Modelling spatio-temporal variability of temperature

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin
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Abstract

Forecasting temperature in time and space is an important precondition for both the design of weather derivatives and the assessment of the hedging effectiveness of index based weather insurance. In this article, we show how this task can be accomplished by means of Kriging techniques. Moreover, we compare Kriging with a dynamic semiparametric factor model (DSFM) that has been recently developed for the analysis of high dimensional financial data. We apply both methods to comprehensive temperature data covering a large area of China and assess their performance in terms of predicting a temperature index at an unobserved location. The results show that the DSFM performs worse than standard Kriging techniques. Moreover, we show how geographic basis risk inherent to weather derivatives can be mitigated by regional diversification.

Keywords: weather insurance, semiparametric model, factor model, Kriging, geographic basis risk

JEL classification: C14, C53, G32

1 Introduction

Weather constitutes an important risk factor for many industries, such as agriculture, the energy sector, and tourism. Firms in these sectors are naturally concerned about unfavorable weather conditions and are interested in the development of risk management tools that can allow them to better cope with weather perils. An important strategy to mitigate negative financial consequences of weather perils is insurance. In fact, in the last decade the insurance industry has developed products that offer protection against weather risks, namely weather derivatives and weather index based insurance. These products are either traded at exchanges, such as at the Chicago Mercantile Exchange (CME), or over the counter (OTC) and can be written on a variety of weather variables such as temperature, rainfall, snow, wind, or indices based on these various parameters. A payoff is granted if a specific weather event occurs at a predetermined weather station. For example, a farmer insured against drought would receive a payoff if the total amount of rainfall reported at an independent weather station undercuts a predetermined threshold. Hellmuth et al. (2009) and Balzer and Hess (2010) provide a comprehensive overview about the characteristics and applications of these instru-

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ments. While a considerable market potential for weather insurance has been predicted by many (e.g., Barnett and Mahul 2007; Skees et al. 2008), thus far the actual acceptance of these products falls short of expectations. A major obstacle for the implementation of weather insurance is the existence of basis risk, which occurs if the insurance payoff does not exactly match the actual losses of the insured (see Deng et al. (2007) and Woodard and Garcia (2008)). Basis risk can occur either because the underlying weather index and losses are not perfectly correlated (production basis risk) or because the weather conditions at the reference station and the location of the policyholder deviate (geographic basis risk). Geographic basis risk can be considerable depending on the weather variable and the geographic situation (Miranda and Vedenov 2001). Knowledge of the geographic basis risk is important from the viewpoint of buyers and sellers of weather insurance products. The insured want to know if and to what extent their weather risk exposure can be reduced by buying weather insurance. In other words, they want to assess the hedging effectiveness of the instrument. Insurers are also interested in assessing geographic basis risk since it helps them to predict the potential demand for weather insurance as well as to estimate if it makes sense economically to launch such a product at a given location (cf. Paulson and Hart (2006) and Paulson et al. (2010)).

Geographic basis risk is mainly a result of two factors. The first is spatial variability of weather conditions and the second is unavailable weather data at locations where this information is needed to assess weather risk. The desire to estimate and to forecast meteorological events is very general and goes beyond the estimation of geographic basis risk. Thus, it is not surprising that a variety of statistical tools exist that support this task. Sluiter (2009) provides an overview of current spatial interpolation methods for climate data. Caruso and Quarta (1998) compare four commonly used methods for point interpolation. They find that Kriging outperforms the other approaches if locations are situated further away from each other. A similar finding is reported by Stahl et al. (2006).

Temperature, however, varies in space and time and thus temporal variation must be taken into account. This can also be achieved by Kriging techniques, such as local space-time Kriging (Gething et al. 2007), or by dynamic models for non-stationary spatio-temporal data (Stroud et al. 2001). As an alternative to standard Kriging procedures, we consider a dynamic semiparametric factor model (DSFM) which has recently been suggested by Song et al. (2010, 2013) for analyzing high dimensional data. A DSFM is able to smooth high dimensional data via dimension reduction and to forecast in both time and space (see Park et al. (2009)). The nature of a DSFM is based on parametric estimation in time and nonparametric estimation in space. So far, DSFMs have been mainly applied to financial data (Fengler et al. 2007; Giacomoni et al. 2009; Choroś-Tomczyk et al. 2013). DSFMs allow us to study dynamics of temperature in the time level as well as high dimensionality in the space level. In this paper, the Kriging technique is applied as the standard procedure.

The contribution of this paper is twofold: First, we adapt a DSFM to a comprehensive set of temperature data and compare this new method with the standard methods of interpolation and forecasting of weather data. Second, we assess the geographic basis risk and the hedging effectiveness of temperature based index insurance for China. In doing so, we provide information on the market potential of weather derivatives in China. Agriculture plays an important role in China, and farmers are highly exposed to weather risks. The need for insurance in the agricultural sector is reflected by the 24.06 billion Yuan insurance premium volume in 2012, which was 38.8% higher than in 2011 (People’s Bank of China 2013). In March 2013, new agricultural insurance regulations came into force and a weather index based insurance pilot project had been launched in the province of Anhui (Balzer...
and Hess 2010). It is, however, difficult to estimate the market potential for weather index based insurance because the hedging effectiveness of the insurance products is unclear to market participants. It is particularly difficult to quantify geographic basis risk because of the low density of weather stations and the lack of reliable historical weather data. Against this background, our objective is to provide statistical methods that allow the spatial interpolation and prediction of temperature data. This would enable the quantification of geographic basis risk and support an assessment of the willingness to pay for weather insurance.

This paper is structured as follows: In the following section, we provide a formal definition of geographic basis risk and describe methodologies for interpolating and forecasting temperature. In Section 3, we apply the models to Chinese temperature data and compare their performance. Furthermore, we analyze the geographic basis risk for an exemplary buyer of a weather derivative in China. Further discussions and conclusions are in Section 4.

2 Methods

2.1 Geographic Basis Risk

To define geographic basis risk, we consider the revenue $R_{0,k}$ of a weather-dependent producer (e.g., a farmer) at location $k$ at time 0. The revenue can be modeled as the product of the weather-dependent production function $Q_t(I_{t,k})$ and the product’s market price $P$, where $I_{t,k}$ denotes a weather index measured at time $t$ at location $k$, which is assumed to describe a producer’s weather dependency. The specific form of the index will be introduced later in Section 3.1. The producer can hedge his/her weather risk by buying a weather derivative or index based weather insurance. The payoff of the weather derivative depends on the weather index $I_{t,l}$ measured at a reference location $l$. At time 0, the producer pays a premium $\pi_{0,t}$ for the derivative and receives a payoff $F_t(I_{t,l})$. Thus, the discounted revenue of the producer holding the weather insurance is given by (cf. Mußhoff et al. (2011) and Ritter et al. (2013)):

$$R_{0,k} = \{Q_t(I_{t,k}) \cdot P + F_t(I_{t,l})\} \cdot e^{-r\Delta t} - \pi_{0,t},$$

(1)

where $e^{-r\Delta t}$ is a discount factor. Basis risk arises from two facts. First, production revenues $Q(I) \cdot P$ and the weather-dependent insurance payoff $F_t(I_{t,l})$ are not perfectly (negatively) correlated. In our paper, we ignore this source of basis risk called production basis risk because it is producer specific and difficult to generalize. Second, if the production location $k$ and the reference station $l$ are not the same, the weather at the two locations might be different, and thus $I_{t,k}$ and $I_{t,l}$ differ. In turn, the producer faces geographic basis risk.

If the weather derivative was offered at location $k$, the hypothetical revenue would be:

$$\bar{R}_{0,k} = \{Q_t(I_{t,k}) \cdot P + F_t(I_{t,k})\} \cdot e^{-r\Delta t} - \pi_{0,k}$$

(2)
The deviation of the hypothetical revenue from the real revenue is a measure of geographic basis risk. We quantify this deviation by the quadratic loss function:

\[
(R_{0,k} - \hat{R}_{0,k})^2
\]

\[
\frac{1}{|T|} \sum_{t \in T} \left( \left(\left[Q_t(I_{t,k}) \cdot P + F_t(I_{t,l})\right] \cdot e^{-r\Delta t} - \pi_{0,l}\right) - \left(\left[Q_t(I_{t,k}) \cdot P + F_t(I_{t,k})\right] \cdot e^{-r\Delta t} - \pi_{0,k}\right) \right)^2
\]

\[
\left( \frac{\text{Difference of payoffs}}{\text{Difference of prices}} \right)^2
\]

Since the prices of the hypothetical derivative are not reachable on the market, the difference in prices is ignored in this paper. Hence, we want to minimize the (quadratic) difference between the payoffs to reduce the geographic basis risk:

\[
\min \left( F_t(I_{t,l}) - F_t(I_{t,k}) \right)^2
\]

From an ex-ante perspective, the actual payoffs \( F_t(I_{t,l}) \) and the hypothetical payoff \( F_t(I_{t,k}) \) have to be predicted with information available at time 0. Thus, assessing geographic basis risk requires a forecasting procedure. Moreover, while weather data are available at the reference station \( l \), this may not hold for the production location \( k \), particularly in developing countries where the density of weather stations is generally low. This means that the calculation of geographic basis risk also requires a spatial interpolation technique. In the next subsection, we describe the approaches supporting these tasks, i.e., forecasting in time and interpolation in space.

### 2.2 Statistical Approaches

To predict the future index values at an unobserved location, we apply four different approaches: A dynamic semiparametric factor model (DSFM, Section 2.3) and Kriging (Section 2.4) are combined with daily modelling and index modelling. A DSFM allows for interpolating in space and time, whereas Kriging techniques only interpolate spatially. Hence, it has to be combined with forecasting methods in time such as a stochastic temperature model (Section 2.4.1) or Burn Analysis (Section 2.4.2). Applying daily and index modelling allows us to additionally evaluate these two different data processing approaches.

Figure 1 depicts the four procedures in detail: When applying DSFM, one can interpolate historical daily temperatures directly, forecast temperatures at the unobserved location, and derive the index values at the end (Daily DSFM approach). Alternatively, the index values can be calculated first for each location before they are used for interpolating and forecasting the index values at the unobserved location using a DSFM (Index DSFM approach). The two approaches applying Kriging are similar: For the Daily Kriging approach, historical daily temperatures are interpolated using Kriging and then future temperatures for all locations are forecasted using a temperature model. The index values are then derived from the forecasted temperature values. In the Index Kriging approach, derived index values for all locations are interpolated by Kriging. The expected index value at the unobserved location is then obtained by averaging the historical index values (burn analysis).
2.3 Dynamic Semiparametric Factor Model (DSFM)

Factor models are usually applied to high-dimensional data. Fengler et al. (2007) and Park et al. (2009) generalize a dynamic semiparametric factor model for high-dimensional time series, in which linear combinations of factors are utilized for a low-dimensional representation of the data. The estimated factors and factor loadings are driven by historical observations and reflect the dynamics of time series and the spatial, time-invariant component, respectively.

The dynamic semiparametric factor model has the form:

\[ Y_{t,i} = m_0(x_{t,i}) + \sum_{l=1}^{L} Z_{t,l} m_l(x_{t,i}) + \epsilon_{t,i}, \quad 1 \leq i \leq I, 1 \leq t \leq T, \quad x_{t,i} \in [0,1]^d \]  

(7)

The index \( t \) denotes the time evolution, \( i \) reflects the spatial variation, and \( L \) is the number of orthogonal factors. For our dataset, \( Y_{t,i} \) are temperature observations on each day \( t \), and \( x_{t,i} \) denotes the coordinates of each station \( i \), which are in the form of longitude and latitude and remain constant over time. Therefore, \( I \) does not depend on \( t \). We can rewrite this equation as:

\[ Y_{t,i} = Z_t^T m(x_{t,i}) + \epsilon_{t,i}, \]  

(8)

where \( m(\cdot) \) is a tuple of unknown smooth functions \((m_0, m_1, \ldots, m_L)^T\), called basis functions, which reflect the time invariant (factor) structure and can be estimated directly from the data; \( Z_t = (1, Z_{t,1}, \ldots, Z_{t,L}) \) is a multivariate time series with a dynamic structure; and \( \epsilon_{t,i} \) are errors that have zero means, finite second moments, and are independent of \( x_{t,i} \). \( Z_t \) and \( m_l \) are also known as common factors and factor loadings, respectively.

The basis functions are estimated via B-spline series (Park et al. 2009):

\[ m_l(x_{t,i}) = \sum_{k=1}^{K} a_{lk} \psi(x_{t,i}), \]  

(9)

where \( \psi(x_{t,i}) \) is a B-spline basis function and \( a_{lk} \) are coefficients. \( K \) describes the number of series expansion functions and serves as bandwidth in the Kernel estimation.
Then, the least square estimator is defined as:

\[
(Z_t' \hat{A}) = \arg\min_{Z_t, A} \sum_{t=1}^{T} \sum_{i=1}^{l} (Y_{t,i} - Z_t A \psi(x_{t,i}))^2,
\]

(10)

where \( A = (a_{ik}) \) is an \((L + 1) \times K\) matrix, which is used to estimate \( m \) by \( \hat{m} = \hat{A} \psi \).

As noted in Park et al. (2009), the estimates of \( Z_t \) and \( m_t \) are not uniquely defined since their product in Eq. (10) can be obtained by different functions. Nevertheless, there always exists a random matrix \( B \) which gives \( B' \hat{Z}_t \) asymptotically the same covariance structure as \( Z_t \). Then, one can estimate \( \hat{m} = \hat{A} \psi \) such that \( \hat{m} \) is orthonormal in \([0,1]^d\), which implies that \( B \) is also an orthogonal matrix. The calibration procedure is implemented in the following way: First, the initial estimate of \( Z_t \) is set as a white noise sequence to estimate the initial \( \hat{a}_{ik} \). Then, with the fixed alpha, we perform an estimation of \( Z \). Finally, the procedure is iterated until it converges. For more details, refer to Park et al. (2009).

For forecasting, the time-variant factors \( Z_t \) have to be extrapolated for the forecasting horizon. For this purpose, the distinct patterns underlying these factors are predicted using functions for the trend, seasonality, and an autoregressive component for the Daily DSFM approach. For the Index DSFM approach, VAR processes are used to predict these factors.

### 2.4 Kriging procedure

#### 2.4.1 Ordinary Kriging

Kriging is a geostatistical interpolation method introduced by Krige (1976) and generalized by Matheron (1965). The spatial relationships are estimated through a semivariogram. Then, the weighted combinations of neighbors are used for interpolation.

We assume that the random data \( Y(x_i) \) at certain points \( x_i : i = 1, ..., n \) exhibit "intrinsic stationarity", which follows the formula:

\[
E\{Y(x_i + h) - Y(x_i)\} = 0
\]

(11)

The intrinsic hypothesis is a weaker form of stationarity than the strong stationarity or the second-order stationarity. It means that the expected value of \( \{Y(x_i + h) - Y(x_i)\} \) is zero for all vectors which are any two points separated by the distance \( h \). Even if the assumption might be not fulfilled for the whole dataset, it can be relaxed to a rather homogenous search neighborhood (Holdaway 1996).

Then, according to Matheron (1965), the classic semivariogram estimator is given by:

\[
\hat{\gamma}(h) = \frac{1}{2n(h)} \sum_{i=1}^{n} (Y(x_i + h) - Y(x_i))^2,
\]

(12)

where \( Y(x_i) \) denotes the value at location \( x_i \) and \( n(h) \) is the number of experimental pairs separated by the distance \( h \) between each pair. Based on Eq. (12), one can compute the experimental variogram from the data.

In practice, there are a number of different theoretical variogram models. The most commonly used models are Gaussian, spherical, power, exponential, and cubic (Olea 1999). There are, however, no
statistical tests to evaluate an experimental variogram and to determine how closely the theoretical variogram is approximated. In this paper, to do automatic fitting we use the least square method and the Akaike information criterion (AIC) over the sum of the squares between the sample variogram and the fitted variogram model (Olea 1999). For our data, the Stein-Matern parameterization developed by Stein (1999) yields the best fit:

\[ \rho(h) = \frac{\pi^{\frac{5}{2}}}{2^{\kappa-1} \Gamma(\frac{k+1}{2}) \alpha^{2k}} (a|h|)^{\kappa} K_{\kappa}(a|h|), \]  

(13)

where \( K_{\kappa} \) is a modified Bessel function, \( \kappa \) is a additional smooth parameter, \( \phi > 0 \), and \( \alpha \) is the range.

According to Cressie (1993), Ordinary Kriging is defined as:

\[ Y(x_i) = \mu + e(x_i), \]  

(14)

where \( \mu \in \mathbb{R} \) is unknown and \( e(\cdot) \) is a white-noise process with mean zero and variance \( \sigma_e^2 \geq 0 \) which describes the spatial random effect.

Estimation of the Ordinary Kriging predictor requires the following conditions (Cressie and Wikle 2011):

1. Linearity:

\[ \hat{Y}(x_0) = \sum_{i=1}^{n} \omega_i Y(x_i), \]  

(15)

where \( \hat{Y}(x_0) \) is the estimator at the new location \( x_0 \), \( Y(x_i) \) are the random variables at locations \( x_i \), and \( \omega_i \) are weights (Kriging coefficients) assigned to each location \( x_i \) that are derived from the variograms.

2. Unbiasedness:

\[ \sum_{i=1}^{n} \omega_i = 1 \]  

(16)

Subject to the unbiasedness condition, we minimize the mean square prediction error over a set of \( \omega \) and obtain the following Lagrange function:

\[ E[\sum_{i=1}^{n} \omega_i Y(x_i) - Y(x_0)]^2 - 2\lambda (\sum_{i=1}^{n} \omega_i - 1), \]  

(17)

where \( \lambda \) is the Lagrange multiplier and \( n \) the number of locations. The mean square prediction error is given by

\[ \sigma_0^2 = E[Y(x_0) - \hat{Y}(x_0)]^2 \]  

(18)

We adopt the cross validation procedure for the estimation, i.e., for each station, the actual mean square prediction error can be derived by the true observation and the estimator.
2.4.2 Daily Temperature Model

In the Daily Kriging approach, Kriging is used to interpolate historical temperatures day by day for the unobserved location. Then, a daily temperature model can be fitted to historical and interpolated data to predict temperatures at each location on the time scale.\(^1\)

In standard temperature models, the time series of temperature is decomposed into four components: trend, seasonality, an autoregressive component, and the residuals with a seasonal variance. For the trend analysis, we use a simple linear trend as Benth and Šaltytė‐Benth (2005) and Campbell and Diebold (2005). To capture the seasonality, Campbell and Diebold (2005) use a low ordered Fourier series, Benth and Šaltytė‐Benth (2005) prefer a truncated Fourier series, and Härdle et al. (2011) propose a local smoothing technique. In this paper, we follow Benth and Šaltytė‐Benth (2005) and Okhrin et al. (2012) to use a simple cosine function for seasonality. Hence, detrending and deseasonalizing of the temperature \(Y_t\) is conducted using the following method:\(^2\)

\[
Y_t = \Lambda_t + X_t, \\
\Lambda_t = a_0 + a_1 t + a_2 \cos\left(\frac{2\pi(t-a_3)}{365}\right) \\
X_t = \sum_{l=1}^{L} \beta_l X_{t-l} + \sigma_t \varepsilon_t, \\
\varepsilon_t \sim N(0,1).
\]

The function \(\Lambda_t\) denotes the combination of the long-term average temperature \(a_0\), the linear trend \(a_1 t\), and seasonality which is captured by the 365-periodic cosine term. Another factor is the autoregressive component, which we capture by an AR(\(L\)) process:

\[
X_t = \sum_{l=1}^{L} \beta_l X_{t-l} + \sigma_t \varepsilon_t,
\]

with \(\varepsilon_t \sim N(0,1)\). The maximum lag \(L\) is determined by the AIC and \(\sigma_t\) denotes temperature volatility which Campbell and Diebold (2005) model as a seasonal GARCH process. In this paper, however, we follow Benth and Šaltytė‐Benth (2005), who explain temperature variance by the distinctive seasonal pattern:

\[
\sigma_t^2 = c_1 + \sum_{k=1}^{K} \left( c_{c,k} \cos\left(\frac{2\pi k t}{365}\right) + c_{s,k} \sin\left(\frac{2\pi k t}{365}\right) \right)
\]

To estimate variance from the observed residuals, we average the squared residuals on the same day of each year for all 365 calendar days and then model the expected squared residuals. After detrending, deseasonalizing, and removing the autoregressive part and the seasonal variance, the residuals are assumed to follow a standard normal distribution \(N(0,1)\).

2.4.3 Burn Analysis

In the Index Kriging model, Ordinary Kriging is directly applied to a temperature index that underlies temperature insurance. In this approach, we follow common actuarial practice and take the average of the interpolated historical indices as the expected index value at the unobservable location for the next year, i.e., we conduct a “burn analysis”.

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\(^1\) The procedure could also be conducted in the opposite way: First, daily temperature models are fitted to the historical data at all locations to forecast the temporal scale. Then, Kriging is used to interpolate the forecasted data for the left-out location on the spatial scale. Although the RMSE results show no significant difference between the two sub-models, the computational effort is much higher.

\(^2\) Note that the temperature model and all coefficients depend on the specific location. To simplify notation, however, we omit the location index \(i\) in this section.
3 Application to Chinese Temperature Data

3.1 Data
Observations of daily temperature from 1957 to 2009 are provided by the China Meteorological Data Sharing Service System. We select 100 stations which are distributed in a rectangular area with lower density towards the north-west (see Fig. 2). Because a temperature index can only be measured once per year, 53 observations are available for each station in the two index models. To reduce computational burden for the daily models, however, we constrain the dataset to the last 11 years to 4,015 observations between 1999 and 2009 for each station. For all models, the data is split into two parts: Data until December 31, 2008 are used for estimation whereas data from 2009 are retained to evaluate forecasts. Moreover, we conduct leave-one-out cross validation to evaluate the methods, i.e., each time we leave one station out of the sample as the unobserved location, interpolate historical values based on data at the other 99 stations in 2008, and forecast for 2009. This procedure is repeated 100 times, so that each location is left out once. In the end, we compare the resulting $\hat{I}_{k,2009}^{\text{interpol}}$ for each location $k$ with the observed index $I_{k,2009}$ calculated from the observations at the benchmark location in 2009 using the root mean square error (RMSE):

$$\text{RMSE}(\hat{I}_{2009}^{\text{interpol}}, I_{2009}) = \frac{1}{K} \sum_{k=1}^{K} (\hat{I}_{k,2009}^{\text{interpol}} - I_{k,2009})^2$$  \hspace{1cm} (23)

Note that Eq. (23) refers to our primary objective, i.e., minimizing the quadratic differences between the insurance payoffs (Eq. (6)). Weather derivative payoffs are usually derived by multiplying the index value with a constant tick value.

Fig. 2: Map of Chinese weather stations in the study area
(The blue squares show the three reference stations: 1=Beijing, 2=Wuhan, and 3=Guangzhou. The black square shows Lushi and is used for illustrating geographic basis risk.)
The weather index used in this paper is the growing degree days (GDD) index, which has a similar structure as the commonly used heating degree days index (HDD) or the cooling degree days index (CDD). Its relevance for agriculture comes from the fact that the accumulation of heat is one of the main factors influencing the development of crops. The GDD index at location $i$ over a growing season is defined as (World Bank 2005):

$$\text{GDD}_i(\tau_M, \tau_N) = \sum_{t=\tau_M}^{\tau_N} \max(0, Y_{i,t} - \bar{Y}),$$

where $Y_{i,t}$ denotes the daily average temperature in degree Celsius at location $i$ at time $t$. $\tau_M$ and $\tau_N$ are the first and the last day of the growing season from March 1st to October 31st. We assume that the base temperature $\bar{Y}$ is 5 °C.

### 3.2 Results

#### 3.2.1 Kriging

To apply Kriging, we first compute the experimental semivariogram according to Eq. (12) and then fit the theoretical Stein-Matern semivariogram (Eq. (13)). Spatial data often exhibit an anisotropic spatial pattern, which means that the experimental semivariogram varies in different directions. Figure A1 shows the directional difference of the experimental semivariogram on January 1st, 1999. As shown, the directional experimental semivariograms show a similar structure except the direction is east-west. The semivariance strongly increases with increasing distance between each pair in most directions, but it rises only slightly in the east-west direction. However, to simplify the subsequent calculations, we ignore this anisotropic pattern and apply Kriging to the data irrespective of the direction. Figure 3 displays the fit of the experimental and the theoretical Stein-Matern semivariogram for the same day.

![Fig. 3: The experimental (circle) and the theoretical Stein-Matern variogram (line) on January 1st, 1999](image)

Figure 4 depicts the Kriging results for the study area. The left panel shows the optimal ordinary Kriging predictor. From south to north, the temperature predictors decrease gradually. The right panel visualizes the corresponding prediction errors $\sigma_0$ according to Eq. (18). As one can expect, the uncertainty of the predictors is high in regions with a low density of weather stations.
3.2.2 Daily Temperature Model

Estimation results for data from 1999 to 2008 indicate a positive trend for most stations, which might be due to development and air pollution, which is discussed by Campbell and Diebold (2005). Nevertheless, the estimated trend coefficients are small, between $3.841 \cdot 10^{-5}$ and $2.047 \cdot 10^{-4}$ leading to a temperature increase between 0.14 °C and 0.75 °C in the observation period 1999-2008. In contrast, a negative trend occurs in a few stations: In Beijing, for example, the average temperature decreased by 0.57 °C.

Temperatures at all stations show the typical seasonal pattern and a strong autoregressive behavior. The orders $L$ of the autoregressive process vary between 3 and 20 for all stations with no distinct geographical pattern. Although Šaltytė-Benth and Benth (2012) mention that choosing large lags is not meaningful from a statistical and a meteorological point of view, we keep using the original lags individually for each station to account for data differences between stations.

Figure A2 illustrates the observed temperature at Quzhou and the fitted conditional mean function (Eq. (20)). The estimated residuals after removing the trend, seasonality, and AR process still have a strong nonlinear dependence (Fig. A3). This behavior is captured by a seasonal variance function (Eq. (22)). Fig. A4 shows the empirical variance and the fitted Fourier series for Quzhou. With the estimated temperature models at hand, we can forecast the (out-of sample) temperatures for 2009. The GDD indices for 2009 can be calculated from the forecasts.

3.2.3 DSFM

For computation convenience, we transform the initial space of the rectangular coordinates area to a unit square $[0,1]^2$. Moreover, we normalize the index data by subtracting the mean and dividing by the standard deviation calculated over all stations and all years. After the model is estimated, the mean and the standard deviation are incorporated again. To choose the number of factors for DFSM, we use the value of $EV(L)$, which can be interpreted as the explained variation of the model among the total variation and is expressed as follows:

$$EV(L) = 1 - \frac{\sum_{L=1}^{T} \sum_{i=1}^{I} (y_{t,i} - \bar{y})^2 - \sum_{L=1}^{I} \sum_{l=1}^{L} 2_{L}(x_{t,i})^2}{\sum_{L=1}^{T} \sum_{i=1}^{I} (y_{t,i} - \bar{y})^2}$$  \hspace{1cm} (25)

The explained variation for the Index DSFM approach is listed in Tables 1 and 2 depending on the values of $K$, $L$, and the order of the splines $P$. Although the values of $EV$ get larger with increasing $K$. 

**Fig. 4:** Ordinary Kriging prediction (left) and prediction error (right) (results for January 1st, 1999; blue crosses denote observed locations)
and \( L \), we estimate \( L = 3 \) basis functions, \( K = 9 \) knots, and set the order of splines \( P \) equal to 3. Showing an EV of 0.85, this combination is a good compromise between computational time and model accuracy. Setting \( L=3 \) is common in the literature (e.g., Giacomini et al. (2009)), and three factors can sufficiently capture the regional variation of temperatures. In the Daily DSFM approach, we chose \( K=9, P=2, \) and \( L=3 \). For this parameter choice, the explained variance amounts to 95.9\% (see Table 3) and indicates that the model fits the data well.

<table>
<thead>
<tr>
<th>( K )</th>
<th>( L =2 )</th>
<th>( L =3 )</th>
<th>( L =4 )</th>
<th>( L =5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 7 )</td>
<td>0.611</td>
<td>0.613</td>
<td>0.614</td>
<td>0.614</td>
</tr>
<tr>
<td>( 8 )</td>
<td>0.734</td>
<td>0.736</td>
<td>0.737</td>
<td>0.737</td>
</tr>
<tr>
<td>( 9 )</td>
<td>0.848</td>
<td>0.850</td>
<td>0.851</td>
<td>0.851</td>
</tr>
<tr>
<td>( 10 )</td>
<td>0.920</td>
<td>0.922</td>
<td>0.923</td>
<td>0.924</td>
</tr>
<tr>
<td>( 11 )</td>
<td>0.962</td>
<td>0.964</td>
<td>0.965</td>
<td>0.965</td>
</tr>
</tbody>
</table>

Table 1: Explained variance of the Index DSFM approach for \( P=3 \)

<table>
<thead>
<tr>
<th>( K )</th>
<th>( L =2 )</th>
<th>( L =3 )</th>
<th>( L =4 )</th>
<th>( L =5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 7 )</td>
<td>0.354</td>
<td>0.355</td>
<td>0.355</td>
<td>0.356</td>
</tr>
<tr>
<td>( 8 )</td>
<td>0.483</td>
<td>0.484</td>
<td>0.484</td>
<td>0.485</td>
</tr>
<tr>
<td>( 9 )</td>
<td>0.592</td>
<td>0.594</td>
<td>0.594</td>
<td>0.595</td>
</tr>
<tr>
<td>( 10 )</td>
<td>0.718</td>
<td>0.719</td>
<td>0.720</td>
<td>0.720</td>
</tr>
<tr>
<td>( 11 )</td>
<td>0.811</td>
<td>0.813</td>
<td>0.813</td>
<td>0.814</td>
</tr>
</tbody>
</table>

Table 2: Explained variance of the Index DSFM approach for \( P=4 \)

<table>
<thead>
<tr>
<th>( K )</th>
<th>( L =2 )</th>
<th>( L =3 )</th>
<th>( L =4 )</th>
<th>( L =5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6, P=2 )</td>
<td>0.954</td>
<td>0.959</td>
<td>0.965</td>
<td>0.968</td>
</tr>
</tbody>
</table>

Table 3: Explained variance of the Daily DSFM approach

Figure 5 displays the estimated factor functions \( \hat{\mu}_0 \) to \( \hat{\mu}_3 \) in the transformed estimation space. \( \hat{\mu}_0 \) is the intercept function and depicts the regional differences in the average temperature. The factors \( \hat{\mu}_1 \) and \( \hat{\mu}_3 \) are complementary factors which determine the main structure of the stochastic variations of the temperature index. The function \( \hat{\mu}_2 \) allows for fine tuning of the model structure.

Figure 6 shows the estimated loading factors series \( \hat{Z}_t \) from 1999 to 2000, which reflect the dynamics on the time scale. Obviously, the loading factors follow seasonal patterns. To model trend and seasonality in the loading factors, we use the same mean function that we elicited from the daily temperature model in Section 2.4.2. The AIC is used to determine the order of the AR process. This procedure translates the high dimensional problem into a low dimensional analysis of \( \hat{Z}_t \). Finally, based on predictions of the loading factors for the year 2009, we obtain temperature forecasts for each location.

Figure 7 illustrates the interpolation results for the year 2008 using the Index DSFM approach. The irregular surface reflects the regional variation in the temperature index. Index values are higher in the south, where the climate is subtropical, and lower in the north and northwest. The figure also displays the true index observations at all 100 weather stations.
Fig. 5: Estimated factor functions $m_0, m_1, m_2, m_3$ for the Daily DSFM approach

Fig. 6: Time series of the weights $\hat{Z}_{t,1}, \hat{Z}_{t,2}, \hat{Z}_{t,3}$ for the years 1999-2000 from the Daily DSFM approach
3.3 Comparing the Approaches

Model performance is based on an out-of-sample forecast of the GDD index values for 2009. Figure 8 depicts absolute forecast errors from the four models. Errors from the Daily Kriging approach differ only slightly from those from the Index Kriging model. Absolute errors from the DSFM models, however, are generally much higher than those in the Kriging models. Moreover, at least for some stations there are large differences between the Daily DSFM and the Index DSFM. Irrespective of the underlying model, stations with large forecast errors, such as station 57 (Wuqiaoling) and 58 (Pingliang), are located in the northwest where temperature data are sparse.
Table 4 lists the RMSE related to the four models. For daily and index modelling, Kriging clearly outperforms the DSFM. One reason for the poor performance of the DSFM is because the DSFM is a semiparametric approach, whereas Kriging is fully nonparametric. Another reason is the high seasonality in temperature data, which is not sufficiently captured in the factors of the DSFM. This explanation is supported by the fact that the Index DSFM approach, in which seasonality plays a minor role, obtains better results. Nevertheless, the RMSE is also rather high for this model. This problem could be mitigated by increasing the number of factors \( L \), however, this would increase the computational effort. Another option is detrending and deseasonalizing the data beforehand and applying a DSFM to the residuals. In our case, modelling the residuals leads to the problem that the seasonality for the unobserved location is unknown and hence forecasts for the temperature cannot be derived. Nevertheless, if seasonality is known, such as from a nearby location or if interpolating is used to fill in missing data, a DSFM may perform better than it did here.

Another finding is that index modelling outperforms daily modelling for the DSFM, whereas the opposite is true for the Kriging model. The difference, however, is rather small and not significant according to Welch’s t test (t-statistic = -0.35 and p-value = 0.5919). Nevertheless, the daily model has a considerable advantage because the forecasts change when the time of calculation is closer to the accumulation period of the index. In the index model, however, we use the average of the past years as the forecast which is just updated once per year after the accumulation period. This means that the forecast from the index model remains the same if we do the same procedure on March 31, 2009 instead of December 31, 2008, but that the forecast from the daily model probably improves.

<table>
<thead>
<tr>
<th>Kriging Model</th>
<th>DSFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index model</td>
<td>Daily model</td>
</tr>
<tr>
<td>GDD</td>
<td>418.4436</td>
</tr>
</tbody>
</table>

Table 4: Root mean square errors for the four approaches
3.4  Illustration of Geographic Basis Risk

Finally, we use the Daily Kriging model to quantify the geographic basis risk that is inherent to a weather insurance based on the GDD index. We prefer to use the daily model because of its flexibility and because it is commonly used for derivative pricing. We start by selecting three hypothetical reference stations in our study area: Beijing (116’28, 39’48), Wuhan (114’08, 30’37), and Guangzhou (113’20,23’10) (shown in Fig. 2). This choice is based on the economic importance of these cities and because of their varying climate conditions. We assume that weather derivatives based on the GDD index are offered for these three stations. Furthermore, we assume that a farmer situated in Lushi (111’02, 34’03), a county in Henan province, wants to insure his production against weather risk by buying a GDD index contract. Henan province is China’s major wheat-growing area and accounts for 1/4 of China’s wheat production (Ye et al. 2007). Wheat is also the main crop in Lushi, which is a mountainous area with shallow soil tilth. Therefore, weather fluctuations strongly influence yields there. In our hypothetical example, there is no weather station adjacent to the farm in Lushi. This means that historical weather data must be constructed via an interpolation of data from existing weather stations. Since Lushi is remote from the three reference weather stations, the farmer will likely face geographic basis risk. In what follows, we consider the geographic basis risk for different portfolios of weather derivatives: A farmer can either buy a single contract for Beijing (Station 1), Wuhan (Station 2), or Guangzhou (Station 3). Alternatively, the farmer can build a portfolio consisting of two or three derivatives. To derive appropriate portfolio weights, we follow Ritter et al. (2013) and forecast index values for the farm’s location and the three reference stations 10,000 times using the temperature model and approximate the farmer’s index value via linear regression. Hence, we obtain optimal portfolio weights and can compare the simulated index outcome at the farmer’s place with the index outcome of the combined reference stations to assess geographic basis risk for the different scenarios.

Table 5 shows the resulting RMSE for the different scenarios. Based on this simulation, the farmer faces a geographic basis risk between 32 and 39 when buying a GDD derivative based on one reference station. By combining two or three reference stations with optimal weights, the geographic basis risk can be reduced to approximately 28. Therefore, these calculations show that the best option for lowering geographic basis risk is to combine reference stations where derivatives are available. Note that these results are based on an in-sample comparison. For longer time periods, the performance of weights based on training data can be evaluated for testing data (see Ritter et al. (2013)). Our results provide a first impression of how geographic basis risk can be mitigated.

<table>
<thead>
<tr>
<th>Station(s)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Beijing)</td>
<td>39.12</td>
</tr>
<tr>
<td>2 (Wuhan)</td>
<td>36.75</td>
</tr>
<tr>
<td>3 (Guangzhou)</td>
<td>31.89</td>
</tr>
<tr>
<td>1+2</td>
<td>36.09</td>
</tr>
<tr>
<td>1+3</td>
<td>30.09</td>
</tr>
<tr>
<td>2+3</td>
<td>28.52</td>
</tr>
<tr>
<td>1+2+3</td>
<td>28.32</td>
</tr>
</tbody>
</table>

Table 5: Expected geographic basis risk of a farmer in Lushi in 2009 for different combinations of the three reference stations
4 Conclusion

This paper examines the performance of two general statistical procedures for interpolating and forecasting spatio-temporal data. From a statistical view, the approaches differ in the following way: the Kriging approach interpolates the data by the weighted sum of the neighbors with the help of a semivariogram; forecasting is then conducted by applying time series analysis. On the other hand, the DSFM approach decomposes spatio-temporal data into the principal spatial and temporal factors. In this paper, these different methods are evaluated for a long time period and over a large area of China, so the analysis is based on a broad database.

When insuring weather risk with weather derivatives, the problem of geographic basis risk occurs because the payoff depends on the weather at the reference station. To assess geographic basis risk, interpolating and forecasting are required. Interpolating is needed if the buyer of a temperature derivative is not situated next to a weather station and thus lacks current and/or historical weather data, whereas forecasting is used to estimate future payoffs and calculate prices of temperature derivatives. The new approach DSFM is able to conduct both tasks at the same time. This approach is compared to the standard interpolation method Kriging, which is combined with a stochastic temperature model. Moreover, we compare daily and index modelling. The results show that because the DSFM fails to capture the strong seasonality present in temperature data, the Kriging approach outperforms the DSFM. For the Kriging approach, we find no significant differences between the daily model and the index model. Furthermore, we illustrate how geographic basis risk can be quantified when insuring temperature risk with the GDD index and three potential reference stations. A combination of two or three suitable weather stations can significantly reduce the expected geographic basis risk.

Nevertheless, the application of the DSFM in the context of temperature modelling should be further investigated. The precision of the forecasts could be increased by having a greater number of factors or by modelling the detrended and deseasonalized residuals instead of the raw data (not applicable in our context). The DSFM provides a better understanding of the geographical factors that are driving the temperature, which can be evaluated on the basis of the \( m \) functions. Being semiparametric by nature, it allows for a fully parametric specification of the temporal evolution of the underlying factors. A denser network of data stations together with more flexible parameters would allow for a better understanding of the data.
Appendix

Fig. A1: Directional experimental semivariogram on January 1st, 1999.

Fig. A2: Ten years of temperature data (black) with the fitted conditional mean function (red) in station Quzhou.
Fig. A3: Estimated model residuals in station Quzhou after removing the trend, seasonality, and the AR components.

Fig. A4: Empirical and fitted volatility function in station Quzhou.
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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".
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