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Strategic Complementarities and Nominal Rigidities

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Strategic Complementarities and Nominal Rigidities[☆]

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Abstract

We reconsider the canonical model of price setting with menu costs by Ball and Romer (1990). Their original model exhibits multiple equilibria for nominal aggregate demand shocks of intermediate size. By abandoning Ball and Romer's (1990) assumption that demand shocks are common knowledge among price setters, we derive a unique symmetric threshold equilibrium where agents adjust prices whenever the demand shock falls outside the thresholds. The comparative statics of this threshold may differ from the one that gives rise to maximal nominal rigidity examined by Ball and Romer (1990). In contrast to their analysis, we find that a decrease in real rigidities can be associated with an increase in nominal rigidities due to the endogenous adjustment of agents' beliefs regarding the aggregate price level.

Keywords: JEL classification codes: E31, C70, D82
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1. Introduction

Price setting with fixed costs to adjustment (menu costs) and strategic complementarities in firms' pricing decisions may give rise to multiple equilibria. Ball and Romer (1990) (henceforth BR) show that monopolistically competitive firms find it more attractive to incur

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menu costs and adjust their prices in response to aggregate demand shocks if other firms do so as well. Two equilibria, sustained by self-fulfilling beliefs, may emerge. In one, all firms adjust prices, while in the other, prices are rigid. We derive a unique threshold equilibrium in BR's model by exploiting ideas from the literature on global games.[1] The equilibrium is such that firms pay the menu cost and adjust their price if and only if the magnitude of the shock exceeds a certain threshold. In their original analysis, BR derive upper and lower bounds for the shock such that multiple equilibria emerge for shocks within these bounds. The equilibrium threshold in our version equals the midpoint of these bounds.

At first glance this seems to strengthen BR's conclusion that the existence of large monetary nonneutralities in a standard representative agent model with menu costs requires implausible values of standard parameters. However, the threshold we derive implies different comparative statics than those presented by BR. More specifically, BR examine only the comparative statics of the upper bound on the region of multiplicity and neglect how price setters' beliefs about the behavior of other price setters adjust in equilibrium. In contrast, we endogenize price setters' beliefs by endowing them with noisy private information regarding the nominal aggregate demand shock and explicitly confining attention to symmetric threshold strategies, i.e., strategies such that agents adjust if and only if their signals exceed a common threshold. We then show that in the limit when information regarding the shock becomes arbitrarily precise, a price setter who is just indifferent between adjusting and not adjusting believes that half of the agents adjust while the remaining half does not. Our comparative statics then reflect the adjustment of beliefs and we provide necessary and sufficient conditions for them to differ qualitatively from BR's original conclusions.

BR's main focus is the role of real rigidities, i.e., forces that render price setters reluctant to fully adjust their relative prices to a change in aggregate demand, in explaining monetary nonneutralities. When price setters' objective functions are very concave in their relative prices, a unilateral price change by one agent is rather costly (in utility terms) and

[1]See Morris and Shin (2003) for an overview of the theory of global games.

real rigidities are high. This implies agents are more inclined to adjust when others adjust in order to keep relative prices stable. Thus, pricing decisions are also characterized by a high degree of strategic complementarities highlighting that real rigidities and strategic complementarities are closely interlinked.[2] Moreover, when menu costs are present, the gain from price adjustment may fall short of the menu cost, implying that it may be optimal to forgo the adjustment. In particular when real rigidities are large, there is a range of shocks where an agent gains from adjusting only if sufficiently many others adjust as well. This is the range of shocks where multiple equilibria, sustained by self-fulfilling beliefs regarding the behavior of others, may occur.

Multiplicity of equilibrium, however, is driven by the indeterminacy of price setters' beliefs. In the literature on global games, such an indeterminacy is seen as resulting from agents sharing common knowledge about the model's fundamentals and each being perfectly aware of what the others do in equilibrium.[3] This implies that agents' actions and beliefs are perfectly coordinated in equilibrium and thus equilibrium multiplicity occurs.[4] To eliminate this multiplicity, the global games approach, pioneered by Carlsson and van Damme (1993), abandons common knowledge about the economy's fundamental and instead endows agents with idiosyncratic noisy signals. We adopt a similar approach to derive a unique equilibrium in BR's model.[5] In the limiting case with uncertainty regarding the fundamental eliminated, agents' beliefs are still subject to strategic uncertainty breaking the perfect coordination of actions and beliefs and eliminating the equilibrium multiplicity of BR's original analysis.

This further implies that the comparative statics in our version may differ from those of BR. Consider a situation where, for some reason, a price setter's optimality condition becomes less sensitive to changes relative prices and at the same time only slightly less sen-

[2]See especially the discussion in Woodford (2003, pp. 158–173).

[3]Morris and Shin (2003)

[4]Morris and Shin (2001)

[5]While, strictly speaking, the model in not a standard global game, the particular information structure constitutes a convenient device to obtain an equilibrium in threshold strategies, which contrasts with the equilibrium arbitrarily selected by BR.

sitive to variations in aggregate demand. This is tantamount to a decrease in real rigidities. In BR's analysis, this implies that the agent will focus relatively more on her response to aggregate demand given that her beliefs regarding the aggregate price level are assumed to remain unchanged, rendering her more inclined to adjust her price. Thus, the decrease in real rigidities implies a decrease in nominal rigidities. This reasoning is, however, incomplete as it neglects an important effect. If real rigidities decrease, price setters are not only more inclined to adjust their prices in response to large shocks even if others do not, but they are also more inclined to tolerate other agents' price adjustments in response to small shocks. Whether a shock is large or small from the point of view of the agent depends on how sensitive her utility function is to the shock in equilibrium. Thus, coming back to the example above, since the agent became much less sensitive to relative prices than to variations in demand, she is more strongly inclined to tolerate price adjustments by others for a shock of a given magnitude. Therefore she may be more reluctant to adjust her price and, as a consequence, the reduction in real rigidities may be associated with a higher degree of nominal rigidities.

The remainder of the paper is organized as follows. Section 2 presents the basic BR framework and shows the existence of multiple equilibria. Section 3 presents our unique threshold equilibrium and a general comparative statics result. Section 4 provides a numerical illustration in BR's baseline model of monopolistic competition. Section 5 concludes. All mathematical proofs and calculations are relegated to the Appendix.

2. Ball and Romer (1990): Multiple Equilibria

BR consider a canonical model of price setting. A representative price setter produces a differentiated good with her own labor and faces fixed costs to price adjustment. Her utility is given by

$$U_i = W\left(Y, \frac{P_i}{P}\right) - zD_i \tag{1}$$

where Y is real aggregate expenditures, $\frac{P_i}{P}$ is agent i's relative price and z is the menu cost—a small resource cost of changing a nominal price; D_i is equal to one if agent i changes her price and zero otherwise.

Assuming a quantity theory approach to expenditures,

$$Y = \frac{M}{P} \tag{2}$$

where M is the nominal money supply,[6] it follows in equilibrium

$$U_i = W\left(\frac{M}{P}, \frac{P_i}{P}\right) - zD_i \tag{3}$$

BR assume $W_2(1,1) = 0$, $W_{22}(1,1) < 0$, and $W_{12}(1,1) > 0$, which, in turn, normalizes the optimal relative price to unity, implies that the second order condition is satisfied at this price, and that the equilibrium is stable.

The optimal price is governed by the first-order condition

$$W_2\left(\frac{M}{P}, \frac{P_i}{P}\right) = 0 \tag{4}$$

A first order expansion in log deviations from a symmetric equilibrium $\overline{P_i} = \overline{P} = \overline{M}$ yields

$$W_2\left(e^{m-p}, e^{p_i - p}\right) \approx W_{21}(1,1)(m-p) + W_{22}(1,1)(p_i - p) = 0 \tag{5}$$

where $M = \overline{M}e^m$, $P = \overline{P}e^p$, and $P_i = \overline{P_i}e^{p_i}$. Agent i's optimal deviation from the symmetric equilibrium absent menu costs becomes

$$p_i^* = \underbrace{-\frac{W_{21}(1,1)}{W_{22}(1,1)}}_{\equiv\beta} m + \underbrace{\frac{W_{21}(1,1) + W_{22}(1,1)}{W_{22}(1,1)}}_{\equiv 1-\beta} p \tag{6}$$

Whenever an agent changes her price, she will do optimally and choose $p_i = p_i^*$. In deciding whether to change her price, she compares the payoff difference between setting the optimal

[6] Thus, following BR we assume without loss of generality that fluctuations in aggregate demand arise from fluctuations in the nominal money supply.

price, $p_i = p_i^*$, and maintaining her old price, $p_i = 0$, to the menu costs z. This payoff difference can be expanded to second order in terms of log deviations around $\overline{P_i} = \overline{P} = \overline{M}$ as[7]

$$PC(m,p,p_i^*) \doteq W\left(e^{m-p}, e^{p_i^* p}\right) - W\left(e^{m-p}, e^{-p}\right) \approx -W_{22}(1,1)\left(p_i^*\right)^2 \tag{7}$$

We are now in a position to assess the range of monetary deviations, m, for which rigidity, $p_i = p = 0$, and adjustment, $p_i = p = p_i^*$, are equilibrium. Note that even if no other agents adjust their prices, agent i always adjusts her price if

$$PC(m,0,p_i^*) > z \Leftrightarrow -\frac{1}{2}W_{22}(1,1)\left(p_i^*\right)^2 > z$$

Agent i's optimal price in this case is $p_i^* = \beta m$. Combining this with the latter inequality yields a threshold value

$$x^* = \frac{1}{\beta}\sqrt{\frac{2z}{-W_{22}(1,1)}} \tag{8}$$

implying that *all* agents consider adjustment their dominant choice whenever $|m| > x^*$ and consequently, adjustment is the only equilibrium.

Conversely, suppose that all agents adjust their prices, i.e. $p^* = m$. Even then, agent i does not adjust her price if

$$PC(m,m,p_i^*) < z \tag{9}$$

This yields another threshold value

$$x^{**} = \sqrt{\frac{2z}{-W_{22}(1,1)}} \tag{10}$$

implying that if $|m| < x^{**}$, not adjusting is dominant for *all* agents and rigidity is the only equilibrium. However, as $\beta \in (0,1)$ and $x^{**} < x^*$, there exists multiple equilibria for intermediate monetary deviations. This partition of equilibria is displayed in figure 1.

[7] See Appendix A.1 for details.

[Figure 1 about here.]

Proposition 1. *Full Information Thresholds*
For large monetary deviations, $|m| > x^*$, a unique equilibrium exists where all agents adjust prices. For small monetary deviations, $|m| < x^{**}$, a unique equilibrium exists where no agent adjusts her price. For intermediate deviations $x^{**} < |m| < x^*$ both, adjustment and rigidity can be sustained as (self-fulfilling) equilibria.

Proof. See BR. □

3. Unique Threshold Equilibrium

The multiplicity of equilibria is due to the indeterminacy of price setters' beliefs, which follows from assuming that monetary deviations are common knowledge among agents and that agents are perfectly coordinated in equilibrium.[8] This multiplicity prohibits the derivation of general comparative statics. BR nonetheless examine the comparative statics of the x^* threshold, ignoring the effects of multiplicity.

To obtain a unique equilibrium and valid set of comparative statics, we assume that m is drawn from a uniform distribution over the real line. This "improper prior" assumption is often made in the literature on global games. Since we confine attention to posterior conditional distributions, it does not present any technical difficulties.[9] The realization of m is not common knowledge but agents observe private signals concerning its realization. These signals take the form

$$x_i = m + \epsilon_i, \quad \text{with} \quad \epsilon_i \sim \mathcal{N}(0, \sigma), \text{ i.i.d.} \tag{11}$$

[Figure 2 about here.]

[8] Morris and Shin (2001)
[9] Morris and Shin (2003, Section 2.1).

We confine ourselves to symmetric threshold strategies. A threshold strategy for agent i can be described by a value $\tilde{x}_i$ such that agent i adjusts if and only if $|x_i| > \tilde{x}_i$. A symmetric threshold strategy is simply described by a common threshold for all agents, $\tilde{x}_i = \tilde{x}_j = \tilde{x}$. We then look for symmetric equilibria in threshold strategies. Such a threshold strategy for agent i is depicted in figure 2. In order to derive the equilibrium threshold, we exploit the indifference of an agent between adjusting and not adjusting when she observes $x_i = \tilde{x}$ and believes that other agents also use the threshold $\tilde{x}$.

Proposition 2. *Unique Threshold Equilibrium*
As fundamental uncertainty regarding the size of the monetary shocks vanishes, $\sigma \to 0$, there exists a unique threshold, $\tilde{x}$, such that all agents adjust if and only if $|x_i| < \tilde{x}$ and refrain from adjusting otherwise. The threshold is given by

$$\tilde{x} = \frac{x^* + x^{**}}{2} \tag{12}$$

Proof. See Appendix A.2. □

To understand the intuition behind proposition 2, consider the equilibrium beliefs under the resulting threshold equilibrium as displayed in figure 3. Now imagine the model with a very small amount of uncertainty regarding the fundamental. An agent's belief is a random variable whose distribution is centered on the signal she receives. As σ is small, for signals far enough away from the thresholds, $\tilde{x}$ and $-\tilde{x}$, the distribution of beliefs will place essentially no mass past the threshold. As fundamental uncertainty vanishes, these distributions collapse and coordination will occur as long as $m \neq \tilde{x}$. That is, an agent expects that others receive almost exactly the same signal as she does and therefore she expects the aggregate price level to equal her signal. However, whenever she receives a signal equal to either threshold, the distribution is split into two half: one to the left and one to the right of the threshold. In other words, the distribution collapses to a point mass divided evenly between rigidity and adjustment. With agents coordinating their beliefs and actions on half of the agents adjusting and half not, the resulting aggregate price level becomes exactly the average of what it would be with an infinitesimally larger or smaller monetary shock. The threshold, then, that splits beliefs half to adjustment and half to rigidity and leaves the agent indifferent to both choices is the average of the thresholds with beliefs wholly coordinated either to adjustment or rigidity.

[Figure 3 about here.]

A direct consequence of propositions 1 and 2 is $\tilde{x} < x^*$ or that the unique threshold equilibrium is associated with less nominal rigidity than the x^* threshold examined by BR would imply. This is not surprising as BR themselves point out that they examine the region with the largest possible nominal rigidities. The crucial question is whether the comparative statics of the $\tilde{x}$ and x^* thresholds differ. Without loss of generality, suppose that the function $W(\cdot,\cdot)$ depends on a finite number of parameters denoted by α_k, $k = 1,\ldots,I$. The following proposition provides necessary and sufficient conditions under which the comparative statics of the threshold x^* are different from those of $\tilde{x}$.

Proposition 3. *Comparative Statics*

For marginal changes in parameters α_k, denoted $\mathrm{d}\alpha_k$, $k = 1,\ldots,I$, the threshold $\tilde{x}$ is increasing and x^* decreasing if and only if

$$\beta \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} \mathrm{d}\alpha_i > \frac{2}{\beta} \sum_{i=1}^{I} \frac{\partial W_{21}}{\partial \alpha_i} \mathrm{d}\alpha_i + \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} \mathrm{d}\alpha_i > 0 \tag{13}$$

And vice versa, the threshold $\tilde{x}$ is decreasing and x^* increasing if and only if

$$\beta \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} \mathrm{d}\alpha_i < \frac{2}{\beta} \sum_{i=1}^{I} \frac{\partial W_{21}}{\partial \alpha_i} \mathrm{d}\alpha_i + \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} \mathrm{d}\alpha_i < 0 \tag{14}$$

Proof. See Appendix A.3. □

To interpret this result, recall that the equilibrium thresholds are defined as the critical signals that render an agent indifferent between adjusting and not adjusting given that she either believes no other agent adjusts (the BR threshold x^*) or that other agents use the same threshold strategy as she does (our threshold $\tilde{x}$). What we are after is understanding when the two thresholds, x^* and $\tilde{x}$, have different comparative statics. Indifference entails that the agent's payoff difference, under either threshold x^* or $\tilde{x}$, equals the menu cost z. Then, if parameter variations increase one of these payoff differences and decrease the other, the former threshold will fall and the latter will rise. Differentiating the payoff difference gives (which is valid for either threshold),

$$\mathrm{d}PC = p^*\left(2(-W_{22})\mathrm{d}p^* - p^*\mathrm{d}W_{22}\right) \tag{15}$$

Since the optimal prices under either threshold are identical, i.e., $p^*|_{x^*} = \beta x^* = \frac{2\beta}{1+\beta}\tilde{x} = p^*|_{\tilde{x}}$, the signs of the differentials will be different if

$$2(-W_{22})\mathrm{d}p^* - p^*\mathrm{d}W_{22} \tag{16}$$

have different signs under the two thresholds in question. Note that all terms in the last equation are independent of the thresholds except for $\mathrm{d}p^*$. Hence, the important question is how the optimal prices $p^*|_{x^*}$ and $p^*|_{\tilde{x}}$ change. Given any signal x_i, we can write

$$\mathrm{d}p^* = x_i\mathrm{d}\beta - E[p|x_i]\mathrm{d}\beta + (1-\beta)\,\mathrm{d}E[p|x_i] \tag{17}$$

where we abbreviate $\mathrm{d}\beta = \sum_{i=1}^{I}\frac{\partial\beta}{\partial\alpha_i}\mathrm{d}\alpha_i$ and $\mathrm{d}W_{22} = \sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}\mathrm{d}\alpha_i$.

At the threshold x^*, given BR's implicit assumption that $E[p|x_i] = \mathrm{d}E[p|x_i] = 0$,

$$\mathrm{d}p^*\big|_{x^*} = x^*\mathrm{d}\beta \tag{18}$$

Yet, at our threshold $\tilde{x}$

$$\mathrm{d}\,p^*\big|_{\tilde{x}} = (\tilde{x} - E[p])\mathrm{d}\beta + (1-\beta)\,\mathrm{d}E[p] = \left(\tilde{x} - \frac{p^*}{2}\right)\mathrm{d}\beta + (1-\beta)\frac{1}{2}\mathrm{d}p^* = \frac{x^*}{1+\beta}\mathrm{d}\beta \tag{19}$$

From equations (18) and (19) follows that $|\mathrm{d}\,p^*|_{\tilde{x}}| < |\mathrm{d}\,p^*|_{x^*}|$. This implies that there exists a range of values for $\mathrm{d}\beta$ such that

$$2(-W_{22})\mathrm{d}\,p^*\big|_{x^*} - p_i^*\mathrm{d}W_{22} > 0 \quad\text{but}\quad 2(-W_{22})\mathrm{d}\,p^*\big|_{\tilde{x}} - p_i^*\mathrm{d}W_{22} < 0.$$

Using the differential for $\mathrm{d}\beta = \frac{\beta}{W_{21}}\left(\mathrm{d}W_{21} + \beta \mathrm{d}W_{22}\right)$, it is straightforward to show that the parameter restrictions under which the above two inequalitites hold simultaneously are those provided in Proposition 3.

The reason is that BR ignore how the expected aggregate price level adjusts to changes in parameters and maintain that the expected aggregate price level equals zero. In our version, the expected aggregate price level is not assumed equal zero and agents take into account how it is affected by parameter variations. This can be seen from expression (17), where the second term in brackets measures the effect of parameter changes on an agent's response to her beliefs regarding the aggregate response of agents and the third component measures how her beliefs themselves respond to these parameter changes. In contrast to BR this also entails that we may find certain combinations of parameter changes which induce, say, a reduction in the degree of real rigidities, but at the same time are associated with a higher degree of nominal rigidities: For example, whenever parameter variations reduce agents' sensitivity towards the shock by less than their sensitivity towards changes in relative prices, they may accept variations in relative prices for a larger range of monetary shocks. Since this holds true for all agents, the aggregate price becomes smaller, thus further reducing agents' incentives to adjust their price. As a consequence, the change in any agent's optimal price is smaller and the gains from adjusting the price may fall short of the menu costs.

4. Ball and Romer's (1990) Baseline Model

4.1. Baseline Model of Monopolistic Competition

Here we consider BR's baseline model of monopolistically competitive yeoman farmers who produce differentiated goods and follow BR closely in presentation and notation. There exists a unit mass of *ex ante* identical agents. Agent i produces her good, Y_i, with the production function $Y_i = L_i$ using her own labor, L_i, to maximize her utility, given by

$$U_i = \left[\int_0^1 C_{ij}^{(\epsilon-1)/\epsilon} dj\right]^{\epsilon/(\epsilon-1)} - \frac{\epsilon-1}{\gamma\epsilon} L_i^{\gamma} - zD_i \tag{20}$$

where C_{ij} is agent i's consumption of good j. The number of parameters is $I = 2$ and the parameter vector contains the elasticity of substitution between goods, $\epsilon > 1$, and $\gamma > 1$ which captures the degree of increasing marginal disutility to labor.

With the transactions technology $Y = M/P$, where aggregate production is given by $Y = \int_0^1 Y_i di$ and the aggregate price level, given by $P = \left[\int_0^1 P_i^{1-\epsilon} dj\right]^{1/(1-\epsilon)}$, utility (20) can be written in the form of (3) as

$$U_i = \left(\frac{M}{P}\right)\left(\frac{P_i}{P}\right)^{1-\epsilon} - \frac{\epsilon - 1}{\gamma\epsilon}\left(\frac{M}{P}\right)^{\gamma}\left(\frac{P_i}{P}\right)^{-\gamma\epsilon} - zD_i \tag{21}$$

where use is made of the agent's budget constraint, $PC_i \doteq P\left[\int_0^1 C_{ij}^{(\epsilon-1)/\epsilon} dj\right]^{\epsilon/(\epsilon-1)} = P_i Y_i$, and the demand function for agent i's produce, $Y_i^D = Y(P_i/P)^{-\epsilon}$.

4.2. Numerical Results

BR consider a numerical experiment by asking how big the menu costs would need to be to just render rigidity an equilibrium given that a 5% monetary shock occurs. That is, we set $x^* = 0.05$ and solve for the associated z from the indifference condition (10) to recover the values reported by BR. In a similar way, we then set $\tilde{x} = 0.05$ to recover the menu costs that correspond to our threshold equilibrium reported in (12). We compare the different menu costs for different values of the markup implied by the substitution elasticity, ϵ, and the labor supply elasticity that follows from γ; reporting both values for the menu costs as well as their ratio.

[Table 1 about here.]

The results are presented in Table 1. Firstly, observe that for rigidity to be an equilibrium, the menu costs needed under the threshold $\tilde{x}$ are always higher than those reported by BR. This is a direct consequence of Proposition 2 since the threshold $\tilde{x}$ equals the average of x^* and x^{**}, thus $\tilde{x} < x^*$. Hence, for a given monetary shock (here 5%), a larger menu cost is required to render agents indifferent under the $\tilde{x}$-threshold compared to the BR-equilibrium.

Secondly, with small markups and inelastic labor supply, the menu costs required by the $\tilde{x}$-equilibrium is 8.66 percent of revenue or 3.65 times larger than those in BR. Increasing the elasticity of labor supply or the markup reduces the required menu costs. Increasing the elasticity of labor supply, however, reduces the menu costs required by the BR-equilibrium relatively more than under the $\tilde{x}$-equilibrium, leading the latter to increase slightly relative to the former. This relationship is reversed when markups are increased. If we do this, the menu costs under the $\tilde{x}$-equilibrium decline so much more quickly than under the BR-equilibrium that their ratio falls by roughly one half.

4.3. Comparative Statics

We now turn to the comparative statics of the model. The results in table 1 seem to indicate that increases in the markup and/or the elasticity of labor supply increase nominal rigidity associated with both the x^* threshold studied by BR and the $\tilde{x}$ threshold that we introduce. This is misleading. A simultaneous increase in the one and decrease in the other parameter of appropriate magnitude will move the two thresholds in opposite directions.

[Figure 4 about here.]

Tracing contours in the thresholds, figure 4 outlines the nonlinear region of parameter changes, starting from a 15% markup and labor supply elasticity of 0.15 and z such that $\tilde{x} = 0.05$, associated with the two thresholds moving in opposite directions. The bounds in proposition 3 give the slopes of these contours at the point associated with the 15% markup and labor supply elasticity of 0.15 and the points in the figure correspond to the pairs of markups and labor supply elasticities considered in table 2. Note that the points are located inside the black curves, which trace out parameter combinations that leave the thresholds unchanged. Movements towards the northeast imply increases in both thresholds, holding the menu costs fixed.

[Table 2 about here.]

For example, increasing the markup from 5% to 100% while simultaneously decreasing the elasticity of labor supply leads the x^* threshold to decrease as the menu costs in the table required to leave an agent indifferent to a 5% monetary shock are increasing. This implies that nominal rigidity is decreasing. However, $\tilde{x}$ is rising as the menu costs that lead to indifference are falling, implying that nominal rigidity is actually increasing. Notice that β, inversely related to the degree of nominal rigidity or strategic complementarities, is increasing here. Thus, BR's x^* threshold has nominal and real rigidities moving moving in concert, whereas our $\tilde{x}$ threshold has the two types of rigidity moving in opposite directions.

This difference in the movements of x^* and $\tilde{x}$ can be understood as follows. A large increase in the markup and small decrease in the elasticity of labor supply leads to a substantial decrease in the concavity of the agent's utility function with respect to her relative price, $-W_{22}$, but only a mild decrease in the sensitivity of the optimality condition to a change in real balances, W_{21}. With agents' beliefs fixed on rigidity under the x^* threshold, agents are relatively more inclined to offset the change in real balances, thus requiring a larger menu cost to leave them indifferent between adjusting and not adjusting for a given monetary shock as in table 2. This same motive, however, implies a stronger reaction of the price level under the $\tilde{x}$ threshold. This reduces the change in real balances faced by agents. Along with the reduced concavity due to the fall in $-W_{22}$, this renders agents' utility less sensitive to monetary shocks. As shown in table 2, this translates to a reduction in the menu costs required for indifference with a 5% shock to the money supply.

5. Conclusion

We have derived a unique threshold equilibrium for the canonical price setting problem studied by Ball and Romer (1990) with fixed costs to price adjustment. By applying ideas often used in the literature on global games, we break the common knowledge of nominal aggregate demand shocks which would sustain the regions of multiple equilibria. We find that BR's comparative statics are sensitive to these multiple equilibria, even in their canonical model of price setting under monopolistic competition. BR's analysis, which assumes the largest possible range of rigidity sustainable as a Nash equilibrium, would conclude that large increases in firms' markups coinciding with small decreases in the labor supply elasticity should decrease nominal rigidity. We find the potential for the opposite conclusion, as the endogenous change in beliefs has a larger impact on price setting than the decrease in real rigidities induced by the parameter changes. While we have restricted ourselves to BR's framework, Dotsey and King (2005) highlight that multiple equilibria resulting from coordination failures are a pervasive feature in the state-dependent pricing literature, which would give our approach the potential for applicability to a broader set of models than examined by BR.

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Appendix A. Appendix

Appendix A.1. Derivation of Second Order Log Payoff Difference Approximation

Noting that terms that do not involve p_i^1 or p_i^2 drop out, the payoff difference can be written as

$$\begin{aligned} PC(m,p,p_i^1,p_i^2) &= W\left(e^{m-p},e^{p_i^1-p}\right) - W\left(e^{m-p},e^{p_i^2-p}\right) \\ &\approx W_2(1,1)\left(p_i^1-p\right) + W_{21}(1,1)(m-p)\left(p_i^1-p\right) + (1/2)W_{22}(1,1)\left(p_i^1-p\right)^2 \\ &- W_2(1,1)\left(p_i^2-p\right) - W_{21}(1,1)(m-p)\left(p_i^2-p\right) - (1/2)W_{22}(1,1)\left(p_i^2-p\right)^2 \end{aligned} \tag{A.1}$$

From the agent's first order condition, (4), $W_2(1,1)=0$, simplifying the foregoing to

$$\begin{aligned} PC(m,p,p_i^1,p_i^2) &\approx W_{21}(1,1)(m-p)\left(p_i^1-p\right) + (1/2)W_{22}(1,1)\left(p_i^1-p\right)^2 \\ &- W_{21}(1,1)(m-p)\left(p_i^2-p\right) - (1/2)W_{22}(1,1)\left(p_i^2-p\right)^2 \end{aligned} \tag{A.2}$$

collecting terms in $W_{21}(1,1)$ and $W_{22}(1,1)$ yields

$$PC(m,p,p_i^1,p_i^2) \approx W_{21}(1,1)(m-p)\left(p_i^1-p_i^2\right) + \frac{1}{2}W_{22}(1,1)\left[\left(p_i^1-p\right)^2 - \left(p_i^2-p\right)^2\right] \tag{A.3}$$

expanding and recollecting the terms in brackets delivers

$$PC(m,p,p_i^1,p_i^2) \approx W_{21}(1,1)(m-p)\left(p_i^1-p_i^2\right) + \frac{1}{2}W_{22}(1,1)\left(p_i^1+p_i^2-2p\right)\left(p_i^1-p_i^2\right) \tag{A.4}$$

which can be rewritten as

$$\begin{aligned} PC(m,p,p_i^1,p_i^2) &\approx \left[W_{21}(1,1)(m-p) + W_{22}(1,1)\left(p_i^*-p\right)\right]\left(p_i^1-p_i^2\right) \\ &+ W_{22}(1,1)\left(\frac{p_i^1+p_i^2}{2} - p_i^*\right)\left(p_i^1-p_i^2\right) \end{aligned} \tag{A.5}$$

the term in brackets is zero according to the expansion of the first order condition, (5), delivering

$$\begin{aligned} PC(m,p,p_i^1,p_i^2) &= W\left(e^{m-p},e^{p_i^1-p}\right) - W\left(e^{m-p},e^{p_i^2-p}\right) \\ &\approx W_{22}(1,1)\left(\frac{p_i^1+p_i^2}{2} - p_i^*\right)\left(p_i^1-p_i^2\right) \end{aligned} \tag{A.6}$$

Evaluating the foregoing at $p_i^1 = p_i^*$, $p_i^2 = 0$ yields the expression in the main text.

Appendix A.2. Proof of Proposition 2

Suppose that all other agents use a threshold strategy around some value k, i.e., they adjust if and only if the magnitude of their signal, $|x_j|$ exceeds the value k. Consider agent i who observes signal x_i. Her optimal price is given by

$$E[p_i^*|x_i,k] = \beta x_i + (1-\beta)E[p|x_i,k] \tag{A.7}$$

Her conditional payoff difference is

$$E[PC(m,p,p_i^1,p_i^2)|x_i,k] \approx W_{22}(1,1)\left(\frac{p_i^1+p_i^2}{2} - E[p_i^*|x_i,k]\right)\left(p_i^1-p_i^2\right) \tag{A.8}$$

as we assume the prices set by each agent are known to her and the expression above is then linear in p_i^* thus allowing us to pass the expectations operator through. Evaluating this at $p_i^1 = E[p_i^*|x_i,k]$, $p_i^2 = 0$ yields

$$E[PC(m,p,E[p_i^*|x_i,k],0)|x_i,k] \approx -W_{22}(1,1)\left(E[p_i^*|x_i,k]\right)^2 \tag{A.9}$$

To simplify notation, we write

$$E[p|x_i,k] \doteq h(x_i,k,\sigma)$$

and abbreviate

$$E[PC(m,p,E[p_i^*|x_i,k],0)|x_i,k] \doteq PC_\sigma(x_i,k)$$

thus making the dependence on σ explicit.

In order to calculate the payoff difference, we first need to calculate the conditionally expected price level $h(x_i,k,\sigma)$. Here we exploit the symmetry of the price setting problem, i.e. we use the fact that agent $j \neq i$ with signal x_j uses an optimal price setting rule of the form given by equation (A.7) and expects the price level $h(x_j,k,\sigma)$.

Conditional on the threshold strategy around k, agent i calculates,

$$h(x_i,k,\sigma) = (1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(x_j,k,\sigma)f(x_j|m)\mathrm{d}x_j f(m|x_i)\mathrm{d}m \tag{A.10}$$

$$-(1-\beta)\int_{\mathbb{R}}\int_{-k}^{k} h(x_j,k,\sigma)f(x_j|m)\mathrm{d}x_j f(m|x_i)\mathrm{d}m \tag{A.11}$$

$$+\beta\int_{\mathbb{R}}\int_{\mathbb{R}} x_j f(x_j|m)\mathrm{d}x_j f(m|x_i)\mathrm{d}m \tag{A.12}$$

$$-\beta\int_{\mathbb{R}}\int_{-k}^{k} x_j f(x_j|m)\mathrm{d}x_j f(m|x_i)\mathrm{d}m \tag{A.13}$$

where the parts (A.11) and (A.13) reflect the definition of the threshold strategy, whereby no agent adjusts her price for signals in the interval $[-k,k]$.

With signal noise terms distributed normally, the distributions of x_j conditional on m and vice versa are

$$f(x_j|m) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(x_j-m)^2}{2\sigma^2}\right), \quad f(m|x_i) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-x_i)^2}{2\sigma^2}\right)$$

We now rewrite parts (A.10) - (A.13) of $h(x_i,k,\sigma)$ to obtain expressions which allow us to study the case where $\sigma \to 0$.

Define $\mu = \frac{x_j - m}{\sigma}$ and the change of variables, $d\mu = \frac{1}{\sigma}dx_j$. The inner integral of (A.10) can be rewritten as

$$\int_{\mathbb{R}} h(x_j,k,\sigma)f(x_j|m)\mathrm{d}x_j = \int_{\mathbb{R}} h(x_j,k,\sigma)\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(x_j-m)^2}{2\sigma^2}\right)\mathrm{d}x_j = \int_{\mathbb{R}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu$$

Inserting this into (A.10) and using $\gamma = \frac{m-x_i}{\sigma}$ and another change of variables, $d\gamma = \frac{1}{\sigma}dm$, part (A.10) is

$$\begin{aligned}(1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(x_j,k,\sigma)f(x_j,m)\mathrm{d}x_j f(m|x_i)\mathrm{d}m &= (1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-x_i)^2}{2\sigma^2}\right)\mathrm{d}m \\ &= (1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(\sigma\mu+\sigma\gamma+x_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\end{aligned}$$

Next, we rewrite the inner integral of (A.11) as

$$\int_{-k}^{k} h(x_j,k,\sigma)f(x_j|m)dx_j \int_{-k}^{k} h(x_j,k)\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(x_j-m)^2}{2\sigma^2}\right)\mathrm{d}x_j = \int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu$$

where we have used the change of variables from before. Part (A.11) then becomes

$$= -(1-\beta)\int_{\mathbb{R}}\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-x_i)^2}{2\sigma^2}\right)\mathrm{d}m$$
$$= -(1-\beta)\int_{\mathbb{R}}\int_{\frac{-k-m}{\sigma}-\gamma}^{\frac{k-m}{\sigma}-\gamma} h(\sigma\mu+\sigma\gamma+x_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$

Note further that signals are unbiased, i.e. $\int_{\mathbb{R}} x_j f(x_j|m)\mathrm{d}x_j = m$ and $\int_{\mathbb{R}} m f(m|x_i)\mathrm{d}m = x_i$. Hence, part (A.12) simplifies to βx_i.

Finally, we rewrite the inner integral of part (A.13) as

$$\int_{-k}^{k} x_j f(x_j|m)\mathrm{d}x_j = \int_{-k}^{k}(x_j-m)f(x_j|m)\mathrm{d}x_j + m\int_{-k}^{k} f(x_j|m)\mathrm{d}x_j$$
$$= \int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} \sigma\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{\mu^2}{2}\right)\sigma\mathrm{d}\mu + m\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{\mu^2}{2}\right)\sigma\mathrm{d}\mu$$
$$= \sigma\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} \mu\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu + m\left[\Phi\left(\frac{k-m}{\sigma}\right)-\Phi\left(\frac{-k-m}{\sigma}\right)\right]$$

using a change of variables from before. Using this for the inner integral, we can express (A.13) as

$$-\beta\left\{\sigma\int_{\mathbb{R}}\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}}\mu\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-x_i)^2}{2\sigma^2}\right)\mathrm{d}m + \int_{\mathbb{R}} m\left[\Phi\left(\frac{k-m}{\sigma}\right)-\Phi\left(\frac{-k-m}{\sigma}\right)\right]\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-x_i)^2}{2\sigma^2}\right)\mathrm{d}m\right\}$$

Use the other change of variables from before, (A.13) is

$$= -\beta\left\{\sigma\int_{\mathbb{R}}\int_{\frac{-k-x_i}{\sigma}-\gamma}^{\frac{k-x_i}{\sigma}-\gamma}\frac{1}{\sqrt{2\pi}}\mu\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma + \int_{\mathbb{R}}(\sigma\gamma+x_i)\left[\Phi\left(\frac{k-x_i}{\sigma}-\gamma\right)-\Phi\left(\frac{-k-x_i}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right\}$$
$$= -\sigma\beta\int_{\mathbb{R}}\left\{\int_{\frac{-k-x_i}{\sigma}-\gamma}^{\frac{k-x_i}{\sigma}-\gamma}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu + \gamma\left[\Phi\left(\frac{k-x_i}{\sigma}-\gamma\right)-\Phi\left(\frac{-k-x_i}{\sigma}-\gamma\right)\right]\right\}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$
$$-\beta x_i\int_{\mathbb{R}}\left[\Phi\left(\frac{k-x_i}{\sigma}-\gamma\right)-\Phi\left(\frac{-k-x_i}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$

Putting all the pieces together, the expectation of the price level conditional on the signal x_i and the threshold strategy around k is

$$h(x_i,k,\sigma) = (1-\beta)\int_{\mathbb{R}}\left[\int_{\mathbb{R}} h(\sigma\mu+\sigma\gamma+x_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)d\mu\right.$$
$$\left.-\int_{\frac{-k-x_i}{\sigma}-\gamma}^{\frac{k-x_i}{\sigma}-\gamma} h(\sigma\mu+\sigma\gamma+x_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)d\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma$$
$$+\beta x_i\left(1-\int_{\mathbb{R}}\left[\Phi\left(\frac{k-x_i}{\sigma}-\gamma\right)-\Phi\left(\frac{-k-x_i}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma\right)$$
$$-\sigma\beta\int_{\mathbb{R}}\left\{\gamma\left[\Phi\left(\frac{k-x_i}{\sigma}-\gamma\right)-\Phi\left(\frac{-k-x_i}{\sigma}-\gamma\right)\right]+\int_{\frac{-k-x_i}{\sigma}-\gamma}^{\frac{k-x_i}{\sigma}-\gamma}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)d\mu\right\}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma$$

Now consider the limit as $\sigma \to 0$. For simplicity we write $\lim_{\sigma\to 0} h(x_i,k,\sigma)$ as $h(x_i,k,0)$. By the monotone convergence theorem we have for $x_i \neq k$,

$$h(x_i,k,0) = \begin{cases} x_i & \text{if } x_i < -k \\ 0 & \text{if } x_i \in (-k,k) \\ x_i & \text{if } x_i > k \end{cases}$$

Moreover for $x_i = k$, we have

$$\begin{aligned} h(k,k,\sigma) =&(1-\beta)\int_{\mathbb{R}}\left[\int_{\mathbb{R}} h(\sigma\mu+\sigma\gamma+k,k)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right. \\ &\left.-\int_{-\frac{2k}{\sigma}-\gamma}^{-\gamma} h(\sigma\mu+\sigma\gamma+k,k)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma \\ &+\beta k\left(1-\int_{\mathbb{R}}\left[\Phi(-\gamma)-\Phi\left(-\frac{2k}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right) \\ &-\sigma\beta\int_{\mathbb{R}}\left[\gamma\left[\Phi(-\gamma)-\Phi\left(-\frac{2k}{\sigma}-\gamma\right)\right]+\int_{-\frac{2k}{\sigma}-\gamma}^{-\gamma}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma \end{aligned} \tag{A.14}$$

And in the limit, by the monotone convergence theorem,

$$\begin{aligned} \lim_{\sigma\to 0} h(k,k,\sigma) = h(k,k,0) = (1-\beta)\int_{\mathbb{R}}&\left[\int_{\mathbb{R}} h(k,k,0)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu - \int_{-\infty}^{-\gamma} h(k,k,0)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right] \\ &\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma + \beta k\left(1-\int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp(-\frac{\gamma^2}{2}\mathrm{d}\gamma\right) \end{aligned}$$

Where the last line in (A.14) is zero. Rewriting the right hand side of the above yields

$$\begin{aligned} &= (1-\beta)h(k,k,0)\int_{\mathbb{R}}\left[1-\Phi(-\gamma)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma + \beta k\left(1-\int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right) \\ &= h(k,k,0)(1-\beta)\left(1-\int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right) + \beta k\left(1-\int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right) \end{aligned}$$

as $\int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma = \frac{1}{2}$,[10]

$$h(k,k,0) = h(k,k,0)(1-\beta)\frac{1}{2} + \beta k\frac{1}{2} \Rightarrow h(k,k,0) = \frac{\beta}{1+\beta}k$$

For the value k to constitute a threshold equilibrium, an agent with the marginal signal has to be just indifferent between adjusting and not adjusting, i.e., the optimal price conditional on observing $x_i = k$ must be such that $PC_0(k,k) = 0$. The optimal price in this case is given by

$$p_i^* = \beta k + \beta\frac{1-\beta}{1+\beta}k = 2\frac{\beta}{1+\beta}k$$

which, upon inserting this into the payoff difference yields

$$PC_0(k,k) = -W_{22}\left(2\frac{\beta}{1+\beta}k\right)^2 \tag{A.15}$$

[10]See Appendix A.4.

Equating the latter with the menu cost z, yields

$$\sqrt{\frac{2z}{-W_{22}(1,1)}} = 2\frac{\beta}{1+\beta}k \tag{A.16}$$

Solving for k yields the critical signal

$$\tilde{x} \doteq k = \frac{1}{2}\frac{1+\beta}{\beta}\sqrt{\frac{2z}{-W_{22}(1,1)}}$$

which can be written in terms of the thresholds under perfect information, (10) and (8), as expressed in the proposition. That $\tilde{x}$ indeed constitutes a threshold equilibrium follows from the fact that for $x_i > k$, $h(x_i,k,0) > h(k,k,0)$ and conversely for $x_i < k$. This implies that we must have $PC_0(x_i,k) > PC_0(k,k) = z$ for $x_i > k$ so that agents adjust their price and conversely for $x_i < k$. Finally, uniqueness of the threshold equilibrium is obvious from the expression for $PC_0(k,k)$ provided in (A.15): $PC_0(0,0)-z<0$ and $PC_0(\infty,\infty)-z>0$ ($PC_0(-\infty,-\infty)-z<0$) and since $PC_0(k,k)$ strictly increases (decreases) in k for $k>0$ ($k<0$) there exists exactly one value for k ($-k$) where $PC_0(k,k)=z$ ($PC_0(-k,k)=z$).

Appendix A.3. Proof of Proposition 3

Differentiate x^* to yield

$$dx^* = -(2z)^{1/2}W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial\alpha_i}d\alpha_i - \frac{1}{2}(2z)^{1/2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i \tag{A.17}$$

and differentiate $\tilde{x}$ to deliver

$$d\tilde{x} = \frac{1}{2}(2z)^{1/2}\left(-W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial\alpha_i}d\alpha_i - \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i + \frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i\right) \tag{A.18}$$

For $d\tilde{x}>0$, it must hold that

$$-W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial\alpha_i}d\alpha_i - \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i + \frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i > 0 \tag{A.19}$$

which can be rearranged as

$$\frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i > W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial\alpha_i}d\alpha_i\frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i \tag{A.20}$$

Note that the right hand side of the inequality is $-dx^*(2z)^{-1/2}$ and so $dx^*<0$ adds

$$\frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i > W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial\alpha_i}d\alpha_i + \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial\alpha_i}d\alpha_i > 0 \tag{A.21}$$

Multiplying through with $2W_{21}(W_{22})^{1/2}$ and recalling the definition of β delivers the expression in the main text. The inequalities for $d\tilde{x}<0$ but $dx^*>0$ follow analogously.

Appendix A.4. Subsidiary Integral Calculation

We prove here that

$$\int_{\mathbb{R}} \Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma = \frac{1}{2}$$

Note first that the left-hand side above is equivalent to (exploiting the symmetry of the standard normal)

$$\int_{\mathbb{R}}(1-\Phi(\gamma))\left(\frac{e^{-\frac{\gamma^2}{2}}}{\sqrt{2\pi}}\right)\mathrm{d}\gamma \Leftrightarrow 1-\int_{\mathbb{R}}\Phi(\gamma)\phi(\gamma)\mathrm{d}\gamma.$$

Hence, we simply need to show $\int_{\mathbb{R}}\Phi(\gamma)\phi(\gamma)\mathrm{d}\gamma = \frac{1}{2}$. This is straightforward if tedious.

Consider the integral

$$I(\mu,\sigma) \doteq \int_{\mathbb{R}}\phi(\gamma)\Phi\left(\frac{\gamma-\mu}{\sigma}\right)\mathrm{d}\gamma.$$

Its derivative w.r.t. μ is given by

$$I_\mu(\sigma,\mu) = \int_{\mathbb{R}}\left(\frac{-1}{\sigma}\right)\phi(\gamma)\phi\left(\frac{\gamma-\mu}{\sigma}\right)\mathrm{d}\gamma$$

Now observe

$$\begin{aligned}
2\pi\phi(x)\phi\left(\frac{\gamma-\mu}{\sigma}\right) &= \exp\left\{-\frac{1}{2}\left(\gamma^2+\frac{1}{\sigma^2}(\gamma-\mu)^2\right)\right\}\\
&= \exp\left\{-\frac{(1+\sigma^2)}{2\sigma^2}\left(\gamma^2-\frac{2\mu\gamma}{1+\sigma^2}+\frac{\mu^2}{1+\sigma^2}+\frac{\mu^2}{(1+\sigma^2)^2}-\frac{\mu^2}{(1+\sigma^2)^2}\right)\right\}\\
&= \exp\left\{-\frac{(1+\sigma^2)}{2\sigma^2}\left(\gamma-\frac{\mu}{1+\sigma^2}\right)^2\right\}\times\exp\left\{-\frac{1}{2}\frac{\mu}{1+\sigma^2}\right\}
\end{aligned}$$

Hence, we have

$$I_\mu(\sigma,\mu) = \frac{-1}{\sigma}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{\mu^2}{1+\sigma^2}\right\}\int_{\mathbb{R}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{1+\sigma^2}{\sigma^2}\left(\gamma-\frac{\mu}{1+\sigma^2}\right)^2\right\}\mathrm{d}\gamma$$

Using $\tau^2 = \frac{\sigma^2}{1+\sigma^2}$, and denote the normal distribution for s with variance u and mean v by $f(s,v,u)$, we can write

$$\begin{aligned}
I_\mu(\sigma,\mu) &= \frac{-1}{\sigma}\frac{1}{\sqrt{2\pi}}\frac{\sigma}{\sqrt{1+\sigma^2}}\exp\left\{-\frac{1}{2}\frac{\mu^2}{1+\sigma^2}\right\}\int_{\mathbb{R}}f\left(\gamma,\frac{\mu}{1+\sigma^2},\tau\right)\mathrm{d}\gamma\\
&= \frac{-1}{\sqrt{1+\sigma^2}}\phi\left(\frac{\mu}{1+\sigma^2}\right).
\end{aligned}$$

Using the fundamental theorem of calculus we have

$$I(\mu,\sigma) = \int_\sigma^\infty \frac{1}{\sqrt{1+\sigma^2}}\phi\left(\frac{s}{\sqrt{1+\sigma^2}}\right)\mathrm{d}s,$$

which, by using $r = \frac{s}{\sqrt{1+\sigma^2}}$ and performing a change of variables, can be expressed as

$$I(\mu,\sigma) = \int_{\sigma/(\sqrt{1+\sigma^2})}^\infty \phi(r)\mathrm{d}r = 1-\Phi\left(\frac{\mu}{(\sqrt{1+\sigma^2})}\right).$$

Hence, $\lim_{\mu\to 0} I(\mu,\sigma) = 1-\Phi(0) = \frac{1}{2}$.

Table 1: Cost of Adjustment, 5% Monetary Shock, Baseline Model

Labor Supply Elasticity		Markup 5%	15%	50%	100%
	Ball and Romer (1990)	2.38	2.16	1.64	1.22
0.05	Threshold Equilibrium	8.66	6.77	3.72	2.20
		(3.65)	(3.13)	(2.27)	(1.81)
	Ball and Romer (1990)	0.79	0.71	0.53	0.39
0.15	Threshold Equilibrium	2.87	2.23	1.22	0.72
		(3.65)	(3.14)	(2.30)	(1.86)
	Ball and Romer (1990)	0.23	0.20	0.14	0.10
0.5	Threshold Equilibrium	0.85	0.65	0.35	0.20
		(3.65)	(3.17)	(2.42)	(2.04)
	Ball and Romer (1990)	0.11	0.10	0.06	0.04
1	Threshold Equilibrium	0.42	0.31	0.16	0.09
		(3.66)	(3.22)	(2.56)	(2.25)

Replicates the private costs in Ball and Romer (1990, Table 1). The entries indicate the size of the menu costs z necessary to leave the agent indifferent between adjustment and non-adjustment as measured in percent of flexible price revenue. Ball and Romer (1990) examine the equilibrium associated with $x^* = 5\%$, i.e., the entry leaves the agent indifferent assuming all other agents adjust. Threshold Equilibrium gives the menu cost that leaves the agent indifferent given the global game equilibrium, $\tilde{x} = 5\%$. The entries in parentheses give the ratio of the entries in the $\tilde{x} = 5\%$ columns to those in the $x^* = 5\%$ columns.

Table 2: Cost of Adjustment, 5% Monetary Shock

Markup	5%	15%	50%	100%
Labor Supply Elasticity	.175	.15	.1	.065
Ball and Romer (1990)	0.67	0.71	0.81	0.93
Threshold Equilibrium	2.46	2.23	1.84	1.69
β	0.0472	0.1279	0.3226	0.4843

The entries indicate the size of the menu costs z necessary to leave the agent indifferent between adjustment and non-adjustment as measured in percent of flexible price revenue. Ball and Romer (1990) examine the equilibrium associated with $x^* = 5\%$, i.e., the entry leaves the agent indifferent assuming all other agents adjust. The line labeled 'Threshold Equilibrium' yields the menu cost that leaves the agent indifferent under the $\tilde{x}$ equilibrium, when $\tilde{x} = 5\%$.

Figure 1: Full Information Thresholds

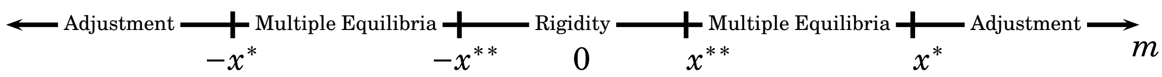


Figure 2: Threshold Strategy

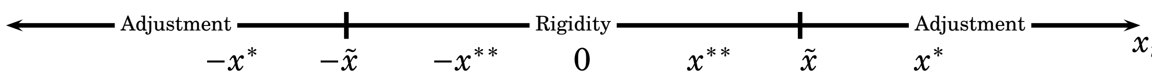
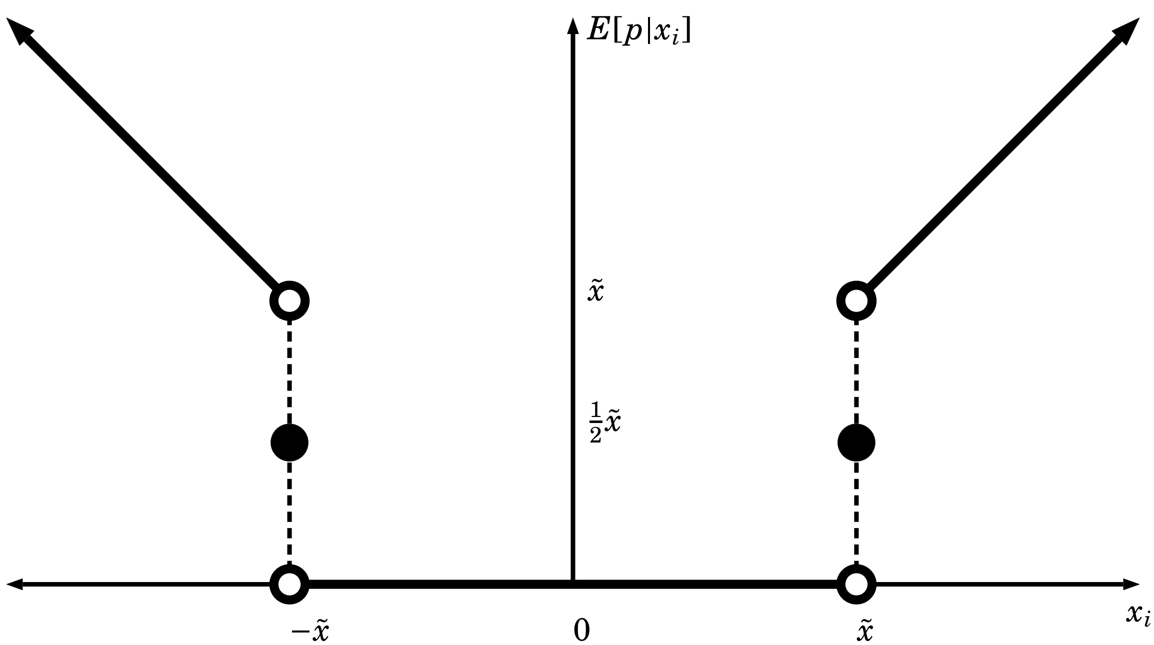
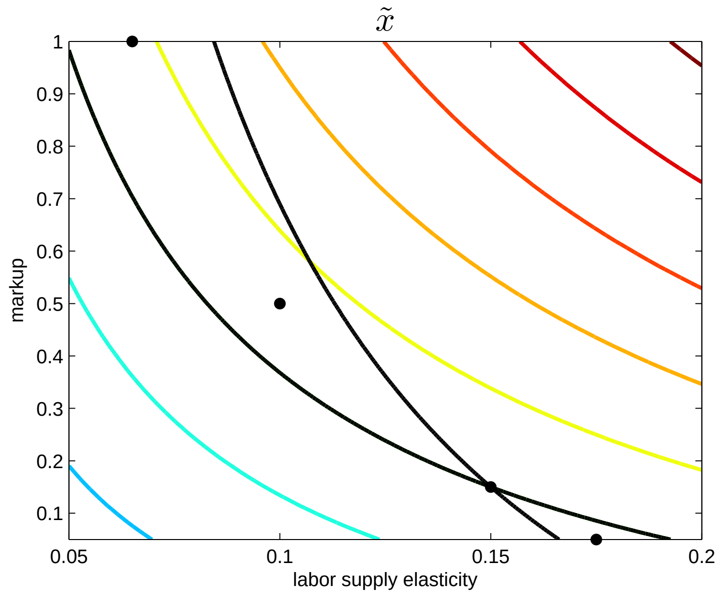
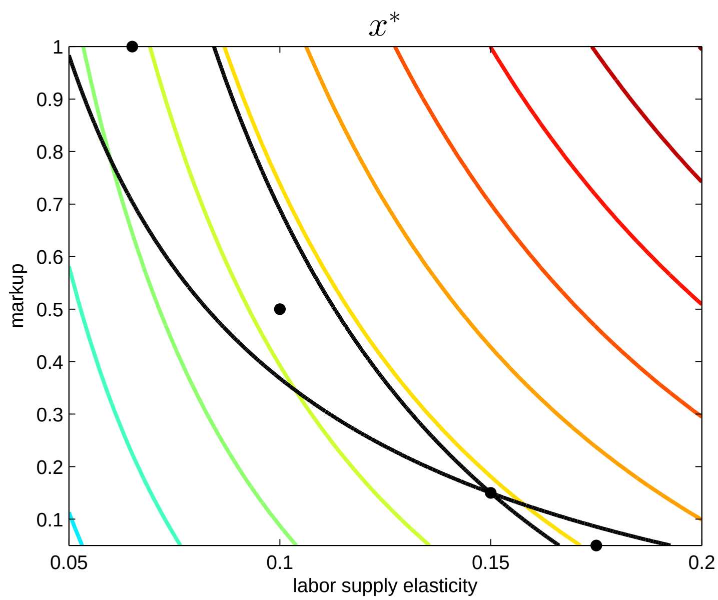


Figure 3: Threshold Equilibrium Beliefs





(a) $\tilde{x}$-equilibrium



(b) BR-equilibrium

Figure 4: Threshold Contour Plot

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