjacent returns appears to be small. In particular, the leptokurtic shape of the density functions indicates the presence of jumps. We aim at detecting these jumps to assess the predictability of monetary policy.

4 Economic Hypotheses

To assess the impact of the guidance strategy on the predictability of monetary policy, we define three economic hypotheses. Each hypothesis refers to a specific classification of monetary policy announcement and non-announcement (all other days) days. In particular, we distinguish between a qualitative and a quantitative guidance period. The criterion to assess the predictability are jump probabilities.

**News-hypothesis:** Monetary policy announcements contain relevant news. The announcements induce significant target and path surprises.

We call target and path surprises significant if jump probabilities on announcement days are significantly larger than on non-announcement days. The verification of the News-hypothesis across the qualitative and quantitative guidance samples already provides a first indication about an impact of the guidance strategy. However, shifts in target and path surprises can be driven by a change of jump probabilities on non-announcement days. Therefore, our main focus is on the predictability across the qualitative and quantitative guidance periods.

**Guidance-hypothesis:** Quantitative guidance enhances the predictability of monetary policy. Quantitative guidance induces significantly less target and path surprises than qualitative guidance.

Less target surprises imply a decrease in jump probabilities of the 1 month rate. A significant decrease determines improvements in the short term predictability of monetary policy. In contrast, less path surprises are identified by a decrease in jump probabilities of the 1, 3 and 5 year interest rates. A significant decrease enhances the longer-term predictability of monetary policy. As a control for an overall decrease of jump probabilities, we state the Jump-level-hypothesis.

**Jump-level-hypothesis:** The overall level of jump probabilities of interest rates is constant across the qualitative and quantitative guidance sample.

We evaluate the Jump-level-hypothesis via jump probabilities on non-announcement days. The three hypotheses are further formalized in Table 2. If both, the Guidance- and Jump-level-hypothesis hold true, we deduce that quantitative guidance significantly increases the short and longer-term predictability of monetary policy.

Important for a valid analysis of our hypothesis: Since 2003 the Norges Bank provides hints about future policy decisions on policy announcement days only, see
Holmsen et al. (2008). Thus, monetary policy surprises are not artificially reduced by information given between announcement days.

In the next section we propose a jump detection approach for the daily interest rate data. The jump detection provides a first step in the estimation of jump probabilities.

5 Methodology

5.1 Semimartingales and Locally Stationary Wavelet processes

Following the standard literature on asset price modeling (see e.g. Aït Sahalia and Jacod 2012), we assume the yield $X = (X_t)_{t \in [0,1]}$ of a specific maturity to be a semimartingale.

$$dX_t = \mu_t dt + \sigma_t dB_t + dJ_t, \quad t \in [0,1]$$  \hspace{1cm} (2)

For the present empirical investigation, the $t \in [0,1]$ interval refers to the whole sample period from 2001-2011. The terms on the right hand side of (2) correspond to the drift, diffusion and jump part of $X$. $B_t$ is a standard Brownian motion and the diffusion variance $\sigma_t^2$ is called the spot volatility. To detect jumps in $X$ which reflect surprise elements, it is informative to consider a compound Poisson jump part. Hence, we assume $J_t$ to consist of a Poisson process $N_t$, that counts the number of jumps up to time $t$. The jump size at time $t$ is $\Delta J_t = J_t - J_{t-}$. Estimates of the quantities in (2) are usually build on the quadratic variation,

$$QV_t = \int_0^t \sigma_s^2 ds + \sum_{\ell=1}^{N_t} (\Delta J_{\ell})^2, \quad t \in [0,1],$$ \hspace{1cm} (3)

including the integrated volatility and the jump variation. The focus of our investigation is on the process $N_t$. In particular, we are interested in daily time intervals on $[0,1]$ where $N_t$ increases by one.

With daily data at hand, we approximate the semimartingale at $T$ equally spaced, discrete time points $t_i = i/T, i = 1, ..., T$. The sampling of the increments of the semimartingale gives a (mean zero) sequence $\Delta X = (\Delta X_{t_i})_{i, \in [0,1]}$, with daily dif-
ferences $\Delta X_{t_i} = X_{t_i} - X_{t_{i-1}}, \ i = 1, \ldots, T$. Following the definition of Locally Stationary Wavelet (LSW) processes of Nason et al. (2000) and the generalization of van Bellegem and von Sachs (2008), we can write the sampled increments of the semimartingale in terms of a LSW process. The mean-square representation of the sampled increments from (2) is given by:

$$
\Delta X_{t_i} = \sum_{j=-\infty}^{-1} 2^{j/2} \sum_{k=-\infty}^{\infty} W_j(t_k) \psi \left( \frac{t_k - t_i}{2^j} \right) \xi_{j,t_k}, \quad t_i = i/T, \ i = 1, \ldots, T,
$$

with a scale $j$ and location $t_i$ dependent transfer function $W_j : (0,1] \to \mathbb{R}$, a non-decimated wavelet system $(2^{j/2} \psi(\cdot))_{j,t_k}$ generated via dilation ($j$) and translation ($t_k$) of a mother wavelet $\psi$ and a zero mean orthonormal identically distributed random process $\xi_{j,t_k}$. For ease of presentation, LSW processes are usually build on the simplest discrete non-decimated system, called the Haar system. However, in general any function that satisfies time-frequency localization properties can be used as a mother wavelet, compare Fryzlewicz and Nason (2006) and the admissibility conditions in Daubechies (1992, Sec. 1.3).\(^5\) Out of (4), the quantity of interest is the wavelet spectrum

$$
S_j(t_i) = W_j^2(t_i), \ j = -1, -2, \ldots.
$$

Paralleling classical Fourier spectral analysis, the wavelet spectrum provides a decomposition of the process’ variance. The larger the wavelet spectrum at scale $j$ and time point $t_i$, the more dominant is the contribution of scale $j$ in the variance at time $t_i$. In the present context of semimartingales, the spectrum $S_j(t_i)$ is considered a decomposition of the quadratic variation. Given (4), from Proposition 1 of van Bellegem and von Sachs (2008) and the wavelet integrated volatility of Høg and Lunde (2003) and Fan and Wang (2007), a representation of the quadratic variation in terms of the LSW spectrum carries over:

$$
QV_{t_i}^{(LSW)} = \sum_{s=1}^{i} \sum_{j=-\infty}^{-1} S_j(t_s), \quad t_i = i/T, \ i = 1, \ldots, T.
$$

\(^5\)Small scales $j$ correspond to low frequencies, while scales approaching -1 correspond to high frequencies. For an introduction to wavelets see Daubechies (1992) and Vidakovic (1999). In our application, $\Delta X$ outside the $(0,1]$ interval are captured by the reflection boundary condition. However, to ease notation, in the following boundaries are treated in the sense of zero padding.
The LSW representation transmits the local regularity of \( X \) to the wavelet spectrum \( S_j(t_i) \) at scale \( j = -1, -2, \ldots \) and time point \( t_i \). Since the jump part \( J_t \) constitute a high frequent characteristic of the underlying process, it materializes at the fine scales of the wavelet spectrum. Lower-resolution levels capture the integrated volatility. The different decay orders of wavelet coefficients of the Brownian and jump part formulated by Fan and Wang (2007) translate to the wavelet spectrum and allow the jump detection via thresholding.

5.2 Jump localization on wavelet spectrum estimates

As introduced by Nason et al. (2000), the expansion of an observed process \( \Delta X \), on a non-decimated wavelet system \( (2^{j/2} \psi(\cdot))_{j,t_k} \) defines an estimator of the wavelet spectrum. The wavelet periodogram at time point \( t_i \) and resolution level \( j \) is given by:

\[
I_j(t_i) = 2^j \left( \sum_{k=1}^{T} \Delta X_{t_k} \psi \left( \frac{t_k - t_i}{2^j} \right) \right)^2, \quad j = -J, \ldots, -1, \; t_i = i/T, \; i = 1, \ldots, T, \; (7)
\]

with \(-J = \lceil -\log_2 T \rceil \) the coarsest scale. As in the Fourier case, the periodogram is not a consistent estimator of the spectrum (5). For fixed scales \( j \), its expected value is

\[
E(I_j(t_i)) = \sum_{\ell=-J}^{-1} A_{j,\ell} \tilde{S}_\ell(t_i),
\]

where \( A_{j,\ell} \) are elements of the invertible, inner product matrix of the autocorrelation wavelets, see van Bellegem and von Sachs (2008).\(^6\)

Compared to the decimated wavelet transform, the bias is the price to pay for the better time resolution of the non-decimated transform.\(^7\) However, as the expected value of the periodogram suggests, (asymptotic) unbiasedness can be achieved by pre-multiplying (7) with the elements of the inverted, inner product matrix:

\[
\hat{S}_j^{(UB)}(t_i) = \sum_{\ell=-J}^{-1} A_{j,\ell}^{-1} I_\ell(t_i)
\]

is the unbiased spectrum estimator. The consistent estimator of the spectrum is an appropriately smoothed version of (8) across time, denoted by \( \hat{S}_j(t_i) \), see e.g.

\(^6\)Note, that \( A \) is not simply the identity matrix since the non-decimated system is not orthogonal.

\(^7\)The use of the decimated wavelet transform gives values of \( I_j \) at dyadic time points \( t_i 2^j, i = 1, \ldots, T, j = -J, \ldots, -1, \) only. The decimated transform does not allow the local variance to be written as a wavelet spectrum, see van Bellegem and von Sachs (2008).

We simply deduce, since \( \hat{S}_j(t_i) \) is a consistent estimator of the local variation for each scale \( j \) and time point \( t_i \), \( \sum_j \hat{S}_j(t_i) \) is a consistent estimator of the increment in the quadratic variation of \( X \) at \( t_i \). Additional summation across time, \( \sum_t \sum_j \hat{S}_j(t) \), provides the LSW estimator of the quadratic variation (6). The estimator parallels the expression of the wavelet realized volatility in Fan and Wang (2007). However, in contrast to Fan and Wang (2007), the summation object \( \hat{S}_j(t_i) \) has the crucial advantage of a more detailed time resolution and a direct statistical meaning as a consistent decomposition of the quadratic variation. In particular, the detailed time resolution of \( \hat{S}_j(t_i) \) allows for a pointwise localization of jumps.

To detect the jumps, we define an upper bound of a spectrum \( \tilde{S}_j(t_i) \) at time point \( t_i \) and scale \( j \) that contains the drift and diffusion part of (2) only, i.e. \( \hat{S}_j(t_i) \) belongs to a process without jumps. Focusing on high resolution levels \( j \to -1 \), we localize jumps in \( X \) by

\[
\hat{N}_{j,t_i} = \sum_{s=1}^{i} \mathbb{1} \left( \hat{S}_j(t_s) \geq \tilde{S}_j(t_s) \right), \quad t_i = i/T, \ i = 1, ..., T. \tag{9}
\]

At fine resolution levels \( j \), the indicator function \( \mathbb{1}(\cdot) \) equals one whenever the estimated spectrum \( \hat{S}_j(t_i) \) exceeds the upper bound of the spectrum \( \tilde{S}_j(t_i) \) without jumps. \( \tilde{S}_j(t_i) \) can be considered a smooth version of the time varying threshold of von Sachs and MacGibbon (2000).

\[
\tilde{S}_j(t_i) = (n + 1)^{-1} \sum_{k=-n/2}^{n/2} \hat{S}_j(t_{i-k}) \delta^2(n), \tag{10}
\]

with \( n << T \) a positive, even number and \( \delta(n) \) a reasonable threshold criterion. One choice of criterion is that of the universal threshold, i.e. \( \delta(n) = \sqrt{2 \log n} \), see e.g. Wang (1995). Since the threshold \( \tilde{S}_j(t_i) \) is time varying and depends on the spectrum, the number of detected jumps is not artificially increasing on local time intervals where the integrated volatility is high. The adaptiveness appears as an advantage compared to usually employed global tuning parameters. In the empirical application we do not consider a comparison of different scales \( j \), see the discussion of Raimondo (1998), but follow Wang (1995) and Fan and Wang (2007) and refer to the finest scale \( j = -1 \). Thus, we detect a jump at \( t_i \) if \( \hat{N}_{-1,t_i} - \hat{N}_{-1,t_{i-1}} = 1 \).
Based on the spectrum estimates, we are now in the position to localize the jump points of the observed interest rate data. In order to test the economic hypotheses formulated in Section 4, we next define an estimator of jump probabilities.

### 5.3 Jump probabilities

The main statistic to test for an impact of increased forward guidance are jump probabilities $p_M$ on different subsets of the date vector, $\mathcal{M} \subset [t_1, \ldots, t_T]$. The sets belong to either policy announcement days or days without policy announcements. Furthermore, we distinguish between a qualitative and a quantitative guidance period. Table 1 gives a precise formulation of the different sets $\mathcal{M}$. To estimate the jump probabilities for each set, we localize the jumps from the daily interest rate data via wavelet spectrum estimates. The number of jumps for the whole observation period is given by $\hat{N}_{j=-1,t_T=1}$ from (9). Restricting the detected number of jumps to sets $\mathcal{M}$, provides the jump probability estimator

$$\hat{p}_M = h^{-1} \hat{N}_{-1,1}^M,$$

with $h$ the number of days that belong to $\mathcal{M}$. Since the counting process $(N_{-1,t_i})_{t_i \in (0,1]}$ is defined as a Poisson process, we approximate the occurrence of jumps at a particular day by the Bernoulli distribution. The variance of the estimator (11) is, therefore, given by $\text{Var}(\hat{p}_M) = \hat{p}_M(1 - \hat{p}_M)/h$. As a natural choice, we build the hypothesis tests on the principles of a simple t-test.

### 6 Empirical results

#### 6.1 Wavelet spectra of Norwegian interest rates

In order to get deeper insights into the variation and jump characteristics of the observed processes, we estimate wavelet spectra of the interest rates as described in Section 5.2. Computational steps closely follow Nason et al. (2000). Results are based on Daubechies’ least asymmetric wavelets, i.e. Symmlets, and a minimax threshold criterion, see Vidakovic (1999, Sec 3.4.5 and 6.4) for further details. In contrast to the differencing operations of Haar wavelets, Symmlets are closer related to averaging filter functions, thus, can be considered to provide local averages of the
Figure 3: Wavelet spectrum estimates for 1 month and 3 year interest rates.

A) Wavelet Spectrum estimates

B) Finest scale and threshold

C) Truncated finest scale

Notes: Spectrum estimates of 1 month (left) and 3 year (right) interest rates based on Symmlets with six vanishing moments and a wavelet shrinkage method (hard thresholding). Part A depicts estimates of wavelet spectra from the finest scale $j = -1$ up to the coarse scale $j = -12$ (color code: the darker the shading the stronger the impact of the particular scale and time point). Part B highlights the finest scale $j = -1$ (gray line) and the minimax threshold with a window width of six month (black line). Part C shows values at $j = -1$ above the threshold.
return series. We chose a bandwidth of 12 days at the finest scale, i.e. six vanishing moments, to achieve some degree of smoothness. We apply the minimax threshold criterion since it is less conservative than the universal threshold. As a robustness check, Appendix B reports results on Haar wavelets and the universal threshold. We also consider results for different numbers of vanishing moments.

Figure 3 depicts estimated spectra of the 1 month (left column) and the 3 year (right column) interest rates. Part A shows wavelet spectrum estimates on a time-scale plane. Non-surprisingly, the color code of the plots indicates that most of the return variation can be attributed to fine scale components (dark shading). On average, the finest scale \( j = -1 \) accounts for more than 50 percent of the total variation (6). Coarser resolution levels gain in importance around 2008. The darker shading displays the volatility increasing effects of the global financial crisis. However, due to the local adaptiveness of the threshold, we do not distinguish between crisis and non-crisis periods. As Part B of Figure 3 shows, the threshold (black line) adjusts to the local increase of the variation (gray line). Since spectrum estimates at scale \( j = -1 \) above the threshold localize jumps, the number of detected jumps is not artificially increasing during times of financial stress. Truncated estimates of the finest scale above the threshold are shown in Part C of Figure 3. The truncated estimates indicate the heterogeneity of squared jump sizes across time. Our focus on jump probabilities avoids handling this heterogeneity.\(^8\)

To analyze changes in target and path surprises we now focus on jump probabilities and the hypotheses formulated in Section 4.

### 6.2 Quantitative guidance and monetary policy surprises

In the previous subsection we estimate spectra of the different interest rate series and detect jump points according to the thresholding rule presented in Section 5.2. We now take advantage of the localized jump points to verify the economic hypotheses discussed in Section 4. We simply compare different jump probabilities by a standard approximation of the t-test. Tests are titled according to their respective hypothesis. The estimated jump probabilities are presented in Table 1, Table 2 reports the test results.

\(^8\)See further assessments of the jump points in Appendix A.
Table 1: Jump probabilities on policy announcement and non-announcement days.

<table>
<thead>
<tr>
<th>Guidance period</th>
<th>Sets</th>
<th>Observations</th>
<th>( \hat{p}_{M_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 month</td>
</tr>
<tr>
<td>Qualitative</td>
<td>( M_1 = { {\text{Announcement days</td>
<td>Qual}} } )</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5.65)</td>
</tr>
<tr>
<td></td>
<td>( M_2 = { {\text{Non-announcement days</td>
<td>Qual}} } )</td>
<td>1129</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.58)</td>
</tr>
<tr>
<td>Quantitative</td>
<td>( M_3 = { {\text{Announcement days</td>
<td>Quant}} } )</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3.94)</td>
</tr>
<tr>
<td></td>
<td>( M_4 = { {\text{Non-announcement days</td>
<td>Quant}} } )</td>
<td>1498</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.56)</td>
</tr>
</tbody>
</table>

Notes: Estimates of jump probabilities \( \hat{p}_{M_i} \) of set \( i = 1, 2, 3, 4 \), and corresponding standard deviations are given in percent. Jump detection refers to (9) and Symmlets with six vanishing moments and an adaptive, six month minimax threshold.

Table 2: Results of hypothesis tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>Hypothesis</th>
<th>( p )-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 month</td>
</tr>
<tr>
<td>News-test(Qual)</td>
<td>( H_0 := \hat{p}<em>{M_1} \leq \hat{p}</em>{M_2} ), ( H_1 := \hat{p}<em>{M_1} &gt; \hat{p}</em>{M_2} )</td>
<td>0.03</td>
</tr>
<tr>
<td>News-test(Quant)</td>
<td>( H_0 := \hat{p}<em>{M_3} \leq \hat{p}</em>{M_4} ), ( H_1 := \hat{p}<em>{M_3} &gt; \hat{p}</em>{M_4} )</td>
<td>0.14</td>
</tr>
<tr>
<td>Guidance-test</td>
<td>( H_0 := \hat{p}<em>{M_1} \leq \hat{p}</em>{M_3} ), ( H_1 := \hat{p}<em>{M_1} &gt; \hat{p}</em>{M_3} )</td>
<td>0.20</td>
</tr>
<tr>
<td>Jump-level-test</td>
<td>( H_0 := \hat{p}<em>{M_2} = \hat{p}</em>{M_4} ), ( H_1 := \hat{p}<em>{M_2} \neq \hat{p}</em>{M_4} )</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Notes: Jump probabilities \( \hat{p}_{M_i} \) of set \( i = 1, 2, 3, 4 \) are taken from Table 1. \( p \)-values refer to the approx. normally distributed test statistic \( z_{ij} = (\hat{p}_{M_i} - \hat{p}_{M_j})/\sqrt{\text{Var}(\hat{p}_{M_i}) + \text{Var}(\hat{p}_{M_j})} \). We reject the Null hypotheses at a confidence level of 10%.
During the period of qualitative guidance Norwegian interest rates show a pronounced response to monetary policy announcements. On average, jumps on policy announcement days are three times more likely than on all other days. For example, Table 1 shows that the three year rate jumps on announcement days with an estimated probability of 12.5%, i.e. five jumps out of 40 policy days. In contrast, on non-announcement days the probability is 3.45%, corresponding to 39 jumps out of 1129 non-policy days. As the \( p \)-values of the News-tests in Table 2 indicate, at a confidence level of 10%, all maturities have significantly larger jump probabilities on monetary policy announcement days. This suggests significant target and path surprises during the period of qualitative guidance.

In the quantitative guidance period jump probabilities of the 1 and 5 year maturities are no longer larger on policy announcement days than on non-announcement days. In general, the difference between jump probabilities is much smaller than during the period without key rate projections. For the example of the three year rate, the probability of a jump on policy announcement days has decreased to 3.70%. With a total number of 54 policy days this implies halving the absolute number of jumps and a decline by 8.8 percentage points. At the same time, jumps on non-announcement days decline by 0.5 percentage points only. \( p \)-values of the News-tests show that jump probabilities are no longer significantly larger on monetary policy days. Consequently, no significant target and path surprises can be observed during the period of quantitative guidance.

Since we find significant policy surprises during the qualitative guidance period but non-significant policy surprises since key rate projections are published, evidence so far suggests that quantitative guidance increases the predictability of monetary policy. Utilizing the Guidance- and Jump-level-test, we now test across the qualitative and quantitative guidance sample to verify whether target and path surprises decrease significantly.

Regarding the short term predictability of monetary policy, the Guidance-test applied to the 1 month rate indicates no significant reduction in target surprises. The jump probabilities on policy announcement days decline from 15% to 9.26%. However, with a \( p \)-value of 0.2, the guidance-test suggests no further improvements in the predictability of current policy decisions. In contrast, the Guidance-tests for longer-term maturities of 1, 3 and 5 years reflect significantly less jumps on monetary policy announcement days. Jumps of the 1 and 3 year rates decline from 12.5%
to 3.70%. With the introduction of quantitative guidance, path surprises became significantly less likely, implying that market participants revise their expected path of future key rates less frequent. Quantitative guidance consequently enhances the longer-term predictability of monetary policy.

Finally, results of the Jump-level-test strengthen the outcomes of our empirical study. The Jump-level-test indicates no significant changes in jump probabilities on non-announcement days between the two guidance samples. Thus, the outcome of the Guidance-test is not driven by an overall decline in jump probabilities. We rule out an overall structural break of jump intensities between the two guidance periods and interpret changes on policy announcement days as driven by changes in policy guidance.

7 Conclusion

This paper’s contributions are twofold: First, we introduce a new methodology to detect jumps in time series data. We propose to formulate Locally Stationary Wavelet processes in the context of semimartingales. The connection allows a decomposition of the quadratic variation of semimartingales by means of a wavelet spectrum. We detect jumps on wavelet spectra via an adaptive thresholding rule. The pointwise time resolution and a direct statistical meaning as a consistent decomposition of the quadratic variation appear as benefits of the spectrum estimator. Second, based on the Norwegian example, we provide new evidence in favor of quantitative guidance compared to qualitative guidance. While our empirical results suggest that quantitative guidance does not improve the predictability of actual policy decisions, we find that switching from qualitative to quantitative guidance stabilizes expectations about future policy decisions. We conclude that key rate projections improve the longer-term predictability of monetary policy.

Extension of the present work appear promising. We already indicate that the jump detection approach can be easily extended to define estimates of the jump variation and integrated volatility. Furthermore, the formulation of semimartingales under additive noise appears as a simple extension since the wavelet transform can be considered a smoothing operator and a further de-noising step is involved in the spectrum estimate. For empirical investigations, longer histories of key rate projections would be helpful. Studying market adjustments to central bank’s revisions at
particular projection horizons would further increase the understanding about central banks’ ability to drive markets’ expectations. The analysis of common jumps (cojumps) of particular interest rate maturities appears as a promising technique to investigate that relation.

References


A Evaluation of jump days

Based on the data introduced in Section 3 and the setup of Section 5, we investigate the size of returns on days where jumps occur (jump days). On average, the absolute change in interest rates should be relatively large on jump days. Table 3 reports these sample averages and contrasts jump days with returns of the whole data set.

Table 3: Average absolute changes in interest rates.

<table>
<thead>
<tr>
<th>Sample</th>
<th>1 month</th>
<th>1 year</th>
<th>3 year</th>
<th>5 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole data set</td>
<td>3.75</td>
<td>3.09</td>
<td>3.69</td>
<td>3.60</td>
</tr>
<tr>
<td>Jump days</td>
<td>19.42</td>
<td>12.3</td>
<td>13.3</td>
<td>11.9</td>
</tr>
<tr>
<td>Announcement-days</td>
<td>3.54</td>
<td>3.10</td>
<td>2.33</td>
<td>4.97</td>
</tr>
<tr>
<td>Jump announc.-days</td>
<td>15.60</td>
<td>10.4</td>
<td>24.0</td>
<td>19.6</td>
</tr>
</tbody>
</table>

Notes: The average of absolute changes in interest rates for different maturities are given in basis points. The whole sample includes 2721 observations and 94 policy announcement days. Announcement days that induce jumps range from 5 jumps (5 year rate) to 11 jumps (1 month rate), compare Table 1.

B Robustness analysis

We document the sensitivity of our main results with respect to the specific choice of wavelets and thresholds. Table 4 shows estimated jump probabilities for the different subsets $M$ of the date vector as defined in Table 1. We present results for the Haar wavelet and Symmlets. The tuning-parameter is the threshold criterion and its specific window size (i.e. six month and three month). We also tried different numbers of vanishing moments for Symmlets (4 and 8), however the impact on jump probabilities is rather negligible, thus, we do not report these results here.

Non-surprisingly, our study shows that the different setups have an impact on the number of detected jumps. Thus, jump probabilities depend on the chosen threshold and wavelet. However, the choice of setup has only a small impact on the relative frequency of jumps between the different sets of the date vector (e.g. comparing the qualitative guidance and quantitative guidance period). Since our economic hypotheses are defined to elucidate relations between jump probabilities (given one
specific setup), our empirical conclusions do not crucially depend on the chosen setup.

Table 4: Jump probabilities for different estimation setups.

<table>
<thead>
<tr>
<th>Set</th>
<th>Minimax(6)</th>
<th>Universal(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 month</td>
<td>1 year</td>
</tr>
<tr>
<td></td>
<td>1 month</td>
<td>1 year</td>
</tr>
<tr>
<td>Haar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{p}_{M_1})</td>
<td>22.5 (6.60)</td>
<td>30.0 (7.25)</td>
</tr>
<tr>
<td>(\hat{p}_{M_2})</td>
<td>4.52 (0.62)</td>
<td>8.59 (0.83)</td>
</tr>
<tr>
<td>(\hat{p}_{M_3})</td>
<td>24.1 (5.82)</td>
<td>20.4 (5.48)</td>
</tr>
<tr>
<td>(\hat{p}_{M_4})</td>
<td>7.88 (0.70)</td>
<td>6.54 (0.64)</td>
</tr>
<tr>
<td>Symmlet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{p}_{M_1})</td>
<td>15.0 (5.65)</td>
<td>12.5 (5.23)</td>
</tr>
<tr>
<td>(\hat{p}_{M_2})</td>
<td>5.31 (0.67)</td>
<td>5.93 (0.70)</td>
</tr>
<tr>
<td>(\hat{p}_{M_3})</td>
<td>14.8 (4.83)</td>
<td>5.56 (3.12)</td>
</tr>
<tr>
<td>(\hat{p}_{M_4})</td>
<td>4.07 (0.59)</td>
<td>5.14 (0.57)</td>
</tr>
</tbody>
</table>

Notes: Estimates of jump probabilities and standard errors in parentheses are given in percent. Sets \(M_i\) refer to \(i = 1, 2, 3, 4\) different subsets of the date vector: \(M_1\) : Policy announcement days during the qualitative guidance period, \(M_2\) : Non-announcement days during the qualitative guidance period, \(M_3\) : Announcement days during the quantitative guidance period, \(M_4\) : Non-announcement days during the quantitative guidance period.
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