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Assortative matching through signals

Friedrich Poeschel*

* Humboldt-Universität zu Berlin, Germany

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Friedrich Poeschel§

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Abstract

When agents do not know where to find a match, they search. However, agents could direct their search to agents who strategically choose a certain signal. Introducing cheap talk to a model of sequential search with bargaining, we find that signals will be truthful if there are mild complementarities in match production: supermodularity of the match production function is a necessary and sufficient condition. It simultaneously ensures perfect positive assortative matching, so that single-crossing property and sorting condition coincide. As the information from signals allows agents to avoid all unnecessary search, this search model exhibits nearly unconstrained efficiency.

JEL Classification Numbers: J64, D83, C78
Key words: Assortative matching, sorting, search, signals, information

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§ Humboldt-Universität zu Berlin, Institut für Wirtschaftstheorie II, Spandauer Straße 1, 10178 Berlin, Germany; friedrich.poeschel@googlemail.com

1 Introduction

Search is a reaction to insufficient information: if we knew where to find the best available option, we would not search for it. If a lack of information is thus the problem, exchange of information can be the solution. When agents form matches on decentralised markets such as the labour market, they can exchange a great deal of information before they even meet - using job advertisements and applications. Rapid and large information flows through email and internet have the potential to eliminate any lack of information, and they can thereby reduce search unemployment to a minimum. However, especially when made online, advertisements and applications can be truthful just as well as they can be full of lies. Indeed, given that it matters with whom an agent matches, one would expect that agents manipulate information so as to attain particular matches.

This paper proposes a simple market design that leads agents to exchange truthful information. We build a model with heterogeneous agents whose type is private information, and we let them choose the marketplace where they look for a match. The market design consists of two requirements: first, each marketplace has to be publicly designated for a specific set of types. Second, agents are asked to declare their type before entry to a marketplace and they are only granted entry if they declare a type that the marketplace is designated for. For example, only if they claim to hold a degree will they be invited to a graduate job fair. Entering a certain marketplace then becomes a (costless) signal of an agent's type.[1] Other agents can never observe the true type, so that agents' bargaining prior to a match is based on these signals. That is, agents who claim to have a high type will be expected to produce like a high type. With only mild complementarities in match production, any incentive to lie then disappears: too much would be expected of low types for them to still gain from matching incognito with high types.

As a real-world example for the role of complementarities, suppose a low-skilled worker faces the choice between working at McDonalds and working at McKinsey. While McKinsey would pay a higher salary, the worker would have to perform there like her high-skilled colleagues. The sheer effort and the extra hours needed to reach this performance can outweigh the benefit of a higher salary, so that the low-skilled worker actually prefers working at McDonalds. In our model, a low type can conceal the difference between expected and actual match output by reducing her net share of the output accordingly, but this reduction can outweigh her gain from higher match production with a high type. If the match production function is *supermodular*, so that agents are themselves more productive when matched to a higher type, the reduction will outweigh the gain: then the output expected from a match of two higher types rises disproportionately, while actual match output with one higher type only rises proportionately.

The types in our model can only match with certain higher types if they themselves

[1] In fact, it is irrelevant for our results whether agents choose a signal and thereby join a marketplace or whether they choose a marketplace and thereby send a signal.

behave like those higher types. When the match production function is supermodular, this behaviour does not pay off, and all agents instead signal their type truthfully. Supermodularity ensures truthful signals by introducing a single-crossing property into agents' marginal productivity, rather than into signal costs as in Spence (1973). Hence, none of our results relies on differential signal costs; after all, writing down an invented CV for an application is as costly as writing down a truthful CV, and painting an advertised job in unduly bright colours is as costly as honestly laying out its dull nature. We therefore normalise signal costs to zero, so that the signals in our model are *cheap talk*.

In practice, lies in applications and job advertisements certainly occur. Yet they seem much less frequent than one might think, given how easy it would be to lie. This suggests that most real-world agents consciously choose not to lie. If so, it remains an open question whether the choice not to lie is rather intrinsically motivated by agents' preferences over matches, as in this paper, or extrinsically motivated, as when agents' claims are rigorously assessed. Circumstantial evidence, such as recurrent incidences of fake doctors, documents that effective assessment is often missing even where strong qualifications are essential. This evidence supports intrinsic motivation, which includes our argument that expected performance levels would appear too demanding to underqualified candidates (apart from a few individuals with boundless self-confidence).

The model can in turn account for real-world behaviour that might puzzle a search theorist. When a worker is found out to have lied in the application, why is the worker then typically fired (or not hired in the first place) rather than being kept on at a different wage? This paper points to asymmetric information: while the worker's exact qualification remains unknown to the employer, it is very likely that a worker who lied is underqualified rather than overqualified. It may then be easier to find replacement rather than to disentangle lies from truth, thereby determine the worker's actual qualifications, and then - if possible at all - adjust the job design to fit these qualifications. Correspondingly, if a low type in our model signals like a high type, meets a high type, but then does not behave like a high type, bargaining will fail because bargaining strategies are based on the signals. All the high type can infer is to be facing some lower type, as higher types cannot gain from such behaviour. A sufficiently high type then prefers meeting another agent to a second round of bargaining with the lower type. Therefore, agents do not get away with reneging on their signals.

When signals thus provide full information, agents know where to find the best available option and their first meeting results in a match. As long as search frictions are not so high that agents are even discouraged from one meeting (and therefore do not participate), the outcome is the same as in the frictionless case: in a setting with frictions and full information, agents find the best option at first try, and in a frictionless setting with imperfect information, nothing keeps agents from searching until they find the best option. Hence, the separating equilibrium of our search model achieves benchmarks set by Becker's (1973) frictionless matching model: not just *positive assortative matching* (likes tend to match with likes) but even perfect positive assortative matching (only equal types match), a stable matching, and maximised aggregate output.

Through the market design proposed here, policy makers can therefore achieve both a shorter search duration and a more efficient sorting. When interpreted in the context of frictional labour markets, the design reduces search unemployment: agents are unemployed only until their first meeting. The delay and the costs associated with the first meeting thus constitute the only difference between the decentralised outcome in the separating equilibrium and the first-best outcome that could be centrally imposed by a social planner. In other words, full information allows agents to avoid almost all search costs (concretely, the costs associated with second and further meetings), so that unconstrained efficiency is almost achieved here despite frictions.

Finally, this paper makes a technical contribution. Our model and Becker's (1973) frictionless setting also have the mild condition for positive assortative matching (PAM) in common: in both models, the necessary and sufficient condition for perfect PAM is supermodularity of the match production function. A search model comparable to ours, but without signals, has been analysed by Shimer and Smith (2000). They establish (imperfect) PAM under the condition that the match production function, the logarithm of its first derivative, and the logarithm of its cross-partial derivative are all supermodular. These conditions are directly comparable to our condition and are unambiguously more restrictive. From an empirical perspective, one would rather expect a mild condition because PAM is a pervasive phenomenon: across regions and cultures, more productive workers tend to be hired by more productive firms and more educated women tend to marry more educated men.[2] Note that the conditions for PAM and for truthful signals exactly coincide in our model: supermodularity of the match production function here ensures both sorting and single-crossing.

The paper proceeds as follows. After further related literature has been discussed in section 2, section 3 specifies a frictional matching market and the procedures of search. Section 4 defines equilibrium in the model and proposes a separating equilibrium in which supermodularity suffices for perfect PAM. Its existence is proven step by step through a series of lemmas in section 5. The separating equilibrium is found to be unique as well as efficient in section 6. There we also discuss the role played by supermodularity and by the model's priors before section 7 concludes. All proofs are provided in the appendix.

2 Relation to the literature

Signals in the context of search have typically been analysed in models of *directed search*: sellers post offers and commit to them; having observed the offers, buyers then simultaneously choose which seller to visit. As buyers cannot coordinate, queues may result

[2] As an exemplary reference for these stylised facts, see Mare (1991).

and only some buyers can buy. This congestion constitutes the only kind of search friction in directed search models. In a sequential search model such as Shimer and Smith (2000), the frictions are instead due to agents' discounting. While sequential search models analyse agents' decentralised behaviour in continuous time, directed search models feature stylised stages at which all agents move simultaneously. The aim of this paper is to integrate signals into a sequential search model close to Shimer and Smith (2000).

The key difference between our model and directed search, however, concerns the reason why signals are informative. In directed search, the assumption that sellers somehow commit to their posted offers is almost ubiquitous. This ad-hoc assumption is made because sellers might otherwise renege on their signals, as we demonstrate in Poeschel (2012). In other words, the reasons why sellers' signals are reliable are exogenous to directed search models. We argue in this paper that, in sequential search models, a simple market design can lead to truthful signals under mild conditions. The core of our analysis will explore why agents might not have an incentive to renege on their signal, even though agents can freely choose their signal at no cost.

Being an exception in the directed search literature, Menzio (2007) is much closer to this paper. He shows for a directed search model with bargaining that cheap talk can endogenously be informative: expectations created by signals feed back into bargaining, so that a correlation arises between signals and actual behaviour. In effect, agents are bound by their signal. The sequential search model in this paper somewhat similarly embeds strategic bargaining, but signals here are perfectly correlated with the actual types. This perfect correlation then allows sorting to be perfect in our separating equilibrium.

The analysis in Eeckhout and Kircher (2010) of sorting in a directed search model (with the commitment assumption) relates to the technical contribution of this paper. They show that PAM will arise for common meeting technologies if the square root of the match production function is supermodular. This condition is weaker than in Shimer and Smith (2000), but still stronger than in Becker (1973) and this paper. Yet our results confirm the impression from Eeckhout and Kircher (2010) that models with more information in the search process only require weaker complementarities for PAM. While they focus on links between these complementarities and agents' individual matching rates, we focus on links between the complementarities and agents' incentives to signal truthfully. By assuming commitment to posted offers, Eeckhout and Kircher (2010) abstract from the issue of truthful signals; in turn, we abstract from differences in matching rates by allowing for any number of marketplaces with constant returns to scale.[3]

A search model built by Chade (2006) features discounting and noisy signals uncontrolled by the agents. Yet these signals are not observed before agents meet. Rather, when

[3]Several other papers identify conditions for sorting, but are not directly applicable to the set-up considered here. Notably, Smith (2006) finds log-supermodularity to be the sorting condition in a model without bargaining, again a stronger condition than in this paper. In Morgan (1998) and Atakan (2006), supermodularity suffices, but the only search frictions in their models are explicit costs that agents pay out of pocket for each meeting. By contrast, our model includes explicit costs in addition to discounting.

agents do meet, they do not observe each others' true types but only the noisy signal. Hence search is still random in this model, and the noisy signals in fact add information frictions to search frictions. Assuming that the noisy signals carry some information - again for exogenous reasons -, matching is shown to exhibit PAM in a very weak sense: the distribution of types that a high type might match with first-order stochastically dominates this distribution for a low type.[4] This paper primarily differs from Chade (2006) in that signals in our model are not informative by assumption but are deliberately and strategically chosen by agents. Moreover, signals are observed before meetings and allow agents to avoid search costs, thereby tending to reduce the effect of frictions.

Jacquet and Tan (2007) consider a search model with non-transferable utility and a particular log-supermodular match production function. For such an environment, Burdett and Coles (1997) found that types segregate into classes and match exclusively within them. Building on this, Jacquet and Tan (2007) let agents establish any number of marketplaces, as in our model. They find that each marketplace is populated by only one class in equilibrium. By going to the appropriate marketplace, each agent can thus avoid meetings that do not lead to a match and can instead match after the first meeting.

However, perfect PAM cannot be achieved in Jacquet and Tan (2007) because agents still have an incentive to invade the marketplaces of slightly higher types: precisely because of frictions, higher types will accept somewhat lower types rather than continue searching. This incentive is absent in the separating equilibrium our model. The key difference is private information: in our model, sufficiently high types never accept lower types they meet because they cannot tell just how low the type is. Only agents whose own type is sufficiently low expect an unknown lower type to be acceptably close to their type. As a result, marketplaces are in equilibrium only populated by one type.

Finally, contributions by Hoppe et al. (2009) and Hopkins (2012) consider signals and sorting in *matching tournaments*, where match partners are essentially prizes for ex-ante investments in signals. In both models, agents first select a costly signal of their privately observed type and then match without frictions. Hopkins (2012) assumes a single-crossing property and Hoppe et al. (2009) assume a specific multiplicative match production function that satisfies log-supermodularity. In the symmetric equilibrium, agents' signals are then strictly increasing in their types. This leads to perfect PAM at the matching stage - just as one would have expected, given Becker's (1973) findings. However, as there are no frictions in these papers, they cannot explain how truthful signals can arise despite the incentive to lie that prevents perfect PAM in Jacquet and Tan (2007).

[4]The same form of sorting is found in a contribution by Lentz (2010) that does not feature any signals but allows for search on the job (more generally, search while matched), while search is also random. Agents in Lentz (2010) and in the related model in Goldmanis et al. (2009) sort only over time. By contrast, the fundamentally different sorting mechanism in our model can explain PAM already among graduates in their first job, without invoking stronger conditions.

3 Model

The market consists of heterogeneous agents who match among themselves. Agents are indexed by a discrete productivity type $x \in \Theta$, where $\Theta = \{\underline{x}, \ldots, \bar{x}\}$ with $\underline{x} > 0$. Types are exogenously given, but only privately observable. For each discrete type, there is a continuum of agents and the overall mass of agents is normalised to 1. The measure of agents with types weakly below $x \in \Theta$ is denoted $L(x)$, where $L(\cdot)$ is a cumulative distribution function with probability mass function $l(\cdot)$. The mass of agents of type x is thus given by $l(x)$, and we require $l(x) > 0$, $\forall x$.

Time is continuous with an infinite horizon. Each agent is always in one of four states: matched, searching (that is, unmatched but participating), waiting (for continued bargaining, as explained below), and not participating. We denote the mass of waiting agents of type x by $\kappa(x) \leq l(x)$ and that of non-participating agents by $\nu(x) \leq l(x)$.

Searching agents can create marketplaces to meet on. We index the N marketplaces agents use by n, and N may be countably infinite. Agents cannot be on several marketplaces simultaneously (i.e. their search activity is indivisible), but they can always switch between marketplaces without incurring any cost. Let $u^n(x) \leq l(x)$ represent the mass of searching agents of type x on marketplace n. Only searching agents can be met on a marketplace; waiting agents are temporarily unavailable and agents who match immediately leave the marketplace. When indifferent whether to engage in search, whether to accept a match, and whether to stay in a marketplace or switch, an agent respectively searches, accepts the match, and stays.

Before two agents can match, a meeting between them will have to occur. To distinguish between the agents, we will denote one's type by x and the other's by y. Agents can produce together in one of two sectors F and G, where a match between types x and y generates constant flow output $f(x, y)$ and $g(x, y)$, respectively. The flow output generated by an unmatched agent is normalised to zero. We assume that types with low productivity in one sector have a high productivity in the other:

Assumption 1 (Regularity and symmetry). *The match production function $f(\cdot, \cdot)$ is positively valued (i.e. $f : \Theta^2 \mapsto \mathbb{R}_{++}$), strictly increasing, and symmetric (i.e. $f(x, y) = f(y, x)$). Let $g(\cdot, \cdot)$ be the exact mirror image of $f(\cdot, \cdot)$ so that $g(\underline{x}, \underline{y}) = f(\bar{x}, \bar{y})$ and $g(\bar{x}, \bar{y}) = f(\underline{x}, \underline{y})$.*

Agents can influence whom they meet through their choice of marketplace: each marketplace n belongs to one sector and is characterised by a set R^n of types that the marketplace is intended for. The set R^n is public information. By choosing to enter marketplace n, an agent thus sends the (costless) signal $\tilde{x} =$ "$x \in R^n$" to the agents she meets on this marketplace, which may or may not be a true statement about her privately observed type x. As every agent who enters a given marketplace sends the same signal, meetings are random inside a marketplace and are described by a meeting function $m(\cdot)$. With a mass of agents

$$\lambda^n = \sum_{x \in \Theta} u^n(x)$$

the flow of meetings in marketplace n equals $m(\lambda^n) \leq \lambda^n$, and $m(0) = 0$. The meeting rate on the marketplace is

$$\eta^n = \begin{cases} \frac{m(\lambda^n)}{\lambda^n} & \text{if } \lambda^n \neq 0 \\ 0 & \text{if } \lambda^n = 0 \end{cases} \tag{1}$$

We assume constant returns to scale in meeting, so that agent x faces the same meeting rate $\eta^n = \eta$ across all N marketplaces. Then x must choose her marketplace by the agents she wants to meet, as she would meet all agents equally quickly. When indifferent, she randomises over her most preferred marketplaces. Finally, a marketplace can be created at no cost but must attract agents in order to last. The agent creating marketplace n irreversibly chooses the sector it belongs to and R^n.

Meeting an agent y on a marketplace with R^n is equivalent to observing the signal $\tilde{y} =$ "$y \in R^n$". The agent x in the meeting can only form a belief about the true type y, as agents never directly observe each other's types. Let h be the history of the interaction with some agent, i.e. a set of actions such as the observed signal. We represent a belief as a probability distribution $\Psi(\cdot)$. Concretely, for each h, the belief held by agent x of the other agent's true type y is the probability distribution $\Psi(\cdot|h)$ over Θ. Then x, having observed h, believes that the other's type is y with probability mass $\psi(y|h)$. All agents use Bayes' rule whenever possible.

Next, match output must also be unobservable: knowing $f(\cdot, \cdot)$, x could otherwise infer y from observing $f(x, y)$. Let $f^e(x|h)$ (and similarly $g^e(x|h)$) denote the match output that x expects after observing h:

$$f^e(x|h) = \sum_{y \in \Theta} f(x, y)\psi(y|h)$$

Agents in a meeting bargain over the division of the match output that they would produce between them. We model this using a strategic bargaining procedure where only one offer is made per meeting. The players are Nature and the agents x and y who meet. The history h records the actions that x has observed thus far, and we simply index histories in chronological order. When x and y first meet, they already know both signals, so that $h_1 = \{\tilde{x}, \tilde{y}\}$. Nature selects x and y each with probability $\frac{1}{2}$ to move first.

Suppose x is selected. Then $h_2 = h_1 \cup \{x\}$ and x proposes some share $\pi(x|y)$ for herself, according to her bargaining strategy $B(x)$ that assigns an action to every possible history at which she moves. Hence $h_3 = h_2 \cup \{\pi(x|y)\}$ and y responds according to $B(y)$ by choosing an action from the set {"accept", "reject but stay", "reject and walk away"}.[5] Agents who walk away immediately continue searching. If y chooses "accept" or "reject and walk

[5] The fact that agents have met implies that these agents prefer engaging in search to not participating. It is thus without loss of generality that non-participation is not a further outside option here.

away", then h_4 will be a terminal history. If she chooses "reject but stay", then $h_4 = h_3 \cup \{$"reject but stay"$\}$ and x chooses from $\{$"continue", "walk away"$\}$. If x does not walk away, the same two agents will meet at rate $\zeta \geq \eta$ for the next round of bargaining, in which Nature again randomly selects one agent to move first, and so on.

If y accepts, agents match immediately and obtain their respective share as a flow utility for the duration of the match. Given that y bases her response on the share she believes would be left for her, i.e. $f^e(x|h_3) - \pi(x|y)$ in sector F (which may be negative), it can happen that the agreed shares sum to more than $f(x, y)$. However, in this case the match immediately breaks up: let the second mover also be the residual claimant. Then y would in this case protest immediately because she does not obtain her agreed share of the flow payoff. The immediate dissolution of the match is practically the same as bargaining failure, so that agents then either walk away or meet again to bargain anew. Further, as each agent can assure herself flow utility 0 by not participating, negative shares will always be rejected. Shares offered in previous rounds can never be accepted ex post, and if players never agree nor walk away, both will obtain 0.

Matches dissolve exogenously at constant rate δ. All agents are risk-neutral, observe everything except other agents' types and match output, apply a discount rate r (with $0 < r < \infty$), and seek to maximise the present discounted value of their expected utility. Because of discounting, the time that elapses before a meeting makes meetings costly. In addition, we include a second kind of search friction by allowing for explicit cost $c \geq 0$ that an agent incurs each time she attends a meeting. Finally, we only assume a minimum of gains from trade:

Assumption 2 (Gains from trade). *The output produced in a match between two agents of the lowest type, discounted at effective discount rate $r + \delta$, can reimburse both agents' explicit costs of one meeting, i.e. $2c \leq f(\underline{x}, \underline{y})/(r + \delta)$ for sector F.*

4 Equilibrium

4.1 Definition of equilibrium

We begin by defining three expected present values: $U^n(x)$ as the value to x of searching in marketplace n, $V(x|y)$ as the value to x of waiting for another bargaining round with y, and $W(x|y)$ as the value to x from being matched with y. Let the set $A(h)$ comprise of all combinations of bargaining strategies $(B(x), B(y))$ that lead to a subgame-perfect equilibrium (SPE) of the bargaining game given history h, so that an agreement is reached immediately and agents match. Let $\alpha(\cdot, \cdot)$ be an indicator function such that $\alpha(B(x), B(y)) = 1$ if $(B(x), B(y)) \in A(h)$ and 0 otherwise. In exact analogy, also define $\Omega(h)$ as the set of bargaining strategies that lead to another round of bargaining given h, and $\omega(\cdot, \cdot)$ as an indicator function such that $\omega(B(x), B(y)) = 1$ if $(B(x), B(y)) \in \Omega(h)$. Then the following *asset equation* expresses, for one marketplace, the expected return on searching as the expected gain from a meeting net of search cost c:

$$rU^n(x) = \eta^n \Bigg(-c + \sum_{y\in\Theta} \alpha(B(x), B(y)) \left[W(x|y) - U^n(x)\right] \psi(y|h = \{\tilde{x}, \tilde{y}\}) \\ + \sum_{y\in\Theta} \omega(B(x), B(y)) \left[V(x|y) - U^n(x)\right] \psi(y|h = \{\tilde{x}, \tilde{y}\}) \Bigg) \tag{2}$$

where $\psi(y|h = \{\tilde{x}, \tilde{y}\})$ is the probability mass of y that x believes conditional on meeting y in marketplace n. The first summation thus captures the gain agent x expects in case of a match, while the second captures the gain expected in case of continued bargaining.

Let us define $U(x)$ as the value of $U^n(x)$ that x obtains in equilibrium. As is natural when signals are involved, we look for a *perfect Bayesian equilibrium* (PBE) of our model. We will focus our attention on separating equilibria that survive the Intuitive Criterion.[6] Because signals are costless all PBE will necessarily be cheap-talk equilibria. A steady-state PBE of our model, separating or not, requires that the flows into and out of matches balance for every type (a pointwise steady state), that agents choose all their strategies optimally, and that agents' beliefs are consistent with actual equilibrium behaviour.

Definition 1 (Search equilibrium with signals). *In a steady-state PBE of the model, each agent $x \in \Theta$*

(i) engages in search if and only if $U(x) \geq 0$

(ii) optimally chooses a sector-specific marketplace such that $\forall n$, $U(x) \geq U^n(x)$ given $B(x)$, $B(y)$ for all $y \in \Theta$, and $(R^n)_{n=1}^N$, where $U^n(x)$ is determined by equation (2)

(iii) chooses a stationary subgame-perfect bargaining strategy as $\arg\max_{B(x)} rU^n(x)$ given all $B(y)$ and R^n, noting that $W(x|y)$ depends on the share obtained in bargaining

(iv) holds beliefs that are formed using Bayes' rule where possible and that are consistent with equilibrium play: given an equilibrium history h, $\psi(y|h) = u^n(y|h)$ where $u^n(y|h)$ is the true probability mass of y in marketplace n conditional on h

and the matching market is in a pointwise steady state, so that the flows into and out of $\sum_{n=1}^N u^n(x) + \kappa(x)$ balance for each $x \in \Theta$. Marketplaces are created until there is no new marketplace n^0 such that $U^{n^0}(x) > U^n(x)$, $\forall n$ holds for any $x \in \Theta$.

A PBE only requires agents' beliefs to be consistent with equilibrium play, not with actions out of equilibrium. As is well known, a PBE can therefore depend on unreasonable off-equilibrium beliefs because these beliefs are never tested in equilibrium. Since unreasonable beliefs are not needed for any of our results, we rule out beliefs that are

[6] Kübler et al. (2008) report experimental evidence suggesting that pooling equilibria never arise when some types can benefit from the effective use of signals.

unreasonable in the sense of the Intuitive Criterion. To do this formally, let us call the choices of n and $B(x)$ the 'grand strategy' of agent x, denoted $GS(x) = (n, B(x))$. Also define $BR(x|h)$ as the set of continuation strategies $GS(x|h)$ that are best responses for x. To apply the Intuitive Criterion as an equilibrium refinement, we have to define the notion of equilibrium domination in our model:

Definition 2 (Equilibrium domination). *Given a PBE of the model, the continuation strategy $GS(x|h)$ is equilibrium-dominated at history h if*

$$U(x) > \max_{GS(y|h) \in BR(y|h)} U(x|GS(x|h))$$

where $U(x|GS(x))$ is the present value to x of searching with strategy $GS(x|h)$.

The Intuitive Criterion then demands that the beliefs of y assign probability 0 to any type x who would have to pursue equilibrium-dominated strategies to reach the respective history: $\psi(x|h) = 0$ if, at a history up to h, x would have had to play an equilibrium-dominated strategy $GS(x|h)$.

4.2 Putative equilibrium

We next propose that a particular separating equilibrium exists under a simple condition on the match production function $f(\cdot, \cdot)$. All we need is a weak and intuitive form of complementarity known as strict supermodularity (or increasing differences): the marginal product of one agent in a match is strictly increasing in the type of the other agent.

Definition 3 (Supermodularity). *The match production function $f(\cdot, \cdot)$ is strictly supermodular if, for all $x_H > x_L$ and $y_H > y_L$,*

$$f(x_H, y_H) - f(x_L, y_H) > f(x_H, y_L) - f(x_L, y_L)$$

A match production function is strictly submodular if the reverse inequality holds.

Further, we refer to the sorting with $x = y$ in all matches as *perfect positive assortative matching* (PPAM). We can now propose existence of the following PBE in our model:

Proposition 1 (Existence). *Let η and ζ be sufficiently close and let agents' beliefs assign probability 0 to equilibrium-dominated actions. Then for any type distribution $L(x)$, strict supermodularity of $f(\cdot, \cdot)$ is necessary and sufficient for the existence of a separating PBE in which each agent $x \in \Theta$*

(i) engages in search: $U(x) \geq 0$

(ii) chooses a marketplace n for which the signal $\tilde{x} =$ "$x \in R^n$" is truthful, where n can only belong to sector F if $f(x, y) \geq g(x, y)$ for $x = y$

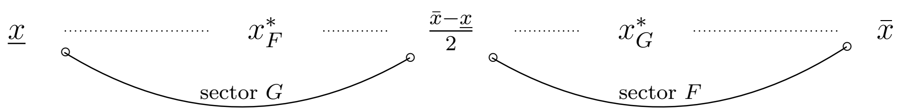
(iii) reaches a bargaining agreement in the first meeting and thus matches:
$\alpha(B(x), B(y)) = 1$ *and* $\omega(B(x), B(y)) = 0$ *for* $x = y$

(iv) correctly believes all signals to be truthful:
$\psi(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = u^n(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = 1$ *for* $y \in R^n$.

The market is in pointwise steady state and is perfectly segmented: $|R^n| = 1$, $\forall n$.

Crucially, the combination of truthful signals and $|R^n| = 1$ means that there is only one type $x \in \Theta$ on each marketplace. When agents meet exclusively agents of their own type and then match, the matching that necessarily results is PPAM.

The figure below depicts the overall symmetric structure of the putative equilibrium. We find in section 5.2 for sector F that all types above a certain threshold x_F^* cannot gain from invading the marketplaces of higher types, while types below x_F^* might. Yet suppose that x_F^* lies below $(\bar{x} - \underline{x})/2$, which will be the case if η and ζ are sufficiently close. Then the types below x_F^* will prefer a marketplace in sector G: as all other types below $(\bar{x} - \underline{x})/2$, they are more productive in sector G. By exact analogy to sector F, their marketplaces in sector G are not invaded by relatively unproductive types above the threshold x_G^* because all types above $(\bar{x} - \underline{x})/2$ prefer a marketplace in sector F. If there is a type $x = (\bar{x} - \underline{x})/2$, the agent randomises over sectors. In short, types choose a marketplace in the sector where they are more productive and they sort perfectly within each sector.



The next section proves proposition 1 through a series of lemmas. Each time, we separately consider a component of proposition 1, taking as given that all other components are indeed as specified in proposition 1. We verify for the component in question, as applicable, that it is optimal for agents to behave as specified, that a steady state results, and that beliefs are consistent with equilibrium play.

5 Existence of the putative equilibrium

5.1 Bargaining, participation, and steady state

We first determine the expected present values in the putative equilibrium situation. Given that beliefs are consistent with equilibrium play (and that $|R^n| = 1$), we have

$$\psi(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = u^n(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\})$$

If x only meets agents of her own type, then

$$u^n(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = 0 \quad \forall y \neq x \tag{3}$$

Since every meeting in the putative equilibrium leads to match,

$$\alpha(B(x), B(y)) = 1 \quad \text{for } y = x \quad \text{and} \quad \omega(B(x), B(y)) = 0 \quad \text{for } y = x \tag{4}$$

For the marketplace chosen in the putative equilibrium, equation (2) thus simplifies to

$$rU(x) = \eta\,[W(x|y) - c - U(x)] \tag{5}$$

with $y = x$. Hence the rate of matches equals the rate of meetings, and an agent effectively incurs costs c each time she matches. Next, the expected return on being matched with y is the expected flow utility while matched and the loss from match dissolution at rate δ:

$$rW(x|y) = \sigma(x|y) - \delta[W(x|y) - U(x)] \tag{6}$$

where $\sigma(x|y)$ denotes the expected share that x obtains when bargaining with y over the flow of match output, which is in effect known from truthful signals: for sector F,

$$\sigma(x|y) = \frac{1}{2}\pi(x|y) + \frac{1}{2}[f(x, y) - \pi(y|x)] \tag{7}$$

One can solve equation (5) for $U(x)$ and equation (6) for $W(x|y)$, then use the latter to substitute for $W(x|y)$ in the former to obtain

$$rU(x) = \beta[\sigma(x|y) - (r + \delta)c] \tag{8}$$

where $\beta = \eta/(r + \delta + \eta)$. Now suppose y has been randomly selected to move first in the bargaining game. In response to the share left for her, x can reject it and continue searching, which carries the value $U(x)$, or she can reject this share and wait for another round of bargaining, which carries a value $V(x|y)$. Note that the first mover y cannot hope to attain a better position than she currently has: at best, she will find herself as first mover again in a later meeting, be it with the same agent x or another agent of the same type. As delay is costly, y seeks to seize the opportunity and to ensure that x accepts her offer. In turn, x will accept any implicitly offered payoff $W^O(x|y)$ that satisfies

$$W^O(x|y) \geq \max[V(x|y), U(x)] \tag{9}$$

as she would otherwise reject the offer. When x moves first, y requires

$$W^O(y|x) \geq \max[V(y|x), U(y)] \tag{10}$$

In case of a second meeting, the same logic as before implies that the first mover seeks to ensure agreement, so that the second meeting can be expected to result in a match. The second meeting happens at rate ζ, so that

$$rV(x|y) = \zeta \left[W(x|y) - c - V(x|y)\right] \tag{11}$$

in the putative equilibrium. Solving equation (11) for $V(x|y)$ and equation (5) for $U(x)$, one finds that $V(x|y) \geq U(x)$ since $\zeta \geq \eta$. Hence the outside option $U(x)$ is not binding. As we also require bargaining strategies to be stationary, the game reduces to a variant of Rubinstein's (1982) set-up, and we have the following result (see appendix for all proofs):

Lemma 1 (Bargaining equilibrium). *Given truthful signals and given marketplace choices as in the putative equilibrium situation, the following stationary strategies form the unique SPE of the bargaining game in sector F:*

(i) for herself, agent x always proposes

$$\pi^*(x|y) = \left(1 - \frac{\phi}{2}\right) f(x,y) + \phi(r+\delta)c \quad \textit{with} \quad \phi = \frac{\zeta - \beta\delta}{r + \zeta} \tag{12}$$

When y proposes $\pi(y|x)$, x always accepts if and only if $\pi(y|x) \leq \pi^*(y|x)$.

(ii) for herself, y always proposes $\pi^*(y|x) = \pi^*(x|y)$. When x proposes $\pi(x|y)$, y always accepts if and only if $\pi(x|y) \leq \pi^*(x|y)$

Agreement is reached in the first round of bargaining. The expressions for sector G are obtained by substituting $g(\cdot,\cdot)$ for $f(\cdot,\cdot)$.

The essence of the bargaining SPE is that each agent makes offers that leave the other indifferent, and each agent accepts offers that make her indifferent or better off: the first-mover takes a share $\pi^*(x|y)$ such that the second-mover share

$$f(x,y) - \pi^*(x|y) = \frac{\phi}{2} f(x,y) - \phi(r+\delta)c$$

is just enough to prevent the second mover from rejecting. The second-mover share will still be weakly positive if

$$\frac{\phi}{2} f(x,y) \geq \phi(r+\delta)c \quad \Leftrightarrow \quad 2c \leq \frac{f(x,y)}{r+\delta}$$

which by assumption 2 even holds for $f(x,y) = f(\underline{x}, \underline{y})$. The two indifference conditions in equations (9) and (10), depending on who moves first, thus together pin down a unique SPE for each sector. Finally, expected shares in the SPE reflect the symmetry of the bargaining situation: for sector F,

$$\sigma(x|y) = \sigma(y|x) = \frac{1}{2}\pi^*(x|y) + \frac{1}{2}\left[f(x,y) - \pi^*(x|y)\right] = \frac{1}{2} f(x,y) \tag{13}$$

To ensure that all agents engage in search, c must not be so high that $U(x)$ becomes negative for some x, since each agent can obtain a payoff 0 by not participating.

Lemma 2 (Participation). *Assumption 2 is necessary and sufficient for all agents to prefer engaging in search to non-participation.*

By definition, the mass of matched agents is $l(x)-\sum_{n=1}^{N} u^n(x)-\kappa(x)-\nu(x)$. As agents in the putative equilibrium prefer search to non-participation and reach an agreement in the first bargaining round, $\nu(x)=\kappa(x)=0$, $\forall x \in \Theta$. Hence agents only flow from searching to being matched (at rate η) and back (at rate δ). Equating these flows, we obtain the pointwise steady state in the putative equilibrium:

$$\delta \left[l(x) - \sum_{\mathcal{N}(x)} u^n(x) \right] = \eta \sum_{\mathcal{N}(x)} u^n(x) \quad \forall x \in \Theta \tag{14}$$

where $\mathcal{N}(x) \equiv \{n | R^n = \{x\}\}$ is the set of all marketplaces on which x meets exclusively her own type when signals are truthful.

5.2 Marketplace choices, signals and beliefs

In this section, we examine whether any one agent in the putative equilibrium has a unilateral incentive to deviate by choosing to enter a marketplace in the same sector but intended for another type, so that the agent's signal is false. There are two reasons why we need to worry about such deviations. First, because true types are only privately observable, agents can perfectly imitate agents of other types by bargaining as these types would. Second, agents might enter another type's marketplace but, once in a meeting, renege on the signal they thereby sent. Since search frictions make switching to another meeting costly, the other agent in the meeting might still accept the match. For example, consider a rather high type y_H in sector F who matches with x_H in the putative equilibrium. If y_H finds herself in a meeting with a type $x_L < x_H$, she might nevertheless grudgingly accept whenever her share of $f(x_L, y_H)$ is not so far below her expected share of $f(x_H, y_H)$ that the costs of another meeting would be justified. Therefore, there can be an incentive to send false signals and invade other types' marketplaces.

Let us focus on marketplaces in sector F for the rest of this section, as all results will analogously apply to sector G. We take as given that all other agents on the marketplace signal truthfully, that all believe signals to be truthful, and that agents choose sectors as in the putative equilibrium: then only agents with a type $x \geq (\bar{x} - \underline{x})/2$ search in sector F. We proceed by identifying first the conditions under which everyone of these agents prefers her match in the putative equilibrium (henceforth the *equilibrium match*) to any other match in sector F that is available to her (i.e. a mutually acceptable match with another agent searching in F). From this, we infer under which conditions there will be no unilateral incentive to deviate from truthful signals.

We first compare the equilibrium match to matches with lower types. Let us take the perspective of some agent with a type $x_H > (\bar{x} - \underline{x})/2$, so that lower types necessarily exist in sector F. We thus want to compare being matched with $y_H = x_H$ to being matched with y_L, where $(\bar{x} - \underline{x})/2 \leq y_L < x_H$. The expected present discounted values of these matches are $W(x_H|y_H)$ and $W(x_H|y_L)$, respectively. In the spirit of the one-deviation principle, x reverts to the putative equilibrium strategies after the deviation. Hence, the asset equations for both $rW(x_H|y_H)$ and $rW(x_H|y_L)$ in analogy to equation (6) depend on the same $U(x_H)$ and thus differ only in the expected shares. Solving these two asset equations respectively for $W(x_H|y_H)$ and $W(x_H|y_L)$, we therefore find that

$$W(x_H|y_H) > W(x_H|y_L) \quad \Leftrightarrow \quad \sigma(x_H|y_H) > \sigma(x_H|y_L)$$

where $\sigma(x_H|y_H)$ and $\sigma(x_H|y_L)$ denote the expected share obtained by x_H in a match with y_H and y_L, respectively.

Thus suppose x_H signals to be of type x_L in order to meet a type y_L. Further suppose that x_H continues to behave like a type x_L so as to conform to the beliefs of y_L, given that all other agents signal truthfully. Recall from section 5.1 that neither agent's signal implies a binding outside option. Hence the bargaining equilibrium described by lemma 1 will be reached in the first round of bargaining. Then the expected flow utility for x_H in the match with y_L is

$$\begin{aligned}\sigma(x_H|y_L) &= \frac{1}{2}\left[f(x_H, y_L) - \frac{\phi}{2}f(x_L, y_L) + \phi(r+\delta)c\right] \\ &\quad + \frac{1}{2}\left[f(x_H, y_L) - \left(1 - \frac{\phi}{2}\right)f(x_L, y_L) - \phi(r+\delta)c\right] \\ &= f(x_H, y_L) - \frac{1}{2}f(x_L, y_L) \qquad (15)\end{aligned}$$

If x_H moves first (with probability $\frac{1}{2}$), she leaves a second-mover share to y_L as if output was $f(x_L, y_L)$ and keeps the rest of the actual output $f(x_H, y_L)$. If y_L moves first, y_L takes the first-mover share of $f(x_L, y_L)$ for herself and x_H obtains the actual remainder. In an equilibrium match, by contrast, x_H would obtain

$$\begin{aligned}\sigma(x_H|y_H) &= \frac{1}{2}\left[\left(1 - \frac{\phi}{2}\right)f(x_H, y_H) + \phi(r+\delta)c\right] + \frac{1}{2}\left[\frac{\phi}{2}f(x_H, y_H) - \phi(r+\delta)c\right] \\ &= \frac{1}{2}f(x_H, y_H) \qquad (16)\end{aligned}$$

Comparing $\sigma(x_H|y_L)$ and $\sigma(x_H|y_H)$, we find the following:

Lemma 3 (Matches with lower types). *In the putative equilibrium, strict supermodularity of $f(\cdot, \cdot)$ is necessary and sufficient for any agent in sector F to strictly prefer the equilibrium match to matching with a lower type while perfectly imitating the lower type.*

Next suppose that x_H has signalled to be of type x_L, has thus met a type y_L, but now wants to renege on the signal. We will find below that x_H has to let at least one round of bargaining fail to actually convince y_L of her true type. Here we ask whether reneging could possibly make the deviation to a match with a lower type worthwhile. By considering the hypothetical extreme case that y_L instantly observes the true type x_H, we obtain an envelope result and thereby a negative answer:

Lemma 4 (Reneging in matches with lower types). *Suppose types were instantly observable in meetings. Consider a type x_H in sector F who deviates from the putative equilibrium situation and meets a type y_L, with $(\bar{x} - \underline{x})/2 \leq y_L < x_H$.*
a) If neither agent's outside option is binding, the following stationary strategies will form the unique SPE of the bargaining game and lead to agreement in the first round:

(i) for herself, agent x_H always proposes

$$\pi^*(x_H|y_L) = \frac{2r+\zeta}{2(r+\zeta)}\left[f(x_H, y_L) + \frac{\beta\delta}{2r}\left[f(x_L, y_L) - \frac{\zeta}{2r+\zeta} f(x_H, y_H)\right]\right] + \phi(r+\delta)c$$

When y_L proposes $\pi(y_L|x_H)$, x_H always accepts if and only if $\pi(y_L|x_H) \leq \pi^*(y_L|x_H)$.

(ii) for herself, y_L always proposes

$$\pi^*(y_L|x_H) = \frac{2r+\zeta}{2(r+\zeta)}\left[f(x_H, y_L) + \frac{\beta\delta}{2r}\left[f(x_H, y_H) - \frac{\zeta}{2r+\zeta} f(x_L, y_L)\right]\right] + \phi(r+\delta)c$$

When x_H proposes $\pi(x_H|y_L)$, y_L always accepts if and only if $\pi(x_H|y_L) \leq \pi^*(x_H|y_L)$.

b) Strict supermodularity of $f(\cdot, \cdot)$ is sufficient for any agent in sector F to strictly prefer the equilibrium match to this deviation.

Part a) of lemma 4 may be regarded as a generalisation of lemma 1 to an asymmetric case. Crucially, part b) finds that even if x_H could immediately convince y_L of her true type, x_H would strictly prefer the equilibrium match, as she does when she would have to imitate some lower type. Based on lemmas 3 and 4, we show below that types $x \geq (\bar{x} - \underline{x})/2$ never have an incentive to deviate from the putative equilibrium to matches with lower types if $f(\cdot, \cdot)$ is supermodular, for any beliefs that lower types might hold about deviants.

In turn, whenever a deviant causes bargaining to fail, the other agent thus knows that she faces a strictly lower type: for a weakly higher type, a deviation would be equilibrium-dominated. The other agent now has to choose between two options:[7] another round of bargaining with an evidently lower type or, as in the putative equilibrium, meeting another agent of her own type (as we consider only a single deviation, another agent signals truthfully). Define x_F^* as the highest one of all thresholds that equalise these

[7] The same holds when a deviation is only detected after the start of the match: it can only be detected when agents' initial bargaining agreement breaks down, so that there is no basis for further production while agents wait for the new round of bargaining required for renegotiation.

two options (and similarly x^*_G for an agent in sector G), so that $rU(x^*_F) = rV(x^*_F|y)$ or equivalently

$$\begin{aligned}\eta[W(x^*_F|y^*_F) - c - U(x^*_F)] \;&=\; \zeta\left(-c + \sum_{y<x^*_F} \alpha(B(x^*_F), B(y))\,[W(x^*_F|y) - U(x^*_F)]\,\psi(y|h)\right.\\ &\qquad\left. + \sum_{y<x^*_F} \omega(B(x^*_F), B(y))\,[V(x^*_F|y) - U(x^*_F)]\,\psi(y|h)\right) \quad (17)\end{aligned}$$

in analogy to equations (5) and (2). If η and ζ were equal, x^*_F would not exist: by lemma 4b), x (whose type was observable from a truthful signal) strictly prefers her equilibrium match to a match with a lower type, so that the left-hand side of equation (17) would always exceed the right-hand side. The only reason to possibly continue bargaining with a lower type is that $\zeta \geq \eta$. Types $x < x^*_F$ are willing to because their own type is sufficiently low: then the expected type of the deviant is not so far below their type to outweigh the difference between ζ and η. By contrast, types $x \geq x^*_F$ walk away to meet another agent. As these arguments are central to our reasoning, we prove them more formally:

Lemma 5 (Equilibrium-dominated strategies). *Let $f(\cdot,\cdot)$ be strictly supermodular and let η and ζ be sufficiently close so that $x^*_F \leq (\bar{x} - \underline{x})/2$.*
a) For any agent in sector F, a deviation such that she meets a weakly lower type with whom bargaining fails is equilibrium-dominated.
b) Also let agents' beliefs assign probability 0 to equilibrium-dominated actions and consider a meeting in the putative equilibrium between some x and y in sector F. If x deviates, y will correctly believe to face a lower type and will walk away.

Let us finally turn to the incentive for lower types to deviate to a match with a higher type. Consider some agent with a type $x_L < \bar{x}$, so that higher types necessarily exist. Now we want to compare being matched with an exactly corresponding type $y_L = x_L$, as in the equilibrium match, to being matched with a higher type $y_H > x_L$. The lower type x_L has two possibilities: she can either perfectly imitate x_H, or she can signal having type x_H in order to meet y_H but then renege on the signal.

We have just shown that, if x_L reneges in a meeting in sector F with a type y_H, then y_H will walk away and x_L does not gain from the deviation.[8] Hence, unless x_L herself walks away (without gain from the deviation), she will have to bargain with a type y_H under two constraints: y_H believes to face a type x_H and bargaining must not fail. Recall that these are exactly the constraints under which the bargaining strategy of x_H in the putative equilibrium is optimal (see lemma 1), so that x_L cannot do better than perfectly imitate x_H: if she is more demanding than x_H, bargaining will fail, and if she is less

[8] If x_L instead simply claims to have a lower type, this will not be credible: also a type x_H has an incentive to downplay her type in order to make y_H propose and accept lower shares for herself.

demanding, she will not be optimising. When x_L therefore perfectly imitates x_H, the expected flow utility for x_L is

$$\begin{aligned}\sigma(x_L|y_H) &= \frac{1}{2}\left[f(x_L,y_H)-\frac{\phi}{2}f(x_H,y_H)+\phi(r+\delta)c\right] \\ &\quad +\frac{1}{2}\left[f(x_L,y_H)-\left(1-\frac{\phi}{2}\right)f(x_H,y_H)-\phi(r+\delta)c\right] \\ &= f(x_L,y_H)-\frac{1}{2}f(x_H,y_H) \qquad (18)\end{aligned}$$

If x_L moves first, she has to leave y_H the second-mover share of $f(x_H, y_H)$ to avoid being found out and can thus take whatever is left of the actual output $f(x_L, y_H)$. If y_H moves first, y_H takes the first-mover share of $f(x_H, y_H)$ for herself and x_L obtains the remainder. By contrast, the expected flow utility for x_L from her equilibrium match would be

$$\sigma(x_L|y_L) = \frac{1}{2}f(x_L, y_L) \qquad (19)$$

A comparison of $\sigma(x_L|y_H)$ and $\sigma(x_L|y_L)$ yields the following result:

Lemma 6 (Matches with higher types). *In the putative equilibrium, strict supermodularity of $f(\cdot,\cdot)$ is necessary and sufficient for any agent in sector F to strictly prefer the equilibrium match to matching with a higher type while perfectly imitating the higher type.*

If also types below $(\bar{x} - \underline{x})/2$ searched in sector F, then x_L might gain from reneging in a meeting with y_H, because a type $y_H < x_F^*$ would not walk away. However, all types below $(\bar{x} - \underline{x})/2$ prefer sector G, as we argue in the next section. Corollary 1 collects the conditions identified in this section and the implications for agents' beliefs and choice of marketplace:

Corollary 1 (Truthful signals). *Let agents' beliefs assign probability 0 to equilibrium-dominated actions and let η and ζ be sufficiently close so that $x_F^* \leq (\bar{x} - \underline{x})/2$. Then strict supermodularity of $f(\cdot,\cdot)$ is necessary and sufficient for each agent in sector F to strictly prefer a marketplace $n \in \mathcal{N}(x)$ among the marketplaces in sector F, so that the signal $\tilde{x} =$ "$x \in R^n$" is truthful. Given $h = \{\tilde{x}, \tilde{y}\}$, the only beliefs consistent with truthful signals are*

$$\psi(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = u^n(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = 1 \quad \textit{for} \quad y \in R^n, n \in \mathcal{N}(x).$$

Each agent in sector F essentially finds it optimal to choose a marketplace $n \in \mathcal{N}(x)$, and to thereby signal truthfully, because this is the only way to obtain her equilibrium match, which she prefers to a deviation. As all agents in sector F therefore indeed signal truthfully, only beliefs that signals are truthful on the marketplaces in the sector can be consistent with equilibrium play.

In conclusion, this section has presented an extensive but essentially simple reasoning. We found that agents in sector F will never deviate from the putative equilibrium to match with lower types if $f(\cdot,\cdot)$ is supermodular. A type y in sector F who detects a deviation should therefore believe to face a lower type; when y can choose between continued bargaining with a lower type and her equilibrium match, she prefers the latter because $y \geq x_F^*$. Lower types can thus only match with y by imitating her type, but they will not gain from such a deviation if $f(\cdot,\cdot)$ is supermodular.

5.3 Sector choice and market segmentation

By choosing a marketplace, agents implicitly also choose the sector it belongs to. The previous section found that agents sort perfectly within sector F, and this result extends to sector G: by the symmetry between $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ (see assumption 1), strict supermodularity of $f(\cdot,\cdot)$ also implies strict submodularity of $g(\cdot,\cdot)$. Further, if η and ζ are sufficiently close so that $x_F^* \leq (\bar{x} - \underline{x})/2$, this simultaneously implies $x_G^* \geq (\bar{x} - \underline{x})/2$. Hence results analogous to lemmas 3 through 6 also apply to the types $x \leq (\bar{x} - \underline{x})/2$ who may search in sector G in the putative equilibrium, while the conditions for signals being truthful in sector G and for agents meeting only agents of the same type are even exactly the same as in corollary 1. It remains to confirm that types optimally self-select into sectors as proposed in the putative equilibrium:

Lemma 7 (Sector choice). *Let η and ζ be sufficiently close so that $x_F^* \leq (\bar{x} - \underline{x})/2$. Any agent in the putative equilibrium with a type $x < (\bar{x} - \underline{x})/2$ then strictly prefers to search in sector G, while any agent with a type $x > (\bar{x} - \underline{x})/2$ strictly prefers sector F.*

Let us now take choices among existing marketplaces as given and concentrate on the creation of marketplaces within a given sector. Consider three types x_L, x_M, and x_H in sector F, with $(\bar{x} - \underline{x})/2 \leq x_L < x_M < x_H \leq \bar{x}$. Suppose these types search in the same marketplace, so that each of them can meet with y_L, y_M, or y_H. We know from lemma 4 that each x_H would prefer a match with y_H to a match with y_M or y_L. The agents of type x_H can profitably set up a new marketplace where $R^n = \{x_H\}$ so that agents of type x_H exclusively meet each other. In the initial marketplace, they would also meet other types although matches with these types would be less desirable, which is not offset by any advantage in meeting rates. By setting up an exclusive marketplace, the congestion externality imposed by these other types is avoided (see Jacquet and Tan (2007) for details of this logic).

Given our results above, other types would not invade this new exclusive marketplace, so that the remaining types x_M and x_L can no longer meet with y_H. Among the possible matches, x_M prefers by lemma 4 the match with y_M, so that all agents of type x_M now set up an exclusive marketplace with $R^n = \{x_M\}$, leaving the initial marketplace to the agents of type x_L. This logic applies to any marketplace with different types in either sector.[9] Hence all types have their own exclusive marketplaces in equilibrium; formally, $|R^n| = 1$ for all n. (We will generalise this logic in section 6.4 to show that it does not only apply in the putative equilibrium, but in any separating equilibrium.) There may be several exclusive marketplaces for the same type in equilibrium ($|\mathcal{N}(x)| \geq 1$), as none of our conclusions is affected by their exact number due to constant returns to scale in meeting.

By way of summary, this subsection and the preceding have each shown a component of the putative equilibrium situation to hold, given the other components. We thus found the pointwise steady state in the PBE. Given a supermodular match production function and beliefs that rule out equilibrium-dominated actions, agents search in the sector where they are more productive and seek to meet only exactly corresponding types. All agents then signal their types truthfully and correctly believe that all other agents on their marketplaces signal truthfully. With optimal bargaining strategies, every meeting leads to a match, as one would expect when truthful signals allow agents to know everything in advance. The matches are only between exactly corresponding types. Our model thus leads to PPAM under the same weak condition as in Becker's (1973) frictionless model, despite two kinds of search frictions. The next section discusses key properties of the separating equilibrium.

6 Equilibrium properties

6.1 Dependence on priors

Let us first clarify why supermodularity is central to our results. Since types are only privately observable and nothing keeps agents from imitating other types, an agent may match incognito with any type she likes. However, because actual match output then differs from the match output suggested by the signals, the deviant will only remain incognito if she bears the necessary adjustment: she has to give up as much of her own share as is necessary to bridge the gap when actual output is lower (otherwise bargaining fails and the other agent walks out), and she quietly pockets the excess output when actual output is higher. To explain why a lower type x_L would then not match incognito with a higher type $y_H > x_L$, supermodularity is key: $f(x_H, y_H) - f(x_L, y_H)$ is the necessary adjustment when y_H otherwise matches with x_H in equilibrium, while $f(x_L, y_H) - f(x_L, y_L)$ is the extra output produced in comparison to the equilibrium match of x_L. With $f(x_L, y_H) = f(x_H, y_L)$ in the latter, as established by equation (27), the necessary adjustment will exceed the extra output if $f(\cdot, \cdot)$ is strictly supermodular. From the perspective of a lower type, any possible gains from higher output with a higher type are therefore more than outweighed

[9] The logic also applies to types who do not search in the sector but for whom the separating equilibrium could be sustained; that is, types x with $x_F^* \leq x < (\bar{x} - \underline{x})/2$ in case of sector F and types x with $(\bar{x} - \underline{x})/2 < x \leq x_G^*$ in case of sector G.

by the costs from adjustment.

It is crucial for this argument that the necessary adjustment falls entirely on the deviant x_L. This happens when the treatment x_L faces is independent of her actual type. Therefore, our results are obtained under the realistic assumption that true types are always only privately observable. With publicly observable types, y_H would be willing to compromise when she bargains with a deviant x_L, in order to avoid bargaining failure. Yet under private information, y_H instead bases her bargaining behaviour on the signal sent by x_L, which creates a link between signals and payoffs. Now given that x_L has to signal like a type x_H in order to meet y_H at all, she will be treated exactly like a type x_H at least in the first round of bargaining (and as failure of this round is bad news, a second round with $y_H \geq x_F^*$ never happens). This way, the supermodularity of the match production function fully translates into supermodularity of the payoffs that determine signal choice. In effect, supermodularity assumes the role of a single-crossing property in our model and we thus obtain a fully separating equilibrium even though signals are costless. Separation is therefore not driven by differences in the cost of signals, but by differences in marginal productivity of the same agent over different matches.

6.2 Efficiency

The separating equilibrium we have identified is efficient in a number of important respects. First and foremost, search costs are minimised, both for each agent individually and overall: every meeting results in a match, so that agents match after an expected search time of $1/\eta$. This is the minimum delay because a meeting necessarily precedes a match. In a random search model, each match would typically be preceded by a number of unsuccessful meetings, and only by chance will the first meeting of an agent result in a match. Therefore, search costs in random search models are at least as high from the individual perspective as in our model with truthful signals, and strictly higher in expectation as well as on aggregate. Second, note that all agents match in equilibrium so that there is no unrealised surplus left in the form of agents who never match. On the contrary, Becker (1973) proved the following result:

Corollary 2 (Output efficiency). *If the match production function is strictly supermodular, PPAM will maximise aggregate output.*

Random search models, be it with or without supermodularity of the match production function, do in general not maximise aggregate match output, as they lead to a certain degree of mismatch instead of PPAM. Finally, among the mutually acceptable matches, agents in the equilibrium we found always obtain the match they most prefer. This again contrasts starkly with random search models, where the match an agent expects is the expectation over the mutually acceptable matches, not the most preferred one of them.

6.3 Stability

In this section, we examine whether the equilibrium matching we found is a *stable matching*. Because this equilibrium is symmetric, our notation can abstract from the distinction between types and individual agents without loss of generality. Suffice to let $\sigma(x)$ denote the expected flow utility that an agent of type x obtains under a particular matching. Recall that $\sigma(x) = \sigma(x|y)$ if x and y are matched in this matching and $\sigma(x) = 0$ if x remains unmatched. We can then define stability as follows:

Definition 4 (Stable matching). *The equilibrium matching is stable if $\sigma(x)$ satisfies $\sigma(x) \geq 0$ for all $x \in \Theta$ and there is no match between any two agents with types x and y such that $\sigma(x|y) > \sigma(x)$ and $\sigma(y|x) > \sigma(y)$.*

It is worth noting that a stable matching in this model is by definition also in the *core*.[10] We find that supermodularity of the match production function is a sufficient condition here for PPAM to be a stable matching:

Corollary 3 (Stability of PPAM). *Whenever it exists, the separating equilibrium described by the putative equilibrium leads to a stable matching.*

A stable matching is a most unusual result in a model with search frictions. In random search models, agents cannot search selectively and accept any type from a certain range because search frictions make continued search undesirable. A stable matching cannot be expected to arise under such circumstances and is very unlikely to arise by chance whenever the number of different types is not trivially small. Stable matchings normally only arise in frictionless models. We attribute the reason that a stable matching is achieved here despite search frictions to the signals: they allow agents to pursue their search almost as if there were no search frictions.

Adachi (2003) shows for a fairly general search model that the set of equilibria will reduce to the set of stable matchings in a model à la Gale and Shapley (1962) if search frictions become negligible. Our result in this section qualifies this finding in so far as search frictions remain in our model because agents do not meet immediately ($\eta < \infty$) and incur costs from meetings ($c \geq 0$), and yet a stable matching results. This suggests that frictions do not prevent a stable matching in a search model as long as they do not keep agents from meeting only specifically chosen types. Intuitively, arbitrarily high frictions do not have any effect as long as agents participate and then find ways to match like in a frictionless environment.

[10] The notion of the core implicitly assumes side payments within a coalition, so that only the coalition's total utility counts. For example, side payments in Becker (1973) ensure that agents end up in the match generating the highest match output, among the available matches. In our model without side payments, each agent's $\sigma(x)$ in the core has to weakly exceed the utility of being single and of any other available match (while a match is available to x if $\sigma(y|x) > \sigma(y)$). These are the requirements in definition 4.

6.4 Uniqueness

While we have shown that a particular separating equilibrium exists, this section argues that it is unique. The first thing to note is that, by its very nature, a separating equilibrium is characterised by truthful signals.[11] In section 5.3, truthful signals lead to marketplaces where agents meet exclusively their own type. This result generalises:

Lemma 8 (Market segmentation). *Agents will meet only their own type in any separating equilibrium if $f(\cdot,\cdot)$ is strictly supermodular.*

Therefore, PPAM is the unique matching that may result in any separating equilibrium of our model. We can now conclude more comprehensively:

Proposition 2 (Uniqueness). *Whenever it exists, the separating equilibrium described by the putative equilibrium is unique up to off-equilibrium beliefs.*

No formal proof is needed, as proposition 2 follows from our earlier results. We know from lemma 8 that any separating equilibrium would have to lead to PPAM, so that other separating equilibria would have to differ in agents' beliefs, their choice of marketplace, their bargaining strategy, or in the steady state. However, lemma 8 implies that choosing a marketplace $n \in \mathcal{N}(x)$ in the sector where one is more productive is the uniquely optimal choice rule for x. When signals are therefore truthful, the unique bargaining SPE in section 5.1 always results. Then only one specification of beliefs about equilibrium actions will be compatible with these choices.

Finally, as the bargaining SPE ensures agreement in the first round of bargaining, $\kappa(x) = 0$ for all $x \in \Theta$ and in any separating equilibrium. Since this agreement to match is reached with an agent of the same type, assumption 2 is sufficient to ensure participation of all types, as shown in section 5.1. Hence also $\nu(x) = 0$, $\forall x \in \Theta$, so that equation (14) applies to the steady state and determines a unique mass for the matched and for the unmatched agents of each type. Hence, separating equilibria other than the putative equilibrium can only differ in beliefs about off-equilibrium actions.

7 Conclusions

This paper has introduced costless signals into a search model with transferable utility. A simple market design has been proposed that leads agents to signal truthfully. We thus find a unique separating equilibrium characterised by perfect sorting, minimised search duration and search costs, and maximised overall match output. These efficiency benchmarks are virtually never met by random search models because frictions lead to lengthy search and to some mismatch. In our model, signals allow agents to avoid this, so

[11] We ignore separating equilibria where signals are not truthful yet still informative because they are linked by a one-to-one mapping to agents' true types, and this mapping forms the basis of agents' correct beliefs. Such equilibria would only be variants of equilibria with truthful signals.

that signals largely offset the effect of frictions on efficiency. This role of signals reflects the pervasive use of effective communication in real-world matching markets that facilitates search.

PAM in the separating equilibrium only requires supermodularity of the match production function, i.e. the same condition as in a frictionless model. While this condition is unambiguously weaker than the conditions in random search models such as Shimer and Smith (2000), it does not merely ensure PAM, but even perfect PAM. To the best of our knowledge, perfect sorting has not resulted before in a model with discounting or explicit search costs. The key is to allow for more information: supermodularity here does not only ensure enough complementarity for sorting but also ensures truthful signals that help agents sort. Supermodularity thereby replaces a single-crossing condition. Hence, compared to models with random search, a model with more information in the search process appears to generate sorting more easily.

Sorting is likely to become more important as technological and societal progress favours specialisation. At the same time, many new means have appeared of effective and rapid communication that might, as in our paper, support sorting. Such means of communication and the greater availability of information may therefore be expected to increase efficiency, but also to deepen segregation. In any case, the interaction of specialisation and communication offers ample scope for further research.

A Proofs

Proof of lemma 1. Since outside options are not binding, y maximises $\pi(y|x)$ subject to $W^O(x|y) \geq V(x|y)$ whenever she moves first. When match output is $f(x,y)$ and y takes $\pi(y|x)$ for herself, $f(x,y) - \pi(y|x)$ would be left for x. Therefore,

$$rW^O(x|y) = f(x,y) - \pi(y|x) - \delta\left[W^O(x|y) - U(x)\right] \tag{20}$$

while $W(x|y)$ and $V(x|y)$ are determined by

$$rW(x|y) = \sigma(x|y) - \delta\left[W(x|y) - U(x)\right] \tag{21}$$

$$rV(x|y) = \zeta[W(x|y) - c - V(x|y)] \tag{22}$$

Use equation (21) to substitute for $W(x|y)$ in equation (22) and solve for $V(x|y)$. After also solving (20) for $W^O(x|y)$, we can rewrite $W^O(x|y) \geq V(x|y)$ as

$$f(x,y) - \pi(y|x) + \delta U(x) \geq \frac{\zeta}{r+\zeta}\left[\sigma(x|y) + \delta U(x) - (r+\delta)c\right] \tag{23}$$

Substituting for $rU(x)$ and then for $\sigma(x|y)$ from equations (8) and (7), respectively, this is

$$(2r + \zeta + \beta\delta)\left[f(x,y) - \pi(y|x)\right] \geq (\zeta - \beta\delta)\left[\pi(x|y) - 2(r+\delta)c\right] \tag{24}$$

after collecting terms. As y raises $\pi(y|x)$ the left-hand side of equation (24) linearly falls, while the right-hand side stays constant. Hence this constraint will hold with equality for the equilibrium value of $\pi(y|x)$. When x moves first, the constraint is analogously found as

$$(2r + \zeta + \beta\delta)\left[f(x,y) - \pi(x|y)\right] \geq (\zeta - \beta\delta)\left[\pi(y|x) - 2(r+\delta)c\right] \tag{25}$$

As binding constraints, equations (24) and (25) are two equations in two unknowns, so that they determine a unique equilibrium. By the symmetry of these equations, we infer $\pi(x|y) = \pi(y|x)$. When we make this substitution in either equation and solve, we obtain the expression in lemma 1. Because both first-mover shares have been derived under the constraint that the second mover accepts, agreement is reached in the first round of bargaining. Finally, subgame perfection as in Rubinstein (1982) holds because present values such as $V(x|y)$ and $U(x)$ incorporate optimising behaviour in every later subgame. The proof for sector G proceeds analogously. □

Proof of lemma 2. As match output is the only source of utility in the model, agents who do not engage in search obtain payoff 0. Then agent x will only engage in search if $U(x) \geq 0$. By equation (8), this requires

$$c \leq \sigma(x|y)/(r+\delta) \quad \Leftrightarrow \quad 2c \leq f(x,y)/(r+\delta)$$

using equation (13). If this holds for $f(\underline{x}, \underline{y})$, as stated in assumption 2, then it will also hold for the output generated in any other match because $f(x,y)$ is strictly increasing in x and y by assumption 1. This carries over to sector G since $f(\underline{x}, \underline{y}) = g(\bar{x}, \bar{y})$. □

Proof of lemma 3. Any type $x_H > (\bar{x} - \underline{x})/2$ in sector F will strictly prefer the equilibrium match to a match with a lower type $y_L \geq (\bar{x} - \underline{x})/2$ if $W(x_H|y_H) > W(x_H|y_L)$. As argued before, this is equivalent to

$$\begin{aligned} \sigma(x_H|y_H) &> \sigma(x_H|y_L) \\ \Rightarrow f(x_H, y_H) - f(x_H, y_L) &> f(x_H, y_L) - f(x_L, y_L) \end{aligned} \tag{26}$$

using equations (15) and (16). Next, note that we can write

$$f(x_H, y_L) = f(y_H, x_L) = f(x_L, y_H) \tag{27}$$

where the first equality holds because $x_H = y_H$ and $y_L = x_L$, while the second equality holds by symmetry of $f(\cdot, \cdot)$ (see assumption 1). Therefore substituting $f(x_L, y_H)$ for $f(x_H, y_L)$ on the left-hand side of equation (26) only, we obtain the equation in definition 3. By this definition, strict supermodularity of $f(\cdot, \cdot)$ is necessary and sufficient for the equation to hold. □

Proof of lemma 4, part a). Agents $x_H \geq (\bar{x} - \underline{x})/2$ and $y_L \geq (\bar{x} - \underline{x})/2$ in sector F would respectively accept if

$$W^O(x_H|y_L) \geq \max[V(x_H|y_L), U(x_H)], \quad W^O(y_L|x_H) \geq \max[V(y_L|x_H), U(y_L)]$$

If outside options are not binding and y_L moves first, she will maximise $\pi(y_L|x_H)$ subject to $W^O(x_H|y_L) \geq V(x_H|y_L)$. As players revert to the putative equilibrium after a match break-up,

$$rW^O(x_H|y_L) = f(x_H, y_L) - \pi(y_L|x_H) - \delta\left[W^O(x_H|y_L) - U(x_H)\right] \tag{28}$$

while $W(x_H|y_L)$ and $V(x_H|y_L)$ are determined by

$$\begin{aligned} rW(x_H|y_L) &= \sigma(x_H|y_L) - \delta\left[W(x_H|y_L) - U(x_H)\right] && (29) \\ rV(x_H|y_L) &= \zeta[W(x_H|y_L) - c - V(x_H|y_L)] && (30) \end{aligned}$$

Use equation (29) to substitute for $W(x_H|y_L)$ in equation (30) and solve for $V(x_H|y_L)$. After also solving (28) for $W^O(x_H|y_L)$, we can rewrite $W^O(x_H|y_L) \geq V(x_H|y_L)$ as

$$f(x_H, y_L) - \pi(y_L|x_H) + \delta U(x_H) \geq \frac{\zeta}{r + \zeta}\left[\sigma(x_H|y_L) + \delta U(x_H) - (r + \delta)c\right] \tag{31}$$

With $\sigma(x_H|y_L)$ defined in analogy to equation (7), equation (31) becomes

$$(2r + \zeta)\left[f(x_H, y_L) - \pi(y_L|x_H)\right] \geq \zeta\pi(x_H|y_L) - 2\left[\delta r U(x_H) + \zeta(r + \delta)c\right] \tag{32}$$

after collecting terms. Using the results from lemma 1 in equation (8),

$$rU(x_H) = \beta\left[\frac{1}{2}f(x_H, y_H) - (r + \delta)c\right] \tag{33}$$

Thus substituting for $rU(x_H)$ in equation (32), we obtain

$$(2r+\zeta)\left[f(x_H,y_L)-\pi(y_L|x_H)\right] \geq \zeta\pi(x_H|y_L) - \beta\delta f(x_H,y_H) - 2(\zeta-\beta\delta)(r+\delta)c \tag{34}$$

As before, the left-hand side of equation (34) linearly falls as y_L raises $\pi(y_L|x_H)$, while the right-hand side stays constant. This constraint will therefore hold with equality. The same applies to the analogous constraint for the case that x_H moves first:

$$(2r+\zeta)\left[f(x_H,y_L)-\pi(x_H|y_L)\right] \geq \zeta\pi(y_L|x_H) - \beta\delta f(x_L,y_L) - 2(\zeta-\beta\delta)(r+\delta)c \tag{35}$$

As a system of two binding constraints in two unknowns, equations (34) and (35) then determine a unique equilibrium. Solving them simultaneously, one obtains the expressions given for $\pi^*(x_H|y_L)$ and $\pi^*(y_L|x_H)$ in lemma 4. The equilibrium is subgame-perfect because the present values incorporate optimising behaviour in following subgames. □

Proof of lemma 4, part b). We want to prove that some $x_H > (\bar{x}-\underline{x})/2$ in sector F strictly prefers the equilibrium match to a match with a type y_L, where $(\bar{x}-\underline{x})/2 \leq y_L < x_H$, when the type x_H is observed before bargaining begins. First suppose the outside option of x_H binds, $V(x_H|y_L) < U(x_H)$, where

$$rV(x_H|y_L) = \zeta[W(x_H|y_L) - c - V(x_H|y_L)], \quad rU(x_H) = \eta[W(x_H|y_H) - c - U(x_H)] \tag{36}$$

Solving equation (36) respectively for $V(x_H|y_L)$ and $U(x_H)$, we write $V(x_H|y_L) < U(x_H)$ as

$$\zeta(r+\eta)[W(x_H|y_L)-c] < \eta(r+\zeta)[W(x_H|y_H)-c] \tag{37}$$

From $\zeta \geq \eta$ it follows that $\zeta(r+\eta) \geq \eta(r+\zeta)$. Equation (37) thus requires $W(x_H|y_L) < W(x_H|y_H)$, which means that x_H strictly prefers her equilibrium match whenever her outside option binds. Therefore suppose instead that neither agent's outside option binds, so that the results from part a) apply. Then

$$\begin{aligned}\sigma(x_H|y_L) &= \frac{1}{2}\pi^*(x_H|y_L) + \frac{1}{2}\left[f(x_H,y_L)-\pi^*(y_L|x_H)\right] \\ &= \frac{1}{2}\left[f(x_H,y_L) + \frac{\beta\delta}{2r}\left[f(x_L,y_L)-f(x_H,y_H)\right]\right]\end{aligned}$$

Recalling that $\sigma(x_H|y_H) = \frac{1}{2}f(x_H,y_H)$, we will thus have $\sigma(x_H|y_H) > \sigma(x_H|y_L)$ if

$$f(x_H,y_H) > f(x_H,y_L) + \frac{\beta\delta}{2r}\left[f(x_L,y_L)-f(x_H,y_H)\right]$$

which holds because $f(x_H,y_H) > f(x_H,y_L)$ and $f(x_L,y_L) - f(x_H,y_H) < 0$. We conclude that x_H strictly prefers her equilibrium match when neither outside option binds. This preference extends to the case when only the outside option of y_L binds, as x_H then cannot be better off than in the case when neither outside option binds. Suppose it did make x_H better off, so that the share for x_H increases. Since agents split output, the share for y_L decreases accordingly. Then y_L would choose not to take her outside option, which therefore cannot be binding. □

Proof of lemma 5, part a). We have to establish that any agent $x_H \geq (\bar{x} - \underline{x})/2$ in sector F always prefers, for any beliefs of some y_L with $(\bar{x} - \underline{x})/2 \leq y_L \leq x_H$, her equilibrium match to a deviation such that she meets y_L with whom bargaining fails. For $x_H = y_L$, lemma 1 implies that x_H would have preferred reaching a bargaining agreement with y_L. For $x_H > y_L$, we have to consider all possible beliefs held by y_L about the potential match output $f(x, y)$ when bargaining fails:

(i) $f^e(y_L|h) = f(x_H, y_L)$ so that y_L believes to face the true type x_H. By lemma 4, x_H strictly prefers her equilibrium match.

(ii) $f^e(y_L|h) > f(x_H, y_L)$ so that y_L overestimates potential match output and thus believes to face a type even higher than x_H. By the same argument as in the proof of part b) of lemma 4, y_L does not believe the outside option of x_H to bind: if it did, x would have had to pursue an equilibrium-dominated strategy. Observe that both $\pi^*(y_L|x_H)$ and $f(x_H, y_L) - \pi^*(x_H|y_L)$ in lemma 4 are non-decreasing in x_H, whether or not the outside option of y_L binds. Hence y_L demands weakly higher shares than under (i). Because x_H strictly prefers her equilibrium match under (i), she still prefers her equilibrium match when y_L is more demanding.

(iii) $f(x_H, y_L) > f^e(y_L|h) > f(x_L, y_L)$ so that y_L underestimates potential match output but still believes to face a higher type. Note that $f(x_L, y_L)$ is then a lower bound for $f^e(y_L|h)$. By lemma 3, x_H strictly prefers her equilibrium match if y_L believes to face x_L (and x_H imitates x_L to avoid bargaining failure). By the same arguments as under (ii), if y_L believes to face a higher type $x_H > x_L$, she will not believe the outside option of x_H to bind and will demand weakly higher shares. Then x_H still prefers her equilibrium match.

(iv) $f^e(y_L|h) = f(x_L, y_L)$ so that y_L believes to face the same type as her own type. By lemma 3, x_H strictly prefers her equilibrium match.

(v) $f^e(y_L|h) < f(x_L, y_L)$ so that y_L believes to face a lower type. By the definition of x_F^* in equation (17), if $x_F^* \leq (\bar{x} - \underline{x})/2$ then y_L prefers meeting another agent rather than continued bargaining with x_H who is perceived as a lower type. Hence y_L walks away and x_H would prefer her equilibrium match to this deviation.

Hence the deviation in question is equilibrium-dominated for weakly higher types than y_L. $\square$

Proof of lemma 5, part b). When we require that agents' beliefs assign probability 0 to equilibrium-dominated actions and that $f(\cdot, \cdot)$ be strictly supermodular, any type $y_L \geq (\bar{x} - \underline{x})/2$ must believe by part a) of lemma 5 to face a lower type when bargaining fails, so that $f^e(y_L|h) < f(x_L, y_L)$. By the argument under (v) in the proof of part a), y_L then walks away when bargaining fails. $\square$

Proof of lemma 6. Any type x_L in sector F, with $(\bar{x} - \underline{x})/2 \leq x_L < \bar{x}$, will strictly prefer the equilibrium match to a match with a higher type y_H if $W(x_L|y_L) > W(x_L|y_H)$, which is equivalent to

$$\begin{aligned} \sigma(x_L|y_L) &> \sigma(x_L|y_H) \\ \Rightarrow f(x_H, y_H) - f(x_L, y_H) &> f(x_L, y_H) - f(x_L, y_L) \end{aligned}$$

using equations (18) and (19). By equation (27), we can replace $f(x_L, y_H)$ on the right-hand side by $f(x_H, y_L)$. Hence strict supermodularity is necessary and sufficient for this equation to hold. Finally, for the type $\bar{x}$, a higher type than in the equilibrium match does not exist. $\square$

Proof of corollary 1. Consider some arbitrary unmatched agent in sector F and call this exemplary type x_E. Recall that $\mathcal{N}(x_E) \equiv \{n|R^n = \{x_E\}\}$. Given the choice of bargaining strategy and given all other agent's choices in the putative equilibrium, an agent of type x_E will obtain her equilibrium match with an agent of type $y_E = x_E$ *if* she chooses a marketplace $n \in \mathcal{N}(x_E)$. Further given that agents meet exclusively their own type in the putative equilibrium, $|R^n| = 1$ for all n. Hence x_E will obtain her equilibrium match *only if* she chooses a marketplace $n \in \mathcal{N}(x_E)$.

By lemmas 3 through 6, x_E will strictly prefer this match to any other match in sector F if $f(\cdot, \cdot)$ is supermodular, η and ζ are sufficiently close, and agents' beliefs rule out equilibrium-dominated actions. Because type x_E was arbitrarily chosen, the reasoning extends to any type in the sector. If signals are therefore truthful, then $u^n(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = 1$ for $y \in R^n$, and agents' beliefs can only be consistent if $\psi(y|h = \{\tilde{x}, \tilde{y} = \text{“}y \in R^n\text{”}\}) = 1$ for $y \in R^n$. $\square$

Proof of lemma 7. Suppose to the contrary that an agent with a type $\underline{x}$ sets up a marketplace in sector F. Instead of $\underline{x}$, we write x_L to keep the proof general. Let us first focus on matches between x_L and some $y < (\bar{x} - \underline{x})/2$, recalling that truthful signals cannot be expected from types below x_F^*. To provide an envelope result, consider as in lemma 4 the most favourable case for x_L that agents instantly observe each others' types. If x_L then matches with another agent of type $y_L = x_L$, the symmetry of the bargaining situation will imply

$$\sigma(x_L|y_L) = \frac{1}{2} f(x_L, y_L)$$

while x_L would obtain $\frac{1}{2} g(x_L, y_L)$ in her equilibrium match in sector G. Since $x_L < (\bar{x} - \underline{x})/2$, we know that $g(x_L, y_L) > f(x_L, y_L)$, and hence x_L strictly prefers sector G. Alternatively, the other agent has a higher type y_H. Suppose again the most favourable case for x_L that the outside option of y_H does not bind. Suppose that the outside option of x_L to search in sector G also does not bind (if it does, the same logic as in the proof of lemma 4, part b) will imply that x_L strictly prefers sector G). Then we can proceed as in the proof of lemma 4, part a) with the exception that we replace equation (33) by the value to x_L of searching in sector G, since this option is always available to her:

$$rU(x_L) = \beta \left[\frac{1}{2} g(x_L, y_L) - (r + \delta) c \right]$$

We find the expressions for the bargaining shares as

$$\pi^*(x_L|y_H) = \frac{2r+\zeta}{2(r+\zeta)}\left[f(x_H, y_L) + \frac{\beta\delta}{2r}\left[g(x_H, y_H) - \frac{\zeta}{2r+\zeta}g(x_L, y_L)\right]\right] + \phi(r+\delta)c$$
$$\pi^*(y_H|x_L) = \frac{2r+\zeta}{2(r+\zeta)}\left[f(x_H, y_L) + \frac{\beta\delta}{2r}\left[g(x_L, y_L) - \frac{\zeta}{2r+\zeta}g(x_H, y_H)\right]\right] + \phi(r+\delta)c$$

The expected share for x_L is in this case

$$\begin{aligned}\sigma(x_L|y_H) &= \frac{1}{2}\pi^*(x_L|y_H) + \frac{1}{2}[f(x_L, y_H) - \pi^*(y_H|x_L)] \\ &= \frac{1}{2}\left[f(x_L, y_H) + \frac{\beta\delta}{2r}[g(x_H, y_H) - g(x_L, y_L)]\right]\end{aligned}$$

Agent x_L will strictly prefer her equilibrium match in sector G if

$$g(x_L, y_L) > f(x_L, y_H) + \frac{\beta\delta}{2r}[g(x_H, y_H) - g(x_L, y_L)]$$

Noting that $g(x_H, y_H) - g(x_L, y_L) < 0$, this holds for any type x_L as long as $y_H < (\bar{x} - \underline{x})/2$ so that $g(x_L, y_L) > f(x_L, y_H)$. As $x_F^* \leq (\bar{x} - \underline{x})/2$, this holds in particular for all $y_H < x_F^*$. Hence type $\underline{x}$ strictly prefers her equilibrium match in sector G. Now consider the second lowest type instead: this type cannot match anymore with $\underline{x}$ in sector F, so that the logic above now applies to this type, who therefore strictly prefers sector G. The argument can be repeated for all types $x < (\bar{x} - \underline{x})/2$.

Let us now focus on matches between x_L and some $y \geq (\bar{x} - \underline{x})/2$. Since therefore $y \geq x_F^*$, the definition of x_F^* implies that y prefers searching for her equilibrium match to meeting any types below x_F^*. The same preference keeps y from searching on a marketplace with $R^n = \{x\}$ for $x_F^* \leq x < \bar{x}$ but $x \neq y$: lemmas 3 through 6 extend to all $y \geq x_F^*$ and were limited to $y \geq (\bar{x} - \underline{x})/2$ only for expositional reasons. As explained in section 5.3, y also does not search on mixed marketplaces for types $y \geq x_F^*$. Finally, if x_L chooses $R^n = \{y\}$ for the marketplace, then y will believe signals to be truthful and will walk away after bargaining fails. Therefore, x_L can only match with any $y \geq (\bar{x} - \underline{x})/2$ by perfectly imitating her. By lemma 6, x_L would then obtain less than $\frac{1}{2}f(x_L, y_L)$, and since $g(x_L, y_L) > f(x_L, y_L)$, y_L strictly prefers her equilibrium match in sector G. Analogous arguments, using $\bar{x}$ instead of $\underline{x}$ above, prove that all types $x > (\bar{x} - \underline{x})/2$ strictly prefer sector F. □

Proof of corollary 2. The proof given in Becker (1973) applies to our set-up and we essentially repeat it here. Let $f(\cdot, \cdot)$ be strictly supermodular and index types in sector F by $1, 2, \ldots I$ such that $x_1 < x_2 < \ldots < x_I$. If PPAM maximises aggregate output, then

$$\sum_{j=1}^{I} f(x_j, y_{i_j}) < \sum_{i=1}^{I} f(x_i, y_i) \quad \text{for all permutations } (i_1, i_2, \ldots i_I) \neq (1, 2, \ldots I)$$

Suppose to the contrary that aggregate output is maximised by some permutation $i_1, i_2, \ldots i_I$ for which $i_1 < i_2 < \ldots < i_I$ does not hold. Then the permutation includes at least one j_0 such that $i_{j_0} > i_{j_0+1}$. By strict supermodularity of $f(\cdot,\cdot)$,

$$f(x_{j_0+1}, y_{i_{j_0}}) - f(x_{j_0}, y_{i_{j_0}}) > f(x_{j_0+1}, y_{i_{j_0+1}}) - f(x_{j_0}, y_{i_{j_0+1}})$$

because $x_{j_0+1} > x_{j_0}$ while $y_{i_{j_0}} > y_{i_{j_0+1}}$. After rewriting this as

$$f(x_{j_0}, y_{i_{j_0+1}}) + f(x_{j_0+1}, y_{i_{j_0}}) > f(x_{j_0}, y_{i_{j_0}}) + f(x_{j_0+1}, y_{i_{j_0+1}})$$

the left-hand side represents the match production under PPAM, while the right-hand side represents the match production under the permutation $i_1, i_2, \dots i_I$. As the former exceeds the latter, the permutation $i_1, i_2, \dots i_I$ does not maximise aggregate output. □

Proof of corollary 3. Recall that the separating equilibrium exists, and that it leads to PPAM, provided $f(\cdot,\cdot)$ is strictly supermodular and η and ζ are sufficiently close. Now suppose that PPAM is not a stable matching. Then there must be a match between unequal types that is preferred by both types to matches with exactly corresponding types. However, given strict supermodularity of $f(\cdot,\cdot)$, matching with a lower type is an equilibrium-dominated action for the higher type in any match between unequal types in sector F, by the proof of lemma 5. Likewise, such a match is an equilibrium-dominated action for the lower type in sector G. By lemma 7, there is no agent who wishes to switch sectors, so that a match between unequal types that is preferred by both does not exist. Finally, lemma 1 implies together with assumption 1 that $\sigma(x) \geq 0\ \forall x \in \Theta$ under PPAM. □

Proof of lemma 8. Suppose there is at least one marketplace $n = \mathcal{M}$ in which, with truthful signals, agents do not only meet their own type, so that two or more types meet. Focus on the lowest type y_L in $\mathcal{M}$. This type must be the most preferred feasible type of some higher type $x_H > y_L$ in $\mathcal{M}$, otherwise the higher types would exclude y_L from $\mathcal{M}$ to reduce congestion.
We will show that such a marketplace $\mathcal{M}$ cannot exist in a separating equilibrium. When x_H and y_L bargain, $V(x_H|y_L) \geq U(x_H)$ because x_H most prefers y_L and continued bargaining guarantees a meeting with y_L at rate η. While $U(y_L)$ is unknown, y_L could choose in any separating equilibrium to meet only agents of her own type on an exclusive marketplace $n = \mathcal{L}$. As part of a separating equilibrium, the situation in $\mathcal{L}$ would correspond to the putative equilibrium situation in sector F, say, and because of the symmetry when y_L and x_L bargain in $\mathcal{L}$,

$$\pi^*(x_L|y_L) = \pi^*(y_L|x_L) \quad \Rightarrow \quad \sigma(y_L|x_L) = \frac{1}{2} f(x_L, y_L)$$

independently of outside options. As $\mathcal{L}$ is always an option for y_L, the payoff y_L would obtain there constitutes a lower bound for $U(y_L)$, denoted $\underline{U}(y_L)$. With equation (8), it is found as

$$r\underline{U}(y_L) = \beta \left[\frac{1}{2} f(x_L, y_L) - (r + \delta)c \right]$$

Next observe that x_H cannot do better in a match with y_L than to leave y_L only with the payoff $\underline{U}(y_L)$ in expectation, so that the payoff to x_H in this case constitutes an upper bound $\overline{W}(x_H|y_L)$. Now suppose that an agent of type $y_H = x_H$ sets up an exclusive marketplace $n = \mathcal{H}$

for her type. If this creates a profitable deviation for x_H who currently most prefers y_L, the supposed marketplace $\mathcal{M}$ cannot exist in equilibrium. The symmetry in $\mathcal{H}$ would lead to

$$\pi^*(x_H|y_H) = \pi^*(y_H|x_H) \quad \Rightarrow \quad \sigma(x_H|y_H) = \frac{1}{2}f(x_H, y_H)$$

again as in the putative equilibrium situation in sector F. As an envelope case, suppose x_H obtains $\overline{W}(x_H|y_L)$ in a match with y_L in $\mathcal{M}$ and now faces the choice between this match and a match with y_H in $\mathcal{H}$. Part b) of lemma 4 applies to this choice (with $U(y_L) = \underline{U}(y_L)$) and establishes a strict preference for the match with y_H over the match with y_L. As x_H meets y_H at rate η and y_L at most at rate η, this preference also translates into a strict preference for marketplace $\mathcal{H}$. Hence x_H has a profitable deviation from $\mathcal{M}$ to $\mathcal{H}$ even when $\overline{W}(x_H|y_L)$ is obtained in $\mathcal{M}$. By the same reasoning, y_H also gains from setting up $\mathcal{H}$. $\square$

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