TENET: Tail-Event driven NETwork risk

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Abstract

A system of risk factors necessarily involves systemic risk. The analysis of systemic risk is in the focus of recent econometric analysis and uses tail event and network based techniques. Here we bring tail event and network dynamics together into one context. In order to pursue such joint effects, we propose a semiparametric measure to estimate systemic interconnectedness across financial institutions based on tail-driven spillover effects in a high dimensional framework. The systemically important institutions are identified conditional on their interconnectedness structure. Methodologically, a variable selection technique in a time series setting is applied in the context of a single-index model for a generalized quantile regression framework. We could thus include more financial institutions into the analysis to measure their tail event interdependencies and, at the same time, being sensitive to non-linear relationships between them. Network analysis, its behaviour and dynamics, allows us to characterize the role of each industry group in the U. S. financial market 2007 - 2012. The proposed TENET - Tail Event driven NETwork technique allows us to rank the systemic risk contributions of publicly traded U.S. financial institutions.

Keywords: Systemic Risk, Systemic Risk Network, Generalized Quantile, Quantile Single-Index Regression, Value at Risk, CoVaR, Lasso
1. Introduction

Systemic risk endangers the stability of the financial market, the failure of one institution may harm the whole financial system. The sources of risk are complex, as both exogenous and endogenous factors are involved. This calls for a study on a financial network which accounts for interaction between the agents in the financial market. Although the notion systemic risk is not novel in the academic literature (see, e.g., Minsky (1977)), it has been neglected both in the academia and in the financial risk industry until the outbreak of the financial crisis in 2008. Some financial institutions collapsed, even some major ones like Lehman Brothers, Federal Home Loan Mortgage Corporation (Freddie Mac), and Federal National Mortgage Association (Fannie Mae). The magnitude of repercussions caused by this financial crisis and its complexity revealed a significant flaw in financial regulation which has been focused primarily on stability of a single financial institution and triggered several political initiatives across the world such as establishment of Financial Stability Board (FSB) after G-20 London summit in 2009, integration of systemic risk agenda into Basel III in 2010 prior to G-20 meeting in Seoul, enacting the Dodd Frank Wall Street Reform and Consumer Protection Act (‘Dodd Frank Act’) in U. S. in 2010 which is said to bring the most radical changes into the U. S. financial system since the Great Depression.

These initiatives created several challenges such as identifying systematically important financial institutions (SIFIs) whose failure may not only impair the functioning of the financial system but also have adverse effects on the real sector of the economy, studying the propagation mechanism of a shock in a system, or in a network formed by financial institutions, investigating the response of a system to a shock as a whole as well as revealing certain structural patterns in evolution and behavior of a network and establishing a theoretical framework for systemic risk as such.

Although systemic risk is a relatively straightforward concept aimed at measuring risk stemming from interaction between the agents, the variety of risk measures employed at estimating systemic risk and diversity of possible methods to model interaction effects leads to a fact that the literature on this topic is highly heterogenous. The relevant literature in this field can be broadly divided into two groups: economic modelling of systemic risk and financial intermediation including microeconomic (e.g., Beale et al. (2011)) and macroeconomic approaches (e.g., Gertler and Kiyotaki (2010)) with the emphasis on theoretical, structural framework, and quantitative modelling with the emphasis on empirical analysis. The quantitative literature can be further classified by statistical methodol-
ogy into quantile regression based modelling such as linear bivariate model by Adrian and Brunnermeier (2011), Acharya et al. (2012), Brownlees and Engle (2012), high-dimensional linear model by Hautsch et al. (2015), partial quantile regression by Giglio et al. (2012) and partial linear model by Chao et al. (2015). Further approaches include principal-component-based analysis, e.g., by Bisias et al. (2012), Rodriguez-Moreno and Peña (2013) and others; statistical modelling based on default probabilities by Lehar (2005), Huang et al. (2009), and others; graph theory and network topology; e.g., Boss et al. (2006), Chan-Lau et al. (2009), and Diebold and Yilmaz (2014).

Our paper belongs to the quantitative group of the aforementioned literature, namely, modelling the tail event driven network risk based on quantile regressions augmented with non-linearity and variable selection in high dimensional time series setting. Our method is in nature different from Acharya et al. (2012) and Brownlees and Engle (2012)’s method. Acharya et al. (2012) has measured the systemic risk relevance without capturing the network effects of liquidity exposure, and Brownlees and Engle (2012) analyze the risk of a specific asset given the distress of the whole system which is a reverse of our system to institution analysis, and their method would capture little spillover effects. Therefore we believe that our method is a good addition to the literature of systemic risk measure. Also compared to Diebold and Yilmaz (2014), we focus more on the tail event driven interconnectedness, which cannot be captured by conditional correlation. As a starting point of our research we take co-Value-at-Risk, or CoVaR, model by Adrian and Brunnermeier (2011) (from here on abbreviation as AB), where ‘co-’ stands for ‘conditional’, ‘contagion’, ‘comovement’. To capture the tail interconnectedness between the financial institutions in the system AB evaluate bivariate linear quantile regressions for publicly traded financial companies in the U. S..

Whereas AB focus on bivariate measurement of tail risk we aim at assessing the systemic risk contribution of each institution conditional on its tail interconnectedness with the relevant institutions. Thus, the primary challenge is selecting the set of relevant risk drivers for each financial institution. Statistically we address this issue by employing a variable selection method in the context of single-index model (SIM) for generalized quantile regressions, i.e. for quantiles and expectiles. We further extend it to a time series variable selection context in high dimensions. The semi-parametric framework due to the SIM allows us to investigate possible non-linearities in tail interconnectedness. Based on identified relevant risk drivers we construct a financial network based on spillover effects across financial institutions. Further we implement the SIM for quantile regression of the system on each single institution and its relevant variables to identify the systemic risk contribution.

The assumption of non-linear relationship between returns of financial companies is motivated by previous work by Chao et al. (2015), who find that the dependency between any
pair of financial assets is often non-linear, especially in periods of economic downturn. Moreover, non-linearity assumption is more flexible especially in a high dimensional setting where the system becomes too complex to support the belief of linear relationships. According to the 2012 U. S. financial company list from NASDAQ, we select 100 financial institutions consisting of top 25 financial institutions from each industry group: Depositories, Insurance companies, Broker-Dealers and Others. These four groups are divided by Standard Industrial Classification (SIC) codes. Our model is evaluated based on weekly log returns of these 100 publicly traded U. S. financial institutions. Firm specific characteristics from balance sheet information such as leverage, maturity mismatch, market to book and size are added into the model as well. Furthermore, the macro state variables are also involved. The time period from January 5, 2007 to January 4, 2013 covers one recession (from December 2007 to June 2009) and several documented financial crises (2008, 2011). Dividing companies by industry groups and including several market perturbations allows not only to select the key players for each time period, but also additionally to highlight the connections between financial industries, which can in turn provide additional information on the nature of market dislocations. The results of this paper motivate the TENET financial risk meter, see: http://sfb649.wiwi.hu-berlin.de/frm/index.html. All the R programs for this paper can be found on http://quantlet.de.

The rest of the paper is organized as follows. In Section 2 our approach to systemic risk modelling is outlined. Section 3 presents the statistical methodology and the related theorems. Section 4 illustrates the empirical application. Section 5 concludes. Appendix A contains proofs and Appendix B contains estimation results.

2. Systemic Risk Modelling

2.1. Basic concepts

Traditional measures assessing riskiness of a financial institution such as Value at Risk (VaR), or expected shortfall (ES) are based either on company characteristics and/or integrate macro state variables which account for the general state of the economy. Thus, for example, the VaR of a financial institution $i$ at $\tau \in (0, 1)$ is defined as:

$$P(X_{i,t} \leq VaR_{i,t,\tau}) \overset{\text{def}}{=} \tau,$$

where $\tau$ is the quantile level, $X_{i,t}$ represents the log return of financial institution $i$ at time $t$. AB propose the risk measure CoVaR (Conditional Value at Risk) which takes spillover effects and macro state of economy into account. The CoVaR of a financial institution $j$
given $X_i$ at level $\tau \in (0, 1)$ at time $t$:

$$ P\{X_{j,t} \leq \text{CoVaR}_{j|i,t,\tau|R_{i,t}} \} \overset{\text{def}}{=} \tau, $$  \hspace{1cm} (2)

where $R_{i,t}$ denotes the information set which includes the event of $X_{i,t} = \text{VaR}_{i,t,\tau}$ and $M_{t-1}$, note that $M_{t-1}$ is a vector of macro state variables reflecting the general state of the economy (see Section 4 for details of macro state variables).

We start with the concept of CoVaR, which is estimated in two steps of linear quantile regression:

$$ X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, $$  \hspace{1cm} (3)

$$ X_{j,t} = \alpha_{j|i} + \gamma_{j|i} M_{t-1} + \beta_{j|i} X_{i,t} + \varepsilon_{j|i,t}, $$  \hspace{1cm} (4)

$F^{-1}_e(\tau|M_{t-1}) = 0$ and $F^{-1}_{e_{j|i,t}}(\tau|M_{t-1},X_{i,t}) = 0$ are assumed. AB propose in the first step to determine VaR of an institution $i$ by applying quantile (tail event) regression of log return of company $i$ on macro state variables. The $\beta_{j|i}$ in (4) has standard linear regression interpretation, i.e. it determines the sensitivity of log return of an institution $j$ to changes in tail event log return of an institution $i$. In the second step the CoVaR is calculated by plugging in VaR of company $i$ at level $\tau$ estimated in (5) into the equation (6):

$$ \hat{\text{VaR}}_{i,t,\tau} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, $$  \hspace{1cm} (5)

$$ \hat{\text{CoVaR}}_{j|i,t,\tau}^{AB} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i} M_{t-1} + \hat{\beta}_{j|i} \hat{\text{VaR}}_{i,t,\tau}, $$  \hspace{1cm} (6)

Thus, the risk of a financial institution $j$ is calculated via a macro state and a VaR of an institution $i$. Here the coefficient $\hat{\beta}_{j|i}$ of (6) reflects the degree of interconnectedness. By setting $j$ to be the return of a system, e.g. value-weighted average return on a financial index, and $i$ to the return of a financial company $i$, we obtain the contribution CoVaR which characterizes how a company $i$ influences the rest of the financial system. By doing the reverse, i.e. by setting $j$ equal to a financial institution and $i$ to a financial system, one obtains exposure CoVaR, i.e. the extent to which a single institution is exposed to the overall risk of a system.

This approach allows to identify the key elements of systemic risk, namely, network effects, a single institution’s contribution to systemic risk and a single institution’s exposure to systemic risk, however, it still has certain limitations. First of all, AB uses average market valued asset returns weighted by lagged market valued total assets to calculate the system.
return, as they point out it may create mechanical correlation between a single financial institution and the value-weighted financial index. Although AB states that no such correlation is detected after certain tests, this approach has to be adopted with caution. Moreover, a linear relationship between system return and a single institution’s return is assumed. The complexity of the financial system however let us doubt that the linear dependence is accurate.

The first step involves control variables for a single financial institution, i.e. a couple of relevant variables for a single institution. If we regress the system return on one institution’s return and its relevant control variables, this mechanical correlation problem will be avoided. Hence a variable selection as a pre-step should be carried out. Hautsch et al. (2015) apply indeed a LASSO based variable selection to select the control variables to estimate the VaR of the system, but both AB and Hautsch et al. (2015) adopt a linear model. To make the estimation more flexible the SIM will be implemented to allow the nonlinear relationship in this case.

2.2. Step 1 VaR Estimation

TENET can be illustrated by three steps. In the first step we estimate VaR for each financial institution by using linear quantile regression as in AB:

\[ X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, \]  
\[ \hat{\text{VaR}}_{i,t,\tau} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, \]  

\( X_{i,t} \) and \( M_{t-1} \) are defined as in section 2.1. Note that the VaR is estimated by linear quantile regression (7) of log returns of an institution \( i \) on macro state variables. This is justified by the analysis of Chao et al. (2015), who found evidence of linear effects in regressing \( X_{i,t} \) on \( M_{t-1} \).

2.3. Step 2 Network Analysis

In this step, TENET builds up a risk interdependence network based on SIM for quantile regression with variable selection. Note that our model can be easily extended to the case of expectiles, which provide coherent risk measures. First the basic element of the network: CoVaR calculation has to be determined. As in equation (2), \( X_j \) represents a single institution, and the CoVaR of institution \( j \) is estimated by conditioning on its information set. This information set will not only include the asset returns of other
firms estimated and the macro variables used in the previous step, but also uses control variables on internal factors of institution \( j \), i.e. the company specific characteristics such as leverage, maturity mismatch, market-to-book and size. This setting will allow us to model the risk spillover channels among institutions mostly caused by liquidity or risk exposure. Our choice of information set is more comprehensive than AB, and a similar motivation can be found in Hautsch et al. (2015). Further, a systemic risk network is built motivated by Diebold and Yilmaz (2014). TENET captures nonlinear dependency as it is based on a SIM quantile variable selection technique. More precisely:

\[
X_{j,t} \overset{\text{TENET}}{\sim} \text{CoVaR} = g(\beta_{j,R_j}^\top R_{j,t}) + \varepsilon_{j,t}, \quad (9)
\]

\[
\text{CoVaR}_{SIM} \overset{\text{def}}{=} \text{CoVaR}_{j|\tilde{R}_j,t,\tau} = \tilde{g}(\beta_{j,\tilde{R}_j}^\top \tilde{R}_j), \quad (10)
\]

\[
\hat{D}_{j|R_j} = \frac{\partial \tilde{g}(\beta_{j,\tilde{R}_j}^\top \tilde{R}_j)}{\partial R_{j,t}} |_{R_{j,t} = \tilde{R}_j} = \tilde{g}'(\beta_{j,\tilde{R}_j}^\top \tilde{R}_j) \beta_{j,\tilde{R}_j}. \quad (11)
\]

Here \( R_{j,t} \overset{\text{def}}{=} \{X_{j,t}, M_{i-1}, B_{j,t-1}\} \) is the information set which includes \( p \) variables, \( X_{j,t} \overset{\text{def}}{=} \{X_{1,t}, X_{2,t}, \ldots, X_{k,t}\} \) are the explanatory variables including the log returns of all financial institutions except for a financial institution \( j \), \( k \) represents the number of financial institutions. \( B_{j,t-1} \) are the firm characteristics calculated from their balance sheet information. Define the parameters as \( \beta_{j,R_j} \overset{\text{def}}{=} \{\beta_{j,-j}, \beta_{j,M}, \beta_{j,B_j}\}^\top \). Note that there is no time symbol \( t \) in the parameters, since our model is set up based on one fixed window estimation, we can then apply moving window estimation to estimate all parameters in different windows. \( \tilde{R}_{j,t} \overset{\text{def}}{=} \{\tilde{VaR}_{-j,t,\tau}, M_{i-1}, B_{j,t-1}\} \), \( \tilde{VaR}_{-j,t,\tau} \) are the estimated VaRs from (8) for financial institutions except for \( j \) in step 1, and \( \tilde{\beta}_{j,R_j} \overset{\text{def}}{=} \{\tilde{\beta}_{j,-j}, \tilde{\beta}_{j,M}, \tilde{\beta}_{j,B_j}\}^\top \). As in equation (10) CoVaR comprises not only the influences of financial institutions except for \( j \), but also incorporates non-linearity reflected in the shape of a link function \( g(\cdot) \). Therefore, we name it \( \text{CoVaR}_{SIM}^{\text{TENET}} \) which stands for Tail-Event driven NETwork risk. Moreover, in (10) the symbol "SIM" indicates that this CoVaR is estimated by the SIM, not linear quantile regression. \( \hat{D}_{j|R_j} \) is the gradient measuring the marginal effect of covariates evaluated at \( R_{j,t} = \tilde{R}_{j,t} \), and the componentwise expression is \( \hat{D}_{j|R_j} \overset{\text{def}}{=} \{\hat{D}_{j|-j}, \hat{D}_{j|M}, \hat{D}_{j|B_j}\}^\top \). In particular, \( \hat{D}_{j|-j} \) allows to measure spillover effects across the financial institutions and to characterize their evolution as a system represented by a network. Note that in our network analysis we only include the partial derivatives of institution \( j \) with respect to the other financial institutions (i.e. \( \hat{D}_{j|-j} \)). The partial derivatives with respect to institution’s characteristic variables \( \hat{D}_{j|B_j} \) and macro state variables \( \hat{D}_{j|M} \) are not included. One reason is that we find the effects from these variables are rather minor than financial institutions’ returns. Moreover, we concentrate on spillover effects among firms in the network analysis. However, these selected variables will still be part of the control variables to estimate the systemic risk contribution in our
final step. Now let us focus on network analysis.

The term *network* refers to a (directed) *graph*, formally written as $G = (V, E)$ where $V$ is a set of vertices and $E$ is a set of links, or edges. We summarize the estimation results in a form of a weighted adjacency matrix. Let $\hat{D}_{ji}$ be one element in $\hat{D}_{j|-j}$, where $j$ represents one financial institution as before, $i$ stands for another institution which is one element in the other financial institutions set $-j$. Then a weighted adjacency matrix contains absolute values of $\hat{D}_{ji}$ and absolute value of $\hat{D}_{ij}$, while $\hat{D}_{ji}$ are the impact from firm $i$ to firm $j$, $\hat{D}_{ij}$ means the impact from firm $j$ to firm $i$, these two values are different. Table 1 shows the adjacency matrix, note that in each window of estimation one has only one adjacency matrix estimated.

![Adjacency Matrix](image)

**Table 1:** A $k \times k$ adjacency matrix for financial institutions at time $t$.

The above $k \times k$ matrix $A_t$ in Table 1 represents total connectedness across variables at each time point $t = 1, \cdots, T$, and $I_i$ represents the name of financial institution $i$. The adjacency matrix, or a total connectedness matrix, is sparse and off-diagonal since our model by construction does not allow for self-loop effects (namely one variable cannot be regressed on itself). The rows of this matrix correspond to incoming edges for a variable in a respective row and the columns correspond to outgoing edges for a variable in a respective column.

### 2.4. Step 3: Identification of Systemic Risk Contributions

In the third step, TENET explains systemic risk measures. SIM quantile regression (without variable selection) is used again to regress a predefined system return on firm $j$ with controlling all the risk contributors for firm $j$ selected in Step 2, then the systemic
risk contribution can be estimated as follows:

\[
X_{s,t} = g(\beta_{s|F}^T F_{j,t}) + \varepsilon_{s,t},
\]

\[
\text{CoVaR}^{\text{SYSTEM}} = \text{CoVaR}^{\text{SIM}} = \hat{g}(\beta_{s|F}^T \tilde{F}_{j,t}),
\]

\[
\hat{D}_{s|F_j} = \frac{\partial \hat{g}(\beta_{s|F}^T F_{j,t})}{\partial F_{j,t}}|_{F_{j,t}=\tilde{F}_{j,t}} = \hat{g}'(\beta_{s|F}^T \tilde{F}_{j,t}) \beta_{s|F_j},
\]

where \(X_{s,t}\) is defined as a weighted sum of the log returns of the financial system:

\[
X_{s,t} = \sum_{i=1}^{k} X_{i,t} \cdot \text{Asset}_{i,t-1},
\]

where \(\text{Asset}_{i,t-1}\) is the most recent total asset of firm \(i\). Moreover \(F_{j,t} = \{X_{j,t}, C_{j,t}\}\). \(C_{j,t} = \{X_{j,t}^*, M_{j,t-1}^*, B_{j,t-1}^*\}\), \(C_{j,t}\) includes control variables selected from step 2, the star symbol "*" means that only those variables which are chosen to be relevant for firm \(j\) by the variable selection procedure are included, i.e. the log returns of selected financial institutions except for firm \(j\), the selected macro state variables and selected firm characteristics, \(\beta_{s|F_j} = \{\beta_{s|j}, \beta_{s|C_j}\}^T\). \(\tilde{F}_{j,t} = \{\tilde{V}aR_{j,t,\tau}, \tilde{C}_{j,t}\}\), \(\tilde{C}_{j,t} = \{\tilde{V}aR_{j,t,\tau}^*, M_{j,t-1}^*, B_{j,t-1}^*\}\), i.e. the estimated VaRs of selected financial institutions except for firm \(j\), the selected macro state variables and selected firm characteristics are included in \(\tilde{C}_{j,t}\). Moreover, \(\hat{\beta}_{s|F_j} = \{\hat{\beta}_{s|j}, \hat{\beta}_{s|C_j}\}^T\), and \(\hat{D}_{s|F_j} = \{\hat{D}_{s|j}, \hat{D}_{s|C_j}\}^T\) is the partial derivative of system CoVaR with respect to the variables in \(F_{j,t}\) evaluated at level \(F_{j,t} = \tilde{F}_{j,t}\), \(\hat{D}_{s|j}\) is the the partial derivative of system CoVaR with respect to institution \(j\). In terms of identification of the system risk contributions we focus here on \(\hat{D}_{s|j}\).

### 3. Statistical Methodology

Let us denote \(X_t \in \mathbb{R}^p\) as \(p\) dimensional variables \(R_{j,t}\) in (9), \(p\) can be very large, namely of exponential rate. We also drop the subscripts of the coefficients \(\beta_{j|R_{j}}\), as we focus on one regression. The SIM of (9) is then rewritten as:

\[
Y_t = g(X_t^T \beta^*) + \varepsilon_t,
\]

where \(\{X_t, \varepsilon_t\}\) are strong mixing processes, \(g(\cdot): \mathbb{R}^l \to \mathbb{R}^l\) is an unknown smooth link function, \(\beta^*\) is the vector of index parameters. Regressors \(X_t\) can be the lagged variables of \(Y_t\). For the identification, we assume that \(\|\beta^*\|^2 = 1\), and the first component of \(\beta^*\) is positive. We assume that there are \(q\) nonzero components in \(\beta^*\).

Note that (16) can be formulated in a location model and identified in a quasi maximum
likelihood framework: the direction $\beta^*$ (for known $g(\cdot)$) is the solution of

$$
\min_{\beta} E \rho_r \{ Y_t - g(X_t^T \beta) \},
$$

with loss function

$$
\rho_r(u) = \tau u \mathbb{1}(u > 0) + (1 - \tau) u \mathbb{1}(u < 0),
$$

$$
E[\psi_r \{ Y_t - g(X_t^T \beta^*) \}|X_t] = 0 \quad a.s.
$$

where $\psi_r(\cdot)$ is the derivative (a subgradient) of $\rho_r(\cdot)$. It can be reformulated as $F_{\varepsilon|X_t}^{-1}(r) = 0$.

The model is similar to the location scale model considered in Franke et al. (2014). Note that it may be extended it to a quantile AR-ARCH type of single index model,

$$
Y = g(X_t^T \beta^*) + \sigma(X_t^T \gamma^*) \varepsilon_t
$$

To estimate the shape of a link function $g(\cdot)$ and selected $\beta$ coefficients we adopt minimum average contrast estimation approach (MACE) with penalization outlined in Fan et al. (2013). The estimation of $\beta^*$ and $g(\cdot)$ is as following:

$$
\hat{\beta}_r, \hat{g}(\cdot) \overset{\text{def}}{=} \arg \min_{\beta, g(\cdot)} -L_n(\beta, g(\cdot))
$$

$$
= \arg \min_{\beta, g(\cdot)} n^{-1} \sum_{j=1}^{n} \sum_{t=1}^{n} \rho_r \{ X_t - g(\beta^T X_j) - g'(\beta^T X_j) X_{ij}^T \beta \} \omega_{ij}(\beta)
$$

$$
+ \sum_{l=1}^{p} \gamma(\|\beta_l\| \theta),
$$

where $\omega_{ij}(\beta) \overset{\text{def}}{=} \frac{K_h(X_{ij}^T \beta)}{\sum_{t=1}^{n} K_h(X_{ij}^T \beta)}$, $K_h(\cdot) = h^{-1} K(\cdot/h)$, $K(\cdot)$ is a kernel e.g. Gaussian kernel, $h$ is a bandwidth and $L_n(\beta, g(\cdot))$ is defined as $-n^{-1} \sum_{j=1}^{n} \sum_{t=1}^{n} \rho_r \{ X_t - g(\beta^T X_j) - g'(\beta^T X_j) X_{ij}^T \beta \} \omega_{ij}(\beta) + \sum_{l=1}^{p} \gamma(\|\beta_l\| \theta)$. Since the data is not equally spaced we choose a bandwidth $h$ based on k-nearest neighbor procedure (See Härdle et al. (2004) and Carroll and Härdle (1989)). The optimal $k$, number of neighbors, are selected based on a cross-validation criterion. The implementation involves an iteration between estimating $\beta$ and $g(\cdot)$, with a consistent initial estimate for $\beta$, Wu et al. (2010). $X_{ij} = X_t - X_j$, $\theta \geq 0$, and $\gamma(\cdot)$ is some non-decreasing function concave for $t \in (0, +\infty)$ with a continuous derivative on $(0, +\infty)$. Please note that this MACE functional (with respect to $g(\cdot)$) (20) is in fact only a finite dimensional optimization problem since the minimum over
$g(\cdot)$ is to be determined at $a_j = g(\beta^T X_j)$, $b_j = g'(\beta^T X_j)$. There are several approaches for the choice of the penalty function. These approaches can be classified based on the properties desired for an optimal penalty function, namely, unbiasedness, sparsity and continuity. The $L_1$ penalty approach known as least absolute shrinkage and selection operator (LASSO) is proposed for mean regression by Tibshirani (1996). Numerous studies further adapt LASSO to a quantile regression framework, Yu et al. (2003), Li and Zhu (2008), Belloni and Chernozukov (2011), among others. While achieving sparsity the $L_1$-norm penalty tends to over-penalize the large coefficients as the LASSO penalty increases linearly in the magnitude of its argument, and, thus, may introduce bias to estimation. As a remedy to this problem adaptive LASSO estimation procedure has been proposed (Zou (2006); Zheng et al. (2013)). Another approach to alleviate the LASSO bias is proposed by Fan and Li (2001) known as Smoothly Clipped Absolute Deviation (SCAD):

$$
\gamma_\lambda(t) = \begin{cases} 
\lambda|t| & \text{for } |t| \leq \lambda, \\
-(t^2 - 2a\lambda|t| + \lambda^2)/(a-1) & \text{for } \lambda < |t| \leq a\lambda, \\
(a+1)\lambda^2/2 & \text{for } |t| > a\lambda,
\end{cases}
$$

where $\lambda > 0$ and $a > 2$. Note that for $\lambda = \infty$, this is exactly LASSO.

As for selecting $\lambda$, there are two common ways: data-driven generalized cross-validation criterion (GCV) and likelihood-based Schwartz, or Bayesian information criterion-type criteria (SIC, or BIC), Schwarz (1978); Koenker et al. (1994), and their further modifications. The most commonly used criterion is GCV, however, it has been shown that it leads to an overfitted model. Therefore, we employ a modified BIC-type model selection criteria proposed by Wang et al. (2007) and use GCV criterion only to verify whether GCV and BIC diverge significantly. We need to introduce some more notation to present our theoretical results.

Define $\hat{\beta}_r \overset{\text{def}}{=} (\hat{\beta}^{(1)}_r, \hat{\beta}^{(2)}_r)^T$ as the estimator for $\beta^* \overset{\text{def}}{=} (\beta^{*T(1)}, \beta^{*T(2)})^T$ attained by the loss in (20). Here $\hat{\beta}^{(1)}_r$ and $\hat{\beta}^{(2)}_r$ refer to the first $q$ components and the remaining $p-q$ components of $\hat{\beta}_r$ respectively. The same notional logic applies to $\beta^*$. If in the iterations, we have the initial estimator $\hat{\beta}^{(0)}_r$ as a $\sqrt{n/q}$ consistent one for $\beta^{*T(1)}$, we will obtain with a very high probability, an oracle estimator of the following type, say $\tilde{\beta}_r = (\tilde{\beta}^{(1)}_r, 0^T)^T$, since the oracle knows the true model $\mathcal{M}_r \overset{\text{def}}{=} \{ l : \beta^*_l \neq 0 \}$. The following theorem shows that the penalized estimator enjoys the oracle property. Define $\hat{\beta}_0 \in \mathbb{R}^p$ as the minimizer with the same loss in (20) but within subspace $\{ \beta \in \mathbb{R}^p : \beta_{\mathcal{M}_r} = 0 \}$.

With all the above definitions and conditions, see Appendix, we may present the following theorems.

**THEOREM 3.1.** Under Conditions 1-7, the estimators $\hat{\beta}_0$ and $\hat{\beta}_r$ exist and coincide
on a set with probability tending to 1. Moreover,
\[
P(\hat{\beta}^0 = \hat{\beta}_r) \geq 1 - (p - q) \exp(-C'n^\alpha) \tag{21}
\]
for a positive constant \(C'\), where \(\hat{\beta}^0\) is the “ideal” estimator with nonzero elements correctly specified.

This theorem implies the sign consistency.

**THEOREM 3.2.** Under Conditions 1-7, we have
\[
\|\hat{\beta}_{r(1)} - \beta^*_r(1)\| = O_p\{(D_n + n^{-1/2})\sqrt{q}\} \tag{22}
\]
For any unit vector \(b\) in \(\mathbb{R}^q\), we have
\[
b^\top C_{0(1)}^{1/2} C_{0(1)}^{-1/2} C_{0(1)}^{1/2} \sqrt{n}(\hat{\beta}_{r(1)} - \beta^*_r(1)) \xrightarrow{L} \mathcal{N}(0, 1) \tag{23}
\]
where \(C_{0(1)} \overset{\text{def}}{=} E\{E\{\psi_\tau^2(\varepsilon_i)\}|Z_t\} [g'(Z_t)]^2 [E(X_{t(1)}|Z_t) - X_{t(1)}] [E(X_{t(1)}|Z_t) - X_{t(1)}]^\top\}, \) and \(C_{0(1)} \overset{\text{def}}{=} E\{\partial E\psi_\tau(\varepsilon_i)|Z_t\} \{[g'(Z_t)]^2 [E(X_{t(1)}|Z_t) - X_{t(1)}] [E(X_{t(1)}|Z_t) - X_{t(1)}]^\top\}. \) Note that \(E(X_{t(1)}|Z_t)\) denotes a \(p \times 1\) vector with \(j\)th element \(E(X_{j(1)}|Z_t), j = 1, \cdots, q\), and \(Z_t \overset{\text{def}}{=} X_t^\top \beta^*_r, \psi_\tau(\varepsilon)\) is a choice of the subgradient of \(p_\tau(\varepsilon)\) and \(\sigma_r^2 \overset{\text{def}}{=} E[\psi_\tau(\varepsilon_i)]^2/\partial E\psi_\tau(\varepsilon_i)^2\), where
\[
\partial E\psi_\tau(\cdot)|Z_t = \frac{\partial E\psi_\tau(\varepsilon_i - v)^2|Z_t}{\partial v^2}_{v=0}. \tag{24}
\]
Let us now look at the distribution of \(\hat{g}(\cdot)\) and \(\hat{g}'(\cdot)\), estimators of \(g(\cdot), g'(\cdot)\).

**THEOREM 3.3.** Under Conditions 1-7, for any interior point \(z = x^\top \beta^*_r, f_Z(z)\) is the density of \(Z_t, t = 1, \ldots, n\), if \(nh^3 \to \infty\) and \(h \to 0\), we have
\[
\sqrt{nh} \sqrt{f_Z(z)/(\nu_0 \sigma_r^2)} \left\{ \hat{g}(x^\top \beta) - g(x^\top \beta^*_r) - \frac{1}{2} h^2 g''(x^\top \beta^*_r) \mu_2 \partial E\psi_\tau(\varepsilon) \right\} \xrightarrow{L} \mathcal{N}(0, 1),
\]
Also, we have
\[
\sqrt{nh^3} \sqrt{\{f_Z(z)\mu_2^2/(\nu_2 \sigma_r^2)} \left\{ \hat{g}'(x^\top \beta) - g'(x^\top \beta^*_r) \right\} \xrightarrow{L} \mathcal{N}(0, 1).
\]
The dependence doesn’t have any impact on the rate of the convergence of our non-parametric link function. As the degree of the dependence is measured by the mixing coefficient \(\alpha\), it is weak enough such that Condition 7 is satisfied. In fact we assume exponential decaying rate here, which implies the (A.4) in Kong et al. (2010).
4. Results

4.1. Data

Since SIC code can be applied to classify the industries, according to the company list 2012 of U.S. financial institutions from NASDAQ webpage and corresponding four-digit SIC codes from 6000 to 6799 for these financial institutions in COMPUSTAT database, we divide the U.S. financial institutions into four groups: (1) depositories (6000-6099), (2) insurance companies (6300-6499), (3) broker-dealers (6200-6231), (4) others (the rest codes). For instance, Goldman Sachs Group is classified as broker-dealers based on its SIC code 6211. We select top 25 institutions in each group according to the ranking of their market capitalization (like Billio et al. (2012) they apply a similar selection method), so that we can compare the difference among industry groups more clearly. Our analysis focuses on the panel of these 100 publicly traded U.S. financial institutions between January 5, 2007 and January 4, 2013, see Table 2 in Appendix B for a complete list. The weekly price data are available in Yahoo Finance.

To capture the company specific characteristic we adopt the following variables calculated from balance sheet information as proposed in AB: 1. leverage, defined as total assets / total equity (in book values); 2. maturity mismatch, calculated by (short term debt - cash)/ total liabilities; 3. market-to-book, defined as the ratio of the market to the book value of total equity; 4. size, calculated by the log of total book equity. The quarterly balance sheet information is available in COMPUSTAT database, linear interpolation is implemented in order to obtain the weekly data.

Apart from the data on the financial companies we use weekly observations of macro state variables which characterize the general state of the economy. These variables are defined as follows: (i) the implied volatility index, VIX, reported by the Chicago Board Options Exchange; (ii) short term liquidity spread denoted as the difference between the three-month repo rate (available in Bloomberg database) and the three-month bill rate (from Federal Reserve Board) to measure short-term liquidity risk; (iii) the changes in the three-month Treasury bill rate from the Federal Reserve Board; (iv) the changes in the slope of the yield curve corresponding to the yield spread between the ten-year Treasury rate and the three-month bill rate from the Federal Reserve Board; (v) the changes in the credit spread between BAA-rated bonds and the Treasury rate from the Federal Reserve Board; (vi) the weekly S&P500 index returns from Yahoo finance, and (vii) the weekly Dow Jones U.S. Real Estate index returns from Yahoo finance.
4.2. Estimation Results

Then TENET analysis is performed in three steps: first, the Tail Event VaR of all firms are estimated. Secondly, the NETwork analysis based on SIM with variable selection technique is performed. Finally, the systemic risk contributions are measured based on SIM technique.

To estimate VaR as in (7) and (8), we regress weekly log returns of each institution on macro state variables at the quantile level \( \tau = 0.05 \), the whole period is \( T = 266 \), the rolling window size is set to be \( n = 48 \) corresponding to one year’s weekly data. Figure 1 is an example of estimated VaR for J P Morgan (with SIC code 6020).

In the second step a CoVaR based risk network are estimated by applying SIM with variable selection, see (20). Figure 2 shows the CoVaR\(^{TENET}\) of J P Morgan. Then the network analysis induced by the CoVaR\(^{TENET}\) is shown from Figure 3 to Figure 6. Recall adjacency matrix of Table 1 constructed from \( |\hat{D}_{j|i}| \) and \( |\hat{D}_{ij}| \). To aggregate the results over windows, we take the componentwise sum of the absolute values of the adjacency matrices. With the aggregation we will be able to understand the risk channels and the relative role of each firm or each sector in the whole financial network.

For this propose, we define three levels of connectedness: the overall level, the group level and the firm level. The overall level of risk is characterized by the total connectedness of the system and the averaged value of the tuning parameter \( \lambda \). The total connectedness of links is defined as \( TC_t = TC_{tIN} = TC_{tOUT} \) def \( \sum_{i=1}^{k} \sum_{j=1}^{k} |\hat{D}_{j|i}| \), where \( TC_{tIN} \) and \( TC_{tOUT} \) are the total incoming and outgoing links in this matrix respectively. The solid line of Figure 3 shows the evolution of the total connectedness, and the dashed line of Figure 3 shows the averaged \( \lambda \) values of the CoVaR estimations, where \( \lambda \) is the estimated penalization parameter, see section 3. While in the beginning of 2008 there were lower connectedness and smaller averaged \( \lambda \), from third quarter of 2008 both connectedness and averaged \( \lambda \) began to increase sharply which corresponds to Lehman brother’s bankruptcy, evidence also can be found by the government takeover of Fannie Mae and Freddie Mac.

As the crisis was unfolding, the averaged \( \lambda \) stayed at peak level in the beginning of 2009, the system became more heavily interconnected and reached its peak in the second quarter of 2009, which can be seen as the influence of the European sovereign debt crisis. Then the downward trend dominated the whole market, and lasted until end of 2011, the financial institutions are most less connected to each others in second quarter of 2011. From the first quarter of 2011 the averaged \( \lambda \) began to increase and lasted until beginning of 2012 which attributes to the impact of US debt-ceiling crisis in July 2011. Total connectedness series increased again in second quarter of 2011. After the middle of 2012, both averaged \( \lambda \) and total connectedness series went down. After this insight we may state that the total connectedness is lagged to the averaged \( \lambda \): the spillover effects
are lagged to the real state of economy. Since the evolution of averaged $\lambda$ represents the variation of the systemic risk, the CRC 649 proposed a Financial Risk Meter (FRM): http://sfb649.wiwi.hu-berlin.de/frm/index.html.

The group connectedness with respect to incoming links is defined as follows: $GC_{g,t}^{IN} \overset{\text{def}}{=} \sum_{i=1}^{k} \sum_{j \in g} |\hat{D}_{j|i}|$, where $g = 1, 2, 3, 4$ corresponds to the four aforementioned industry groups. The group connectedness with respect to outgoing links is defined as $GC_{g,t}^{OUT} \overset{\text{def}}{=} \sum_{i \in g} \sum_{j=1}^{k} |\hat{D}_{j|i}|$. Figure 4 shows the incoming links for these four groups. The patterns of these four groups are almost identical, i.e. there are more links during the end of 2008 and beginning of 2010 for all groups. While the depositories sector (solid line) received on average more risk, the insurance companies (dashed line) are less influenced by others. This can be seen as evidence for the report of Systemic Risk in Insurance–An analysis of insurance and financial stability published by Geneva Association in 2010 stating that the losses in the insurance industry have been only a sixth of those at banks. In contrast to the incoming links the outgoing links in Figure 5 are more volatile. It is not surprising that the depositories sector dominates the others in the outgoing links, i.e. the bank group emits more risk to others. Broker-dealers and others fluctuate very much in the whole period, but they send out less risk compared with banks. And the insurers emit averagely less risk over all period than the other groups.

Next we turn to analyzing firm level interconnectedness. First of all we define the directional connectedness from firm $i$ to the firm $j$: $DC_{j|i,t} \overset{\text{def}}{=} |\hat{D}_{j|i}|$. The network in Figure 6 shows one example of the firm level directional connectedness on June 12 2009 which was in the financial crisis. We see that there are several strong connections, for example, the link from Bank of America Corporation (BAC) to Janus Capital Group (JNS), from Principal Financial Group (PFG) to MetLife, Inc. (MET) and from Lincoln National Corporation (LNC) to American International Group (AIG). Moreover there are also a couple of weak connections from Morgan Stanley (MS) to others. The ranking of the directional connectedness is calculated by the sum of absolute value of $\hat{D}_{j|i}$ over windows. The strongest link on average is from NewStar Financial, Inc. (NEWS) to Oppenheimer Holdings, Inc.(OPY), see Table 3. Secondly, the firm connectedness with respect to incoming links is defined as $FC_{j,t}^{IN} \overset{\text{def}}{=} \sum_{i=1}^{k} |\hat{D}_{j|i}|$. Finally, the firm connectedness with respect to outgoing links: $FC_{j,t}^{OUT} \overset{\text{def}}{=} \sum_{j=1}^{k} |\hat{D}_{j|i}|$. From Table 4 and 5 we have the top 10 firms in terms of incoming links and outgoing links respectively. The most connected firm with incoming links is Oppenheimer Holding, Inc. (OPY) which is an investment bank and the most connected firm with outgoing links is Lincoln National Corporation (LNC) which is a multiple insurance and investment management company. We have found out that among the top 10 IN-link and OUT-link companies, there are several big firms, such as J P Morgan (JPM) and Wells Fargo (WFC) with IN-link, and Citigroup Inc.(C) and Morgan Stanley (MS) with OUT-link. However, there are also firms with moderate or
small sizes e.g. Oppenheimer Holding, Inc. (OPY) and Safeguard Sciences, Inc. (SFE) with IN-link, and Ladenburg Thalmann Financial Services Inc. (LTS) and Federal Agricultural Mortgage Corporation (AGM) with OUT-link. It is justifiable to have firms with moderate or small sizes, and also reasonable to have non bank institutions, as this is connected with the Global Financial Stability Report (GFSR) in the April 2009 which states that the crisis has shown that not only the banks but also other nonbank financial intermediaries can be systemically important and their failure can cause destabilizing effects. It also emphasizes that not only the largest financial institutions but also the smaller but interconnected financial institutions are systemic important and need to be regulated. "Too connected to fail" is an important issue, this is reflected in our network analysis. We believe therefore that our methodology in this network analysis is good to identify those "too connected to fail" firms who have moderate or small sizes but still of great interest in the systemic risk network.

In addition, based on our network analysis we have the following findouts: (1) the connections between institutions tend to increase before the financial crisis, (2) the network is characterized by numerous heavy links at the peak of a crisis, (3) the connections between institutions reflected by absolute value of partial derivatives get weaker as the financial system stabilized, (4) the incoming links are far less volatile than the outgoing links. Whereas banks dominate both incoming and outgoing links, the insurers are less affected by the financial crisis and exhibit less contribution in terms of risk transmission. The broker-dealer and others are highly volatile with respect to the risk contribution. (5) Several institutions with moderate or small sizes and also some non bank institutions received or transmitted more risk, as there are "too connected" firms.

While in the second step we detect connectedness by applying network analysis, in the third step we provide an exact systemic risk measure for each firm involved using the quantile regression for SIM without variable selection. As mentioned in Section 2.4, the relevant variables selected in the network analysis are set to be control variables for each firm in this step. The system return is the response variable and the covariates will be an individual firm together with its relevant variables. We take the average value of the systemic risk contribution for each firm over windows. Recall that this is calculated by taking the partial derivative with respective to a specific firm return. To illustrate the variation of the systemic risk over time, we present in Figure 7 the \( \hat{\text{CoVaR}} \) of J P Morgan (lower thicker line), its partial derivatives (upper thinner line) and the log returns of the system (points). We see that the partial derivatives are more volatile during the crisis period of 2008 and 2010. The maximum value occurred in the end of 2008. To compare the systemic risk contribution among firms, we list the top ten firms with largest averaged systemic risk contribution in Table 6. Compared with the result of global systemically important banks (G-SIBs) published by Financial Stability
Board 2012, five of our top ten systemically risk contributors appear in this report: J P Morgan (JPM), Bank of America Corporation (BAC), Wells Fargo & Company (WFC), Citigroup Inc. (C) and Goldman Sachs Group (GS). Also we compare our result with the global systemically important insurers (G-SIIs) published by Financial Stability Board 2013, MetLife, Inc. (MET) is present in their list. We also compare with the list of all domestic systemically important banks (D-SIBs) in USA published by Board of Governors of the Federal Reserve System 2014, American Express Company (AXP) and M&T Bank Corporation (MTB) are on that list. In total there are eight systemically important institutions identified. We find out that the large systemic risk contributors calculated in this step are mostly big firms which represent "too big to fail" and need to be well supervised and regulated.

4.3. Model Validation

To evaluate the accuracy of the estimated VaR in the first step, we count the firms’ VaRs violations, which is meant to be the situation when the stock losses exceed the estimated VaRs. In Figure 1 there is no violation in the series of estimated VaR for J P Morgan. The average violation rate for 100 financial institutions is \( \hat{\tau} = 0.0006 \), which is much smaller than the nominal rate \( \tau = 0.05 \).

In step 2 and step 3 we apply SIM with variable selection to calculate CoVaR. We also compare our results with linear quantile LASSO models in both step 2 and step 3 to justify the necessity of having a nonlinear model. The benchmark linear LASSO model is written as follows:

\[
X_{j,t} = \beta_{R_j}^L R_{j,t} + \varepsilon_{j,t},
\]

(25)

\[
\hat{\text{CoVaR}}_{\text{Step}2}^L \overset{\text{def}}{=} \hat{\text{CoVaR}}_{j,\text{Step}2}^L = \hat{\beta}_{R_j}^L R_{j,t},
\]

(26)

where \( R_{j,t}, X_{-j,t}, B_{j,t-1}, \overline{\text{VaR}}_{-j,t,\tau} \) and \( \tilde{R}_{j,t} \) are defined in section 2.3. The parameters \( \beta_{R_j}^L \overset{\text{def}}{=} \{ \beta_{jR_j}^L, \beta_{jM}^L, \beta_{jB_j}^L \}^\top \) and \( \hat{\beta}_{R_j}^L \overset{\text{def}}{=} \{ \hat{\beta}_{jR_j}^L, \hat{\beta}_{jM}^L, \hat{\beta}_{jB_j}^L \}^\top \) which are estimated by using linear quantile regression with variable selection. Then \( \hat{\text{CoVaR}}_{\text{Step}2}^L \) can be simply calculated.

Recall that we denote our estimated CoVaR in step 2 as \( \hat{\text{CoVaR}}^\text{TENET} \). Now we compare the performance of \( \hat{\text{CoVaR}}^\text{TENET} \) and \( \hat{\text{CoVaR}}_{\text{Step}2}^L \). Figure 8 is the \( \hat{\text{CoVaR}}_{\text{Step}2}^L \) of J P Morgan, there are 41 violations during the whole time period of \( T = 266 \), whereas there are only 2 violations in the estimated \( \hat{\text{CoVaR}}^\text{TENET} \) series in Figure 2. The number of averaged violation for \( \hat{\text{CoVaR}}^\text{TENET} \) over all firms is about 4.73, but \( \hat{\text{CoVaR}}_{\text{Step}2}^L \) indicates on average a large number of 41.03, see Table 7. Moreover, the averaged estimated violation rate of \( \hat{\text{CoVaR}}^\text{TENET} \) is around \( \hat{\tau} = 0.02 \) which is closer to the quantile level.
\( \tau = 0.05 \) than the VaR estimation in previous step, indicating that \( \text{CoVaR}^{\text{TENET}} \) picks up the desired level better. However, for \( \text{CoVaR}_{\text{Step2}}^{L} \), the \( \hat{\tau} = 0.15 \) which is much larger than the quantile level 0.05. We apply then the CaViaR test proposed by Berkowitz et al. (2011). While 96% of our \( \text{CoVaR}^{\text{TENET}} \) passed the CaViaR test at the 0.05 significance level, only 2% of \( \text{CoVaR}_{\text{Step2}}^{L} \) passed this test which indicates the invalidation of linear LASSO model, see Table 7 for the \( p \)-values.

Further, we examine the shape of the link functions in the crisis period as well as in the period of relative financial stability. We find out that for almost all firms in a financial crisis period, the link functions are in most of the windows non-linear, while in a stable period, the link functions tend to be more linear. Take the \( \text{CoVaR}^{\text{TENET}} \) for J P Morgan as an example. The left panel of Figure 9 displays the shape of the estimated link function in one window in crisis time and its 95% confidence bands, see Carroll and Härdle (1989). In a stable period one observes in some windows the shape of the link function as on the right panel of Figure 9.

In step 3 the system return can be estimated by applying linear quantile regression (without variable selection) as well:

\[
X_{s,t} = \beta_{s|F_j}^{LT} F_{j,t} + \varepsilon_{s,t},
\]

\[
\text{CoVaR}_{\text{Step3}}^{L} \overset{\text{def}}{=} \text{CoVaR}_{s|F_j,t}^{L} = \beta_{s|F_j}^{LT} \tilde{F}_{j,t},
\]

where \( X_{s,t}, F_{j,t}, C_{j,t}, \tilde{C}_{j,t} \) are same as defined in section 2.4. \( \beta_{s|F_j}^{L} = \{ \beta_{s|F_j}^{L}, \beta_{s|C_j}^{L} \}^{\top} \) and \( \tilde{\beta}_{s|F_j}^{L} = \{ \tilde{\beta}_{s|F_j}^{L}, \tilde{\beta}_{s|C_j}^{L} \}^{\top} \) can be estimated by applying linear quantile regression. \( \text{CoVaR}_{\text{Step3}}^{L} \) can be then estimated easily.

In step 3 we also distinguish linear CoVaR and SIM CoVaR by \( \text{CoVaR}_{\text{Step3}}^{L} \) and \( \text{CoVaR}^{\text{SYSTEM}}_{\text{AB}} \) respectively. For comparison, we estimated the systemic risk measure by \( \text{CoVaR}^{\text{SYSTEM}}_{\text{AB}} \) in (4) in this step as well. The averaged violation of \( \text{CoVaR}^{\text{SYSTEM}}_{\text{AB}} \) is around 2, while there are 20 violations of \( \text{CoVaR}_{\text{Step3}}^{L} \), and 29 violations of \( \text{CoVaR}^{\text{SYSTEM}}_{\text{AB}} \). The averaged \( p \)-values for CaViaR test of \( \text{CoVaR}^{\text{SYSTEM}}_{\text{AB}} \) over windows is 0.52, for \( \text{CoVaR}_{\text{Step3}}^{L} \) is only 0.02, and for \( \text{CoVaR}^{\text{AB}}_{\text{SYSTEM}} \) is 0.24, see Table 7 for more details. Moreover, we also investigate the shape of the link function. Figure 10 shows the \( \text{CoVaR}^{\text{SYSTEM}}_{\text{AB}} \) of J P Morgan and the estimated indices and its confidence bands in different time periods, it confirms Chao et al. (2015)’s results stating that the nonlinear model performs better especially in financial crisis period (see the left of Figure 10). We see the outperformance of our method over AB and the linear model conditional on the network effects.
5. Conclusion

In this paper we propose TENET based on a semiparametric quantile regression framework to assess the systemic importance of financial institutions conditional on their interconnectedness in tails. The semiparametric model to allow for more flexible modeling of relationship between the variables. This is especially justified in a (ultra) high-dimensional setting when the assumption of linearity is not likely to hold. In order to face these challenges statistically we estimate a SIM in a generalized quantile regression framework while simultaneously performing variable selection. (Ultra) high dimensional setting allows us to include more variables into the analysis.

Our empirical results show that there is growing interconnectedness during the period of a financial crisis, and network-based measure reflecting the connectivity. Moreover, by including more variables into the analysis we can investigate the overall performance of different financial sectors, depositaries, insurance, broker-dealers, and others. Estimations results show relatively high importance of depository industry in the financial crisis. We also observe strong non-linear relationships between the variables, especially, in the period of relative financial instability. We conclude that both the most connected firms as well as the big firms with large systemic risk contributions are systemically important. An interactive Financial Risk Meter is proposed on: http://sfb649.wiwi.hu-berlin.de/frm/index.html.
6. Appendix A: Proof

Condition 1. The kernel $K(\cdot)$ is a continuous symmetric function. The link function $g(\cdot) \in C^2$, let $\mu_j \overset{\text{def}}{=} \int u^j K(u) du$ and $\nu_j \overset{\text{def}}{=} \int u^j K^2(u) du$, $j = 0, 1, 2$.

Condition 2. The derivative (or a subgradient) of $\rho_r(x)$, satisfies $E\psi_r(\varepsilon_t) = 0$ and $\inf_{|u| \leq c} \partial E\psi_r(\varepsilon_t - v) = C_1$ where $\partial E\psi_r(\varepsilon_t - v)$ is the partial derivative with respect to $v$, and $C_1$ is a constant.

Condition 3. The density $f_Z(z)$ of $Z_t = \beta^*^\top X_t$ is bounded with bounded absolute continuous first-order derivatives on its support. Assume $E\{\psi_r(\varepsilon|X)\} = 0$ a.s., which means for a quantile loss we have $F^{-1}_\varepsilon(\tau) = 0$. Let $X_{l(1)}$ denote the sub-vector of $X_t$ consisting of its first $q$ elements. Let $Z_t \overset{\text{def}}{=} X_t^\top /\beta^*$ and $Z_{ij} \overset{\text{def}}{=} Z_t - Z_j$. Define $C_{0(1)} \overset{\text{def}}{=} E\{\psi_r^2(\varepsilon_t)|Z_t\}$, $\{g'(Z_t)\}^2(E(X_{l(1)}|Z_t) - X_{l(1)})^\top (E(X_{l(1)}|Z_t) - X_{l(1)})$, and $C_{0(1)} \overset{\text{def}}{=} E\{\partial E\psi_r(\varepsilon_t)|Z_t\}$, $\{g'(Z_t)\}^2(E(X_{l(1)}|Z_t) - X_{l(1)})^\top (E(X_{l(1)}|Z_t) - X_{l(1)})$ and the matrix $C_{1(1)}$ satisfies $0 < L_1 \leq \lambda_{\min}(C_{0(1)}) \leq \lambda_{\max}(C_{0(1)}) \leq L_2 < \infty$ for positive constants $L_1$ and $L_2$. There exists a constant $c_0 > 0$ such that $\sum_{i=1}^{n} \{\|X_{l(1)}\|/\sqrt{n}\}^{2+c_0} \to 0$, with $0 < c_0 < 1$. $v_{ij} \overset{\text{def}}{=} Y_t - a_j - b_j X_{ij} /\beta$. Also, exists a constant $C_3$ such that for all $\beta$ close to $\beta^* \|\beta - \beta^*\| \leq C_3$.

$$\|\sum_1^t \sum_j X_{(0)ij} \omega_{lj} X_{l(1)j} \partial E\psi_r(v_{ij})\|_{2,\infty} = O_p(n^{1-a_1}).$$

Condition 4. The penalty parameter $\lambda$ is chosen such that $\lambda = O(n^{-1/2})$, with $D_n \overset{\text{def}}{=} \max\{d_l : l \in M_{\text{a}}\} = \sigma(n^{a_1-a_2/2} /\lambda) = O(n^{-1/2})$, $d_l \overset{\text{def}}{=} \gamma_\lambda(\|\beta_l^*\|)$, $M_{\text{a}} = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \to 0$ and $h^{-1}\sqrt{q/n} = O(1)$ as $n$ goes to infinity, $q = O(n^{a_2})$, $p = O\{\exp(n^{\delta})\}$, $nh^3 \to \infty$ and $h \to 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$.

Condition 5. The error term $\varepsilon_t$ satisfies $\text{Var}(\varepsilon_t) < \infty$. Assume that

$$\sup_l E\left|\psi_r^m(\varepsilon_t)/m!\right| \leq s_0 M^m$$

$$\sup_l E\left|\psi_r^m(x_{ij})/m!\right| \leq s_0 M^m$$

where $s_0$ and $M$ are constants, and $\psi_r(\cdot)$ is the derivative (a subgradient) of $\rho_r(\cdot)$.

Condition 6. The conditional density function $f(\varepsilon_t|Z_t = z)$ is bounded and absolutely continuous differentiable.

Conditions 7. $\{X_{ij}, \varepsilon_t\}_{t=-\infty}^{t=\infty}$ is a strong mixing process for any $j$. Moreover, there exists positive constants $c_{m1}$ and $c_{m2}$ such that the $\alpha$- mixing coefficient for every
\[ j \in \{1, \cdots, p\}, \]
\[ \alpha(l) \leq \exp(-c_{m2} I_{m2}), \quad (29) \]

where \( c_{m2} > 2\alpha \).

Recall (20) and \( \hat{\beta}^0 \) as the minimizer with the loss

\[
\hat{L}_n(\beta) \overset{\text{def}}{=} \sum_{j=1}^{n} \sum_{t=1}^{n} \rho_t(Y_t - a_j^* - b_j^* X_{ij}^\top \beta) \omega_{ij}(\beta^*) + n \sum_{l=1}^{p} d_l |\beta_l|,
\]

but within the subspace \( \{\beta \in \mathbb{R}^p : \beta_{M^*} = 0\} \), and \( a_j^* = g(\beta^* X), b_j^* = g'(\beta^* X) \). The following lemma assures the consistency of \( \hat{\beta}^0 \).

**LEMMA 6.1.** Under Conditions 1-7, recall \( d_j = \gamma(\beta^*_j) \), we have that

\[
\|\hat{\beta}^0 - \beta^*\| = O_p(\sqrt{q/n} + \|d(1)\|)
\]

where \( d(1) \) is the subvector of \( d = (d_1, \cdots, d_p)^\top \) which contains \( q \) elements corresponding to the nonzero \( \beta^*_j \).

**PROOF.** Note that the last \( p - q \) elements of both \( \hat{\beta}^0 \) and \( \beta^* \) are zero, so it is sufficient to prove \( \|\hat{\beta}^0_{(1)} - \beta^*_{(1)}\| = O_p(\sqrt{q/n} + \|d(1)\|) \).

Following Fan et al. (2013), it is not hard to prove that for \( \gamma_n = o(1) \):

\[
P\left[ \inf_{\|u\|=1} \{ \hat{L}_n(\beta^*_{(1)} + \gamma_n u, 0) \} \right] \rightarrow 1.
\]

Then there exists a minimizer inside the ball \( \{\beta_{(1)} : \|\beta_{(1)} - \beta^*_{(1)}\| \leq \gamma_n\} \). Construct \( \gamma_n \to 0 \) so that for a sufficiently large constant \( B_0 \): \( \gamma_n > B_0 \cdot (\sqrt{q/n} + \|d(1)\|) \). Then by the local convexity of \( \hat{L}_n(\beta_{(1)}, 0) \) near \( \beta^*_{(1)} \), there exists a unique minimizer inside the ball \( \{\beta_{(1)} : \|\beta_{(1)} - \beta^*_{(1)}\| \leq \gamma_n\} \) with probability tending to 1. \( \square \)

Let \( X_{(1)ij} \) denote the subvector of \( X_{ij} \) consisting of its first \( q \) components.

Recall that \( X = (X_{(1)}, X_{(0)}) \) and \( M_\ast = \{1, \ldots, q\} \) is the set of indices at which \( \beta \) are nonzero.

Lemma 1 shows the consistency of \( \hat{\beta}^0 \), and we need to show further that \( \hat{\beta}^0 \) is the unique minimizer in \( \mathbb{R}^p \) on a set with probability tending to 1.

**LEMMA 6.2.** Under conditions 1-7, minimizing the loss function \( \hat{L}_n(\beta) \) has a unique
global minimizer \( \hat{\beta} = (\hat{\beta}_1^T, 0^T)^T \), if and only if on a set with probability tending to 1,

\[
\sum_{j=1}^n \sum_{t=1}^n \psi_r(Y_t - \hat{a}_j - \hat{b}_j X_{ij}^T \hat{\beta}_r) \hat{b}_j X_{(1)tj} \omega_{tj}(\beta^*) + n d(1) \circ \text{sign}(\hat{\beta}_r) = 0 \tag{31}
\]

\[
\|z(\hat{\beta}_r)\|_\infty \leq n, \tag{32}
\]

where

\[
z(\hat{\beta}_r) \overset{\text{def}}{=} d(0)^{-1} \circ \left\{ \sum_{j=1}^n \sum_{t=1}^n b_j^* \psi_r(Y_t - a_j^* - b_j^* X_{ij}^T \hat{\beta}_r) X_{(0)tj} \omega_{tj}(\hat{\beta}_r) \right\} \tag{33}
\]

where \( \circ \) stands for multiplication element-wise.

PROOF. According to the definition of \( \hat{\beta}_r \), it is clear that \( \hat{\beta}(1) \) already satisfies condition (31). Therefore we only need to verify condition (32). To prove (32), a bound for

\[
\sum_{i=1}^n \sum_{t=1}^n b_j^* \psi_r(Y_t - a_j^* - b_j^* X_{ij}^T \beta^*) \omega_{tj} X_{(0)ij} \tag{34}
\]

is needed. Define the following kernel function

\[
h_d(X_i, a_j^*, b_j^*, Y_i, X_j, a_i^*, b_i^*, Y_j) = \frac{n}{2} \left\{ b_j^* \psi_r(Y_t - a_j^* - b_j^* X_{ij}^T \beta^*) \omega_{ij} X_{(0)ij} + b_i^* \psi_r(Y_t - a_i^* - b_i^* X_{ij}^T \beta^*) \omega_{ji} X_{(0)ji} \right\}_d,
\]

where \( \{.\}_d \) denotes the \( d \)th element of a vector, \( d = 1, \ldots, p-q \).

According to Borisov and Volodko (2009), based on Condition 5:

Define \( U_{n,d} \overset{\text{def}}{=} \frac{1}{n(n-1)} \sum_{1 \leq i < j \leq n} h_d(X_i, a_j^*, b_j^*, Y_i, X_j, a_i^*, b_i^*, Y_j) \) as the \( U \)-statistics for (34).

We have, with sufficient large \( c_{m2} \) in Condition 7.

\[
P\{|U_{n,d} - EU_{n,d}| > \varepsilon\} \leq c_{m3} \exp(c_{m5} \varepsilon/(c_{m3} + c_{m4} \varepsilon^{1/2} n^{-1/2}))
\]

where \( c_{m3}, c_{m4}, c_{m5} \) are constants. Moreover, let \( \varepsilon = O(n^{1/2+\alpha}) \), as \( \alpha < 1/2 \), we can further have,

\[
P\{|U_{n,d} - EU_{n,d}| > \varepsilon\} \leq c_{m3} \exp(-c_{m6} \varepsilon/2),
\]

Define

\[
F_{n,d} \overset{\text{def}}{=} (n)^{-1} \sum_{i=1}^n \sum_{j=1}^n b_j \psi_r(Y_t - a_j^* - b_j^* X_{ij}^T \beta^*) \omega_{ij} X_{(0)ij},
\]

also it is not hard to derive that \( U_{n,d} = F_{n,d} n/(n-1) \).

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It then follows that
\[
P(\{|F_{n,d} - EF_{n,d}| > \varepsilon\}) = P(\{|U_{n,d} - EU_{n,d}|(n - 1)/n > \varepsilon\}) \\
\leq 2 \exp(-Cn^{a+1/2})
\]
Define \(A_n = \{\|F_n - EF_n\|_\infty \leq \varepsilon\}\), thus
\[
P(A_n) \geq 1 - \sum_{d=1}^{p-q} P(|F_{n,d} - EF_{n,d}| > \varepsilon) \geq 1 - 2(p - q) \exp(-Cn^{a+1/2}).
\]
Finally we get that on the set \(A_n\),
\[
\|z(\hat{\beta}^0)\|_\infty \leq \|d_{M_d}^{-1} \circ F_n\|_\infty + \|d_{M_d}^{-1} \circ \sum_{i=1}^n \sum_{j=1}^n b_j [\psi_\tau(Y_i - a_j - b_j^* X_{ij}^T \beta^0)] - \psi_\tau(Y_i - a_j^* - b_j^* X_{ij}^T \beta^*)\|_{\infty} \omega_{ij} X_{(0)ij}\|_\infty \\
\leq O(n^{1/2+\alpha}/\lambda) + \|d_{M_d}^{-1} \circ \sum_{i=1}^n \sum_{j=1}^n \partial \mathbb{E} \psi_\tau(v_{ij}) b_j X_{(1)ij}^T (\hat{\beta}(1) - \beta(1)^* \omega_{ij} X_{(0)ij}\|_\infty),
\]
where \(v_{ij}\) is between \(Y_i - a_j^* - b_j^* X_{ij}^T \beta^*\) and \(Y_i - a_j^* - b_j^* X_{ij}^T \hat{\beta}^0\). From Lemma 1,
\[
\|\hat{\beta}^0 - \beta(1)^*\|_2 = O_p(\|d_{(1)}\| + \sqrt{q}/\sqrt{n}).
\]
Choosing \(\|\sum_i \sum_j X_{(0)ij} \omega_{ij} X_{(1)ij}^T \partial \mathbb{E} \psi_\tau(v_{ij})\|_\infty = O_p(n^{1-\alpha_1}), q = O(n^{\alpha_2}), \lambda = O(\sqrt{q}/n) = n^{-1/2+\alpha_2/2}, 0 < \alpha_2 < 1, \|d_{(1)}\| = O(\sqrt{q}D_n) = O(n^{\alpha_2/2}D_n)\)
\[
n^{-1}\|z(\hat{\beta}^0)\|_\infty = O\{n^{-1/2+\alpha} + n^{-1-\alpha_1} \sqrt{q}/\sqrt{n} + \|d_{(1)}\| n^{1-\alpha_1}\} \\
= O(n^{-\alpha_2/2+\alpha} + n^{-\alpha_1} + n^{-\alpha_1+\alpha_2/2} D_n/\lambda),
\]
conditions 4 ensures \(D_n = o(n^{\alpha_1-\alpha_2/2}/\lambda)\), and let \(0 < \delta < \alpha < \alpha_2/2 < 1/2, \alpha_2/2 < \alpha_1 < 1\), with rate \(p = O\{\exp(n^\delta)\},\) then \((n)^{-1}\|z(\hat{\beta}^0)\|_\infty = o_p(1).\)

**Proof of Theorem 1.** The results follows from Lemma 1 and 2.

**Proof of Theorem 2.** By Theorem 1, \(\hat{\beta}_{(1)} = \beta(1)^*\) almost surely. It then follows from Lemma 2 that
\[
\|\hat{\beta}_{(1)} - \beta(1)^*\| = O_p\{(D_n + n^{-1/2})\sqrt{q}\}.
\]
This completes the first part of the theorem. The other part of proof follows largely from Fan et al. (2013).
## 7. Appendix B: Tables and Figures

<table>
<thead>
<tr>
<th>Depositories (25)</th>
<th>Insurances (25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WFC Wells Fargo &amp; Company</td>
<td>AIG American International Group, Inc.</td>
</tr>
<tr>
<td>JPM J P Morgan Chase &amp; Co</td>
<td>MET MetLife, Inc.</td>
</tr>
<tr>
<td>BAC Bank of America Corporation</td>
<td>TRV The Travelers Companies, Inc.</td>
</tr>
<tr>
<td>C Citigroup Inc.</td>
<td>AFL Aflac Incorporated</td>
</tr>
<tr>
<td>USB U.S. Bancorp</td>
<td>PRU Prudential Financial, Inc.</td>
</tr>
<tr>
<td>COF Capital One Financial Corporation</td>
<td>CB Chubb Corporation (The)</td>
</tr>
<tr>
<td>PNC PNC Financial Services Group, Inc. (The)</td>
<td>MMC Marsh &amp; McLennan Companies, Inc.</td>
</tr>
<tr>
<td>BK Bank Of New York Mellon Corporation (The)</td>
<td>ALL Allstate Corporation (The)</td>
</tr>
<tr>
<td>STT State Street Corporation</td>
<td>AON Aon plc</td>
</tr>
<tr>
<td>BBT BB&amp;T Corporation</td>
<td>L Loews Corporation</td>
</tr>
<tr>
<td>STI SunTrust Banks, Inc.</td>
<td>PGR Progressive Corporation (The)</td>
</tr>
<tr>
<td>FITB Fifth Third Bancorp</td>
<td>HIG Hartford Financial Services Group, Inc. (The)</td>
</tr>
<tr>
<td>MTB M&amp;T Bank Corporation</td>
<td>PFG Principal Financial Group Inc</td>
</tr>
<tr>
<td>NTRS Northern Trust Corporation</td>
<td>CNA CNA Financial Corporation</td>
</tr>
<tr>
<td>RF Regions Financial Corporation</td>
<td>LNC Lincoln National Corporation</td>
</tr>
<tr>
<td>KEY KeyCorp</td>
<td>CINF Cincinnati Financial Corporation</td>
</tr>
<tr>
<td>CMA Comerica Incorporated</td>
<td>Y Alleghany Corporation</td>
</tr>
<tr>
<td>HBAN Huntington Bancshares Incorporated</td>
<td>UNM Unum Group</td>
</tr>
<tr>
<td>HCBK Hudson City Bancorp, Inc.</td>
<td>WRB W.R. Berkley Corporation</td>
</tr>
<tr>
<td>PBCT People’s United Financial, Inc.</td>
<td>FNF Fidelity National Financial, Inc.</td>
</tr>
<tr>
<td>BOKF BOK Financial Corporation</td>
<td>TMK Torchmark Corporation</td>
</tr>
<tr>
<td>ZION Zions Bancorporation</td>
<td>MKL Markel Corporation</td>
</tr>
<tr>
<td>CFR Cullen/Frost Bankers, Inc.</td>
<td>AJG Arthur J. Gallagher &amp; Co.</td>
</tr>
<tr>
<td>CBSH Commerce Bancshares, Inc.</td>
<td>BRO Brown &amp; Brown, Inc.</td>
</tr>
<tr>
<td>SBNY Signature Bank</td>
<td>HCC HCC Insurance Holdings, Inc.</td>
</tr>
<tr>
<td>Broker-Dealers (25)</td>
<td>others (25)</td>
</tr>
<tr>
<td>-----------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>GS Goldman Sachs Group, Inc. (The)</td>
<td>AXP American Express Company</td>
</tr>
<tr>
<td>BLK BlackRock, Inc.</td>
<td>BEN Franklin Resources, Inc.</td>
</tr>
<tr>
<td>MS Morgan Stanley</td>
<td>CBG CBRE Group, Inc.</td>
</tr>
<tr>
<td>CME CME Group Inc.</td>
<td>IVZ Invesco Plc</td>
</tr>
<tr>
<td>SCHW The Charles Schwab Corporation</td>
<td>JLL Jones Lang LaSalle Incorporated</td>
</tr>
<tr>
<td>TROW T. Rowe Price Group, Inc.</td>
<td>AMG Affiliated Managers Group, Inc.</td>
</tr>
<tr>
<td>AMTD TD Ameritrade Holding Corporation</td>
<td>OCN Ocwen Financial Corporation</td>
</tr>
<tr>
<td>RJF Raymond James Financial, Inc.</td>
<td>EV Eaton Vance Corporation</td>
</tr>
<tr>
<td>SEIC SEI Investments Company</td>
<td>LM Legg Mason, Inc.</td>
</tr>
<tr>
<td>NDAQ The NASDAQ OMX Group, Inc.</td>
<td>CACC Credit Acceptance Corporation</td>
</tr>
<tr>
<td>WDR Waddell &amp; Reed Financial, Inc.</td>
<td>FII Federated Investors, Inc.</td>
</tr>
<tr>
<td>SF Stifel Financial Corporation</td>
<td>AB Alliance Capital Management Holding L.P.</td>
</tr>
<tr>
<td>GBL Gamco Investors, Inc.</td>
<td>PRAA Portfolio Recovery Associates, Inc.</td>
</tr>
<tr>
<td>MKTX MarketAxess Holdings, Inc.</td>
<td>JNS Janus Capital Group, Inc.</td>
</tr>
<tr>
<td>EEFT Euronet Worldwide, Inc.</td>
<td>NNI Nelnet, Inc.</td>
</tr>
<tr>
<td>WETF WisdomTree Investments, Inc.</td>
<td>WRLD World Acceptance Corporation</td>
</tr>
<tr>
<td>DLLR DFC Global Corp</td>
<td>ECPG Encore Capital Group Inc</td>
</tr>
<tr>
<td>BGCP BGC Partners, Inc.</td>
<td>NEWS NewStar Financial, Inc.</td>
</tr>
<tr>
<td>PJC Piper Jaffray Companies</td>
<td>AGM Federal Agricultural Mortgage Corporation</td>
</tr>
<tr>
<td>ITG Investment Technology Group, Inc.</td>
<td>WHG Westwood Holdings Group Inc</td>
</tr>
<tr>
<td>INTL INTL FCStone Inc.</td>
<td>AVHI AV Homes, Inc.</td>
</tr>
<tr>
<td>GFIG GFI Group Inc.</td>
<td>SFE Safeguard Scientifics, Inc.</td>
</tr>
<tr>
<td>LTS Ladenburg Thalmann Financial Services Inc.</td>
<td>ATAX America First Tax Exempt Investors, L.P.</td>
</tr>
<tr>
<td>OPY Oppenheimer Holdings, Inc.</td>
<td>TAXI Medallion Financial Corp.</td>
</tr>
<tr>
<td>CLMS Calamos Asset Management, Inc.</td>
<td>NICK Nicholas Financial, Inc.</td>
</tr>
</tbody>
</table>

**Table 2:** Financial companies with tickers classified by industry: depositories (25), insurance (25), broker-dealers (25) and others (25).
Figure 1: log return of J P Morgan (points) and estimated VaR for J P Morgan (solid line), $\tau = 0.05$, window size $n = 48$, $T = 266$. The black points stand for the log return of J P Morgan, the blue line is the estimated VaR.

Figure 2: log return of J P Morgan (points) and estimated $\hat{\text{CoVaR}}^{\text{TENET}}$ for J P Morgan (solid line), $\tau = 0.05$, window size $n = 48$, $T = 266$. The black points stand for the log return of J P Morgan, the blue line is the estimated CoVaR.
Figure 3: Total connectedness (solid line) and averaged $\lambda$ of 100 financial institutions (dashes line) from 20071207 to 20130105, both of them are standardized on $[0, 1]$ scale. We developed a Financial Risk Meter based on averaged $\lambda$ series, more details can be found on CRC 649 webpage: http://sfb649.wiwi.hu-berlin.de/frm/index.html.

Figure 4: Incoming links for four industry groups. Depositories: solid line, Insurances: dashed line, Broker-Dealers: dotted line, Others: dash-dot line. $\tau = 0.05$, window size $n = 48$, $T = 266$. 
Figure 5: Outgoing links for four industry groups. Depositories: solid line, Insurances: dashed line, Broker-Dealers: dotted line, Others: dash-dot line. $\tau = 0.05$, window size $n = 48$, $T = 266$.

Figure 6: A network formed by 100 financial institutions, a circular representation of a weighted adjacency matrix after the thresholding (the values smaller than average of the partial derivatives are set to 0), Depositories: clockwise 25 firms from WFC to SBNY (red), Insurance: clockwise 25 firms from AIG to HCC (blue), Broker-Dealers: clockwise 25 firms from GS to CLMS (green), Others: clockwise 25 firms from AXP to NICK (violet), date: 20090612, $\tau = 0.05$, window size $n = 48$. 

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<table>
<thead>
<tr>
<th>Ranking of Sum</th>
<th>From Ticker</th>
<th>To Ticker</th>
<th>Total Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NEWS</td>
<td>OPY</td>
<td>33.06</td>
</tr>
<tr>
<td>2</td>
<td>LNC</td>
<td>CBG</td>
<td>32.74</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>MS</td>
<td>28.26</td>
</tr>
<tr>
<td>4</td>
<td>RF</td>
<td>STI</td>
<td>23.72</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
<td>BAC</td>
<td>22.99</td>
</tr>
<tr>
<td>6</td>
<td>LNC</td>
<td>SFE</td>
<td>17.61</td>
</tr>
<tr>
<td>7</td>
<td>MS</td>
<td>LM</td>
<td>16.82</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>OPY</td>
<td>16.36</td>
</tr>
<tr>
<td>9</td>
<td>CBG</td>
<td>JLL</td>
<td>15.54</td>
</tr>
<tr>
<td>10</td>
<td>LNC</td>
<td>CLMS</td>
<td>15.34</td>
</tr>
</tbody>
</table>

Table 3: Top 10 directional connectedness from one financial institution to another. The ranking is calculated by the sum of absolute value of the partial derivatives.

<table>
<thead>
<tr>
<th>Ranking of IN-link</th>
<th>Ticker</th>
<th>Total IN Sum</th>
<th>Ranking of MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OPY</td>
<td>68.63</td>
<td>98</td>
</tr>
<tr>
<td>2</td>
<td>IVZ</td>
<td>67.54</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>SFE</td>
<td>65.38</td>
<td>93</td>
</tr>
<tr>
<td>4</td>
<td>FITB</td>
<td>64.64</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>KEY</td>
<td>64.01</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>JPM</td>
<td>54.81</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>WFC</td>
<td>50.31</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>ZION</td>
<td>48.95</td>
<td>63</td>
</tr>
<tr>
<td>9</td>
<td>COF</td>
<td>48.36</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>STI</td>
<td>47.41</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 4: Top 10 financial institutions ranked according to Incoming links calculated by the sum of absolute value of the partial derivatives, and the Ranking of market capitalization (MC) in this 100 financial institutions’ list is also shown in this table.
<table>
<thead>
<tr>
<th>Ranking of OUT-link</th>
<th>Ticker</th>
<th>Total Out Sum</th>
<th>Ranking of MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LNC</td>
<td>260.72</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>174.46</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>LTS</td>
<td>164.48</td>
<td>97</td>
</tr>
<tr>
<td>4</td>
<td>MS</td>
<td>163.91</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>CBG</td>
<td>121.48</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>AGM</td>
<td>114.38</td>
<td>89</td>
</tr>
<tr>
<td>7</td>
<td>FITB</td>
<td>97.21</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>RF</td>
<td>84.65</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>ZION</td>
<td>84.52</td>
<td>63</td>
</tr>
<tr>
<td>10</td>
<td>NNI</td>
<td>80.87</td>
<td>77</td>
</tr>
</tbody>
</table>

**Table 5:** Top 10 financial institutions ranked according to Outgoing links calculated by the sum of absolute value of the partial derivatives, and the Ranking of market capitalization (MC) in this 100 financial institutions’ list is also shown in this table.

<table>
<thead>
<tr>
<th>Ranking of SRC</th>
<th>Ticker</th>
<th>Averaged Sum</th>
<th>Ranking of MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JPM</td>
<td>0.27</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>BAC</td>
<td>0.26</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>WFC</td>
<td>0.23</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>0.19</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>PRU</td>
<td>0.17</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>L</td>
<td>0.16</td>
<td>28</td>
</tr>
<tr>
<td>7</td>
<td>GS</td>
<td>0.13</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>MET</td>
<td>0.12</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>MTB</td>
<td>0.11</td>
<td>33</td>
</tr>
<tr>
<td>10</td>
<td>AXP</td>
<td>0.10</td>
<td>5</td>
</tr>
</tbody>
</table>

**Table 6:** Top 10 financial institutions ranked according to the systemic risk contribution (SRC) calculated by the averaged sum of partial derivatives, and the Ranking of market capitalization (MC) in this 100 financial institutions’ list is also shown in this table.
Figure 7: The $\hat{\text{CoVaR}} _{\text{SYSTEM}}$ of J P Morgan (thicker blue line), its estimated partial derivatives (thinner red line) and the log returns of the system (black points). $\tau = 0.05$, window size $n = 48$, $T = 266$.

Figure 8: The log return of J P Morgan (points) and estimated $\hat{\text{CoVaR}} _{\text{Step2}}^L$ for J P Morgan (solid line), $\tau = 0.05$, window size $n = 48$, $T = 266$. 

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<table>
<thead>
<tr>
<th>Model</th>
<th>$\hat{\text{CoVaR}}_{\text{TENET}}$</th>
<th>$\hat{\text{CoVaR}}_{\text{LStep2}}$</th>
<th>$\hat{\text{CoVaR}}_{\text{SYSTEM}}$</th>
<th>$\hat{\text{CoVaR}}_{\text{LStep3}}$</th>
<th>$\hat{\text{CoVaR}}_{\text{AB}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Violation</td>
<td>4.73(2.33)</td>
<td>41.03(29.84)</td>
<td>2.25(1.80)</td>
<td>19.68(5.96)</td>
<td>28.63(8.64)</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.55(0.33)</td>
<td>0.02(0.03)</td>
<td>0.52(0.31)</td>
<td>0.02(0.30)</td>
<td>0.24(0.26)</td>
</tr>
</tbody>
</table>

Table 7: The averaged number of violation and the averaged $p$-value of CaViaR test in $\hat{\text{CoVaR}}_{\text{TENET}}$, $\hat{\text{CoVaR}}_{\text{LStep2}}$, $\hat{\text{CoVaR}}_{\text{SYSTEM}}$, $\hat{\text{CoVaR}}_{\text{LStep3}}$, and $\hat{\text{CoVaR}}_{\text{AB}}$, the standard deviations are given in the brackets.

Figure 9: Left: the estimated link function ($\hat{\text{CoVaR}}_{\text{TENET}}$ of J P Morgan) (solid line) with $h = 0.43$, and estimated the index (points), time period: 20081003-20090828. Right: the estimated link function ($\hat{\text{CoVaR}}_{\text{TENET}}$ of J P Morgan) (solid line) with $h = 0.52$, and estimated the index (points), time period: 20100604-20110506. $\tau = 0.05$, window size $n = 48$, 95% confidence bands (dashed lines).

Figure 10: Left: the estimated link function ($\hat{\text{CoVaR}}_{\text{SYSTEM}}$ of J P Morgan) (solid line) with $h = 0.19$, and estimated the index (points), time period: 20081003-20090828. Right: the estimated link function ($\hat{\text{CoVaR}}_{\text{SYSTEM}}$ of J P Morgan) (solid line) with $h = 0.17$, and estimated the index (points), time period: 20100604-20110506. $\tau = 0.05$, window size $n = 48$, 95% confidence bands (dashed lines).
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